Mathematics 2201
Common Mathematics Assessment

June 12, 2013

Name: _____

Mathematics Teacher: _____

28 Selected Response	28 marks
13 Constructed Response	42 marks
	_____
FINAL	**70 Marks**

TIME: 2 HOURS

NOTE

Diagrams are not necessarily drawn to scale.

FORMULAE

$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$	$a^2 = b^2 + c^2 - 2bccosA$	$cosA = \frac{b^2 + c^2 - a^2}{2bc}$

$\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$	$z = \frac{x - \mu}{\sigma}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Selected Response: Choose the appropriate response on the answer sheet or SCANTRON.

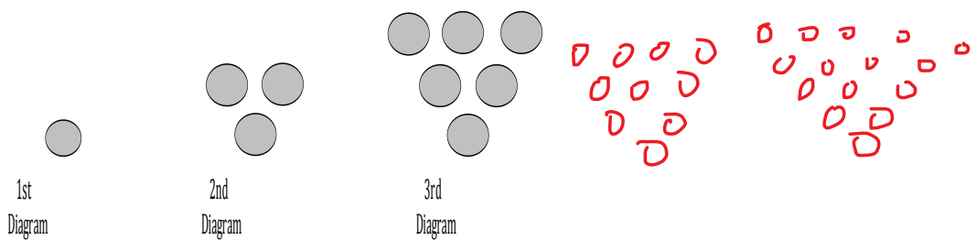
1. What is a statement that is believed to be true but not yet proven?

   (A) Conjecture
   (B) Counterexample
   (C) Deductive Reasoning
   (D) Inductive Reasoning

2. Which is a counterexample to the statement "The sum of two consecutive integers is always greater than each of the two integers"?

   (A) $-4 + (-5) = -9$
   (B) $4 + (-5) = -1$
   (C) $-4 + 5 = 1$
   (D) $4 + 5 = 9$

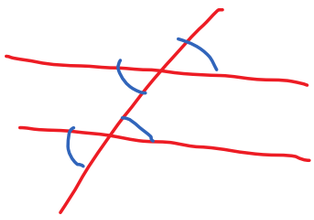
3. How many circles are in the **5th diagram** in the sequence below:



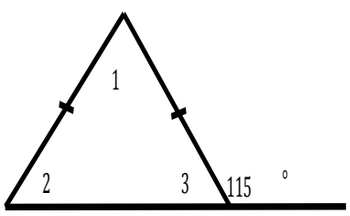
   (A) 9
   (B) 10
   (C) 14
   (D) 15

4. If two non-parallel lines are cut by a transversal, which pair of angles is always equal?

   (A) Alternate Interior
   (B) Corresponding
   (C) Supplementary
   (D) Vertically Opposite



5. A student was asked to find the measure of $\angle 1$. In which step did he make the first error?



Solution

Step 1: $\angle 3 = 180° - 115°$
Step 2: $\angle 3 = 65°$
Step 3: ~~$\angle 1 = \angle 3$~~ $\angle 2 = \angle 3$
Step 4: $\angle 1 = 65°$

   (A) 1
   (B) 2
   (C) 3
   (D) 4

6. How many sides does a convex polygon have if the sum of its interior angles is 1440° ?

$S = 180(n-2)$

$\frac{1440}{180} = \frac{180(n-2)}{180}$

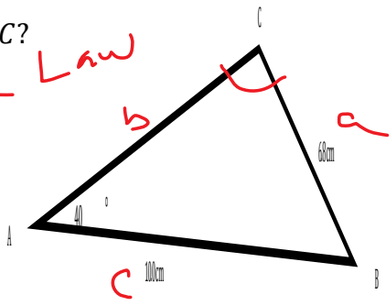
$8 = n-2$

$8+2 = n$

$n = 10$

(A) 4
(B) 6
(C) 8
(D) 10

7. What is the measure of $\angle C$?



Sine Law

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{10} = \frac{\sin 40^\circ}{6.8}$

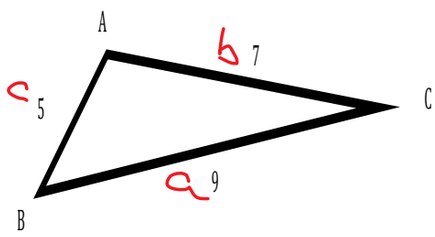
$\frac{\sin C}{10} = 0.0945$

$\sin C = 0.9453$

$C = \sin^{-1}(0.9453) = 71^\circ$

(A) 20°
(B) 26°
(C) 69°
(D) 71°

8. Which equals the measure of $\angle A$?



$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos A = \frac{7^2+5^2-9^2}{2(7)(5)}$

(A) $\cos^{-1}\left(\frac{5^2+9^2-7^2}{2(5)(9)}\right)$

(B) $\cos^{-1}\left(\frac{7^2+5^2-9^2}{2(7)(5)}\right)$

(C) $\cos^{-1}\left(\frac{9^2+5^2-7^2}{2(9)(5)}\right)$

(D) $\cos^{-1}\left(\frac{9^2+7^2-5^2}{2(9)(7)}\right)$

9. Simplify completely: $5\sqrt{7} + 3\sqrt{28}$

$= 5\sqrt{7} + 3\sqrt{4}\sqrt{7}$

$= 5\sqrt{7} + 3\cdot 2\sqrt{7}$

$= 5\sqrt{7} + 6\sqrt{7}$

$= 11\sqrt{7}$

(A) $11\sqrt{7}$
(B) $17\sqrt{7}$
(C) $11\sqrt{14}$
(D) $8\sqrt{35}$

10. Simplify completely: $\sqrt[3]{-8x^{17}}$

$\sqrt[3]{-8}\ \sqrt[3]{x^{17}}$

$= -2\ \sqrt[3]{x^3 \cdot x^3 \cdot x^3 x^3 x^3 \sqrt[3]{x^2}}$

$= -2x^5\sqrt[3]{x^2}$

(A) $-2x^2\sqrt[3]{x^5}$
(B) $-2x^5\sqrt[3]{x^2}$
(C) $2x\sqrt[3]{-2x^8}$
(D) $2x^8\sqrt[3]{-2x}$

11. Write $3x^3\sqrt{5x}$ as an entire radical.

(A) $\sqrt{15x^4}$
(B) $\sqrt{15x^7}$
(C) $\sqrt{45x^4}$
(D) $\sqrt{45x^7}$

$= \sqrt{(3x^3)^2 \cdot 5x}$
$= \sqrt{9x^6 \cdot 5x}$
$= \sqrt{45x^7}$

12. A student was asked to simplify $\frac{x\sqrt{18x^3}}{3}$ but did not complete a correct solution. Which step contains her first error?

Solution:

Step 1: $\frac{x\sqrt{9 \cdot 2 \cdot x^2 \cdot x}}{3}$

Step 2: $\frac{x \cdot 9x^2\sqrt{2x}}{3}$

Step 3: $\frac{9x^3\sqrt{2x}}{3}$

Step 4: $3x^3\sqrt{2x}$

(A) 1
(B) 2
(C) 3
(D) 4

13. Simplify completely: $\frac{5\sqrt{32}}{2\sqrt{3}}$

(A) $\frac{10\sqrt{6}}{3}$
(B) $\frac{40\sqrt{6}}{3}$
(C) $\frac{5\sqrt{96}}{6}$
(D) $\frac{10\sqrt{96}}{12}$

$= \frac{5\sqrt{16}\sqrt{2}}{2\sqrt{3}}$
$= \frac{5 \cdot 4\sqrt{2}}{2\sqrt{3}}$
$= \frac{20\sqrt{3} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}}$
$= \frac{20\sqrt{6}}{2 \cdot 3}$
$= \frac{20\sqrt{6}}{6}$
$= \frac{10\sqrt{6}}{3}$

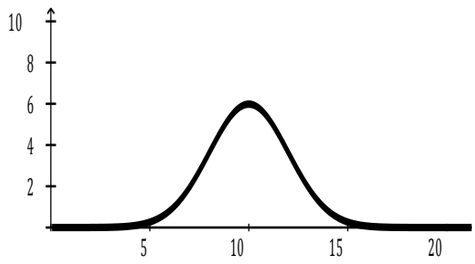
14. What are the restrictions on the variable for $\sqrt{x+2}$?

(A) $x \geq -2$
(B) $x > -2$
(C) $x \geq 2$
(D) $x > 2$

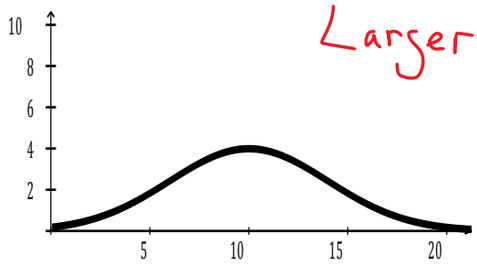
$x + 2 \geq 0$
$x \geq -2$

15. Which represents data with the largest standard deviation?

(A)

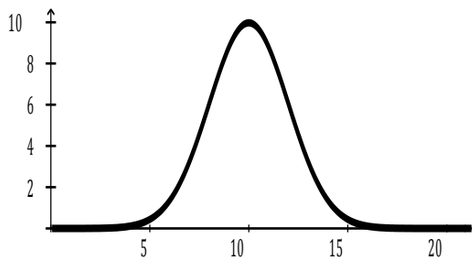


(B)

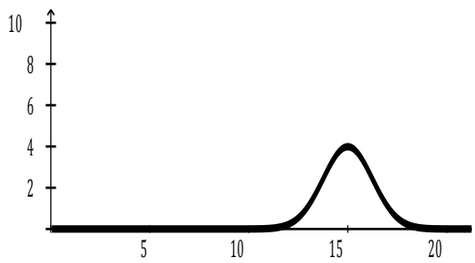


Larger σ = more spread out

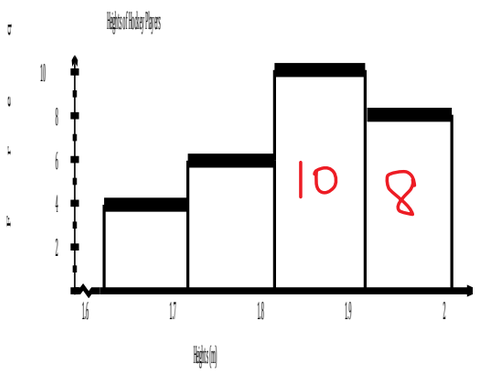
(C)



(D)



16. The histogram shown represents the heights of hockey players on a professional hockey team. How many players have a height between 1.8 m and 2.0 m?

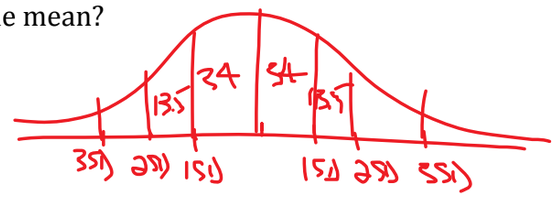


10 + 8 = 18

(A) 10
(B) 18
(C) 24
(D) 28

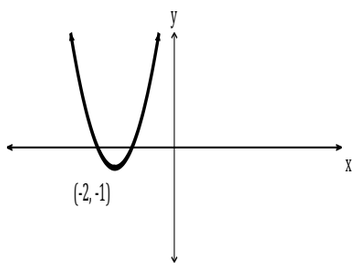
17. A set of data is normally distributed. What percent of the data is within two standard deviations of the mean?

(A) 47.5
(B) 68
(C) 95
(D) 99.7

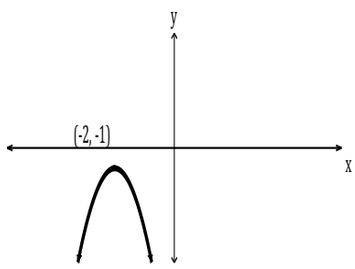


18. The function $y = -3x^2 - 12x - 13$ has axis of symmetry $x = -2$. Which represents the function?

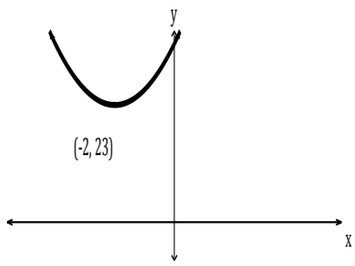
(A)



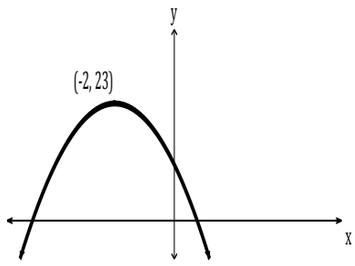
(B)



(C)



(D)

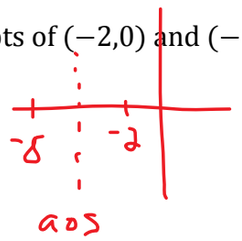


19. What is the domain and range for $f(x) = -2(x + 1)^2 - 3$?

(A) $x \in \mathbb{R}$ and $f(x) \leq -3$
(B) $x \in \mathbb{R}$ and $f(x) \geq -3$
(C) $x \leq -1$ and $f(x) \in \mathbb{R}$
(D) $x \geq -1$ and $f(x) \in \mathbb{R}$

20. A parabola has x-intercepts of $(-2,0)$ and $(-8,0)$. What is the axis of symmetry?

(A) $x = -5$
(B) $x = -3$
(C) $y = -5$
(D) $y = -3$



$\frac{-8+-2}{2} = \frac{-10}{2} = -5$

$x = -5$

21. What is the vertex of $y = 2x^2 + 8x - 5$?

(A) $(-2,-29)$
(B) $(-2,-13)$
(C) $(2,15)$
(D) $(2,19)$

$h = \frac{-b}{2a}$

$h = \frac{-8}{2(2)} = -2$

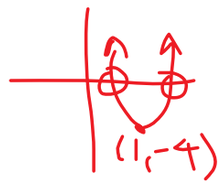
$k = 2(-2)^2 + 8(-2) - 5$

$k = -13$

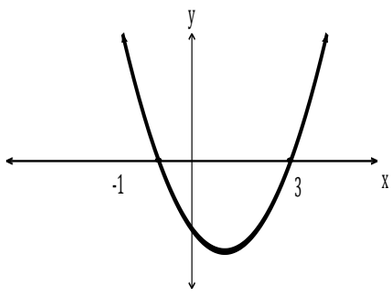
$(-2,-13)$

22. The graph of a quadratic function has vertex $(1,-4)$ and opens upward. How many x-intercepts does it have?

(A) 0
(B) 1
(C) 2
(D) 3



23. What is the equation of the function graphed below?



$x = -1, x = 3$

$x+1=0, x-3=0$

$(x+1)(x-3)$

(A) $y = (x-1)(x-3)$
(B) $y = (x-1)(x+3)$
(C) $y = (x+1)(x-3)$
(D) $y = (x+1)(x+3)$

24. Which is a root of $2x^2 - 5x - 3 = 0$

(A) -3
(B) -1
(C) 1
(D) 3

$2x^2 - 6x + x - 3$

$2x(x-3) + (x-3)$

$(x-3)(2x+1)$

$x = 3, x = -\frac{1}{2}$

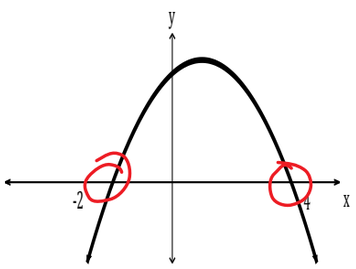
6
$1, 6$
$2, 3$

or sub each answer into the equation and see which equals 0.

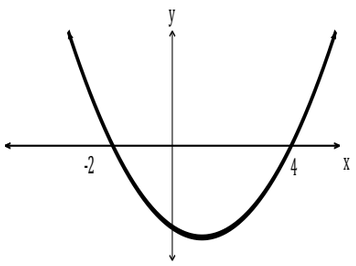
$2(3)^2 - 5(3) - 3 = 0$

25. Which represents a quadratic function with zeros of -2 and 4 and a maximum value?

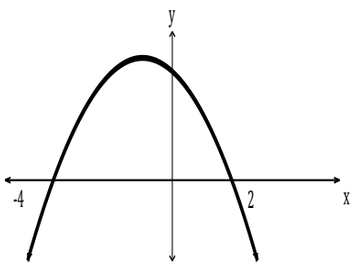
(A)



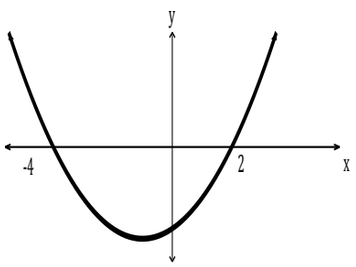
(B)



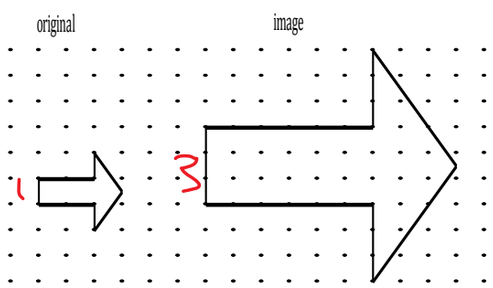
(C)



(D)



26. What is the scale factor in the figure below?



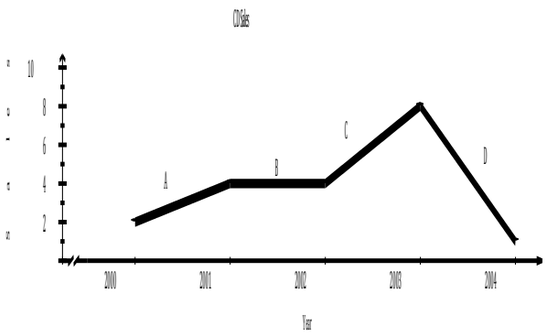
(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 2

(D) 3

27. During which time period was the growth rate of CD sales the greatest in the graph shown?



(A) 2000 – 2001
(B) 2001 – 2002
(C) 2002 – 2003
(D) 2003 – 2004

28. The surface area of a cone is $34\ ft^2$. If the cone is enlarged by a scale factor of 3, what is the surface area, in ft^2, of the image?

(A) 37
(B) 102
(C) 306
(D) 918

$3^2 = \frac{\text{scale}}{\text{original}}$

$3^2 = \frac{x}{34}$

$x = 9 \cdot 34$

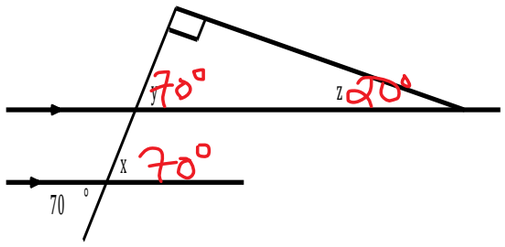
$x = 306$

Constructed Response:
Answers to be written on this paper in the space provided. Show all workings.

29. Use **both** inductive and deductive reasoning to show that the result for the given number trick will always be the original number. [4 marks]

NUMBER TRICK	Inductive Reasoning	Deductive Reasoning
Choose a number.	4	n
Double it.	8	$2n$
Add 6.	14	$2n+6$
Double it	28	$4n+12$
Subtract 4.	24	$4n+8$
Divide by 4.	6	$n+2$
Subtract 2.	4	n

30. Find the measure of each indicated angle. Justify your answer. [3 marks]

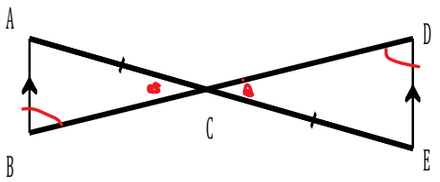


Angle Measure	Justification
$x =$ 70°	vertically opposite angles
$y =$ 70°	Corresponding angles
$z =$ 20°	Sum of angles in triangle equal 180°

31. Use either a paragraph or two-column format to complete the given proof: [3marks]

Given: AB ∥ DE
AC = EC

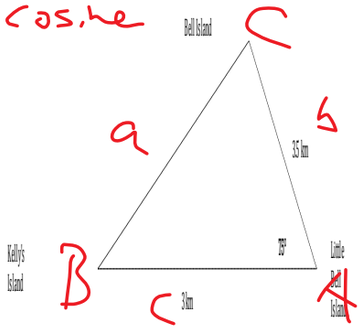
Prove: Δ A B C ≅ Δ E D C



Statement	Reason
AB ∥ DE	Given
AC = EC	Given
∠ABC = ∠EDC	alternate interior
∠ACB = ∠ECD	vertically opposite
ΔABC ≅ ΔEDC	AAS

32. A boat travels from Bell Island to Kelly's Island to Little Bell Island, and returns directly back to Bell Island. What is the total distance travelled? [4 marks]

SAS → cosine



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (3.5)^2 + (3)^2 - 2(3.5)(3)\cos 75^\circ$

$a^2 = 15.026$

$a = \sqrt{15.026}$

$a = 3.9\text{ km}$

Total $= (3 + 3.5 + 3.9)\text{km} = 10.4\text{km}$

33. Simplify completely: $5\sqrt{6}\left(\sqrt{3}+3\sqrt{12}-\sqrt{2}\right)$ [3 marks]

$= 5\sqrt{6}(\sqrt{3}+6\sqrt{3}-\sqrt{2})$
$= 5\sqrt{6}(7\sqrt{3}-\sqrt{2})$
$= 35\sqrt{18}-5\sqrt{12}$
$= 35\sqrt{9}\sqrt{2}-5\sqrt{4}\sqrt{3}$
$= 35\cdot 3\sqrt{2}-5\cdot 2\sqrt{3}$
$= 105\sqrt{2}-10\sqrt{3}$

$3\sqrt{12}$
$= 3\sqrt{4}\sqrt{3}$
$= 3\cdot 2\sqrt{3}$
$= 6\sqrt{3}$

34. State the **restrictions** on x, **solve** the equation, and then **check** for extraneous roots. [4 marks]

$\sqrt{3x+1}-3=-4$

$3x+1\geq 0$
$3x\geq -1$
$x\geq -\frac{1}{3}$

$\sqrt{3x+1}=-4+3$
$\sqrt{3x+1}=-1$
$(\sqrt{3x+1})^2=(-1)^2$
$3x+1=1$
$3x=1-1$
$3x=0$
$\rightarrow x=0$

Check:
$\sqrt{3(0)+1}-3=-4$
$\sqrt{1}-3=-4$
$1-3=-4$
$-2\neq -4$
$\therefore$ No solution

35. A factory produces automotive brake pads with a mean mass of 174 g and a standard deviation of 0.7 g. Quality control expects that the mass of the pads will lie within the acceptable range of 173.9 g and 174.1 g. What is the confidence interval and margin of error this factory uses for its quality control tests? [2 marks]

Confidence Interval: 173.9 – 174.1

$173.9-174=-0.1$
$174.1-174=0.1$

Margin of error: ± 0.1

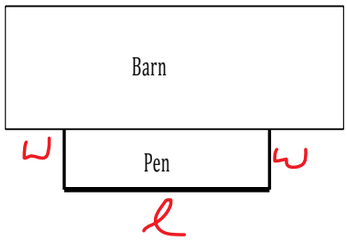
36. Jason scored 82% on a test where the class average was 74% and the standard deviation was 10.6%. If the class was normally distributed, what percentage of the class scored better than Jason? [3 marks]

$z\text{-score} = \frac{x-\mu}{\sigma} = \frac{82-74}{10.6} = 0.75$

from table: 0.7734 or 77.34%

$100\% - 77.34\% = 22.66\%$ scored better than Jason

37. A farmer has 300 m of chain link fencing to create a rectangular pen, using the side of a barn as one side of the pen. Algebraically determine the maximum area that can be enclosed by the pen. [4 marks]



① $2w + l = 300$

② $A = l \cdot w$

$l = -2w + 300$

$A = (-2w + 300)w$

$A = -2w^2 + 300w$

$w_v = \frac{-b}{2a} = \frac{-300}{2(-2)} = 75_m$

$l = -2(75) + 300 = 150_m$

Max area:

$(75m)(150m)$

$= 11250 m^2$

38. Algebraically determine the **vertex** and **x-intercepts** for the function $y = x^2 - 2x - 8$. Sketch the graph, labelling all key points. [3 marks]

$h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$k = (1)^2 - 2(1) - 8 = -9$

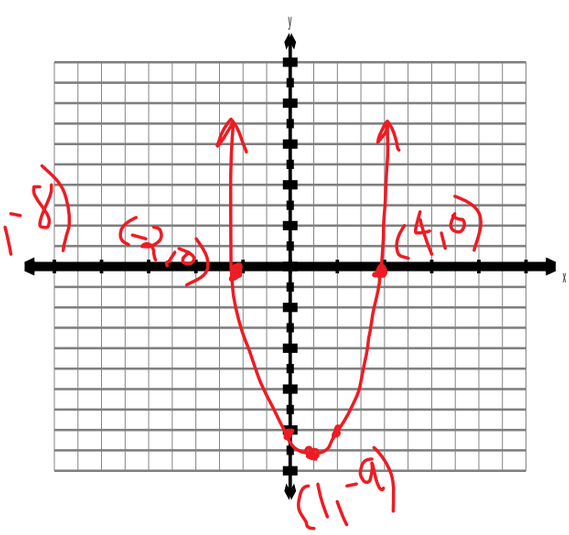
vertex $(1, -9)$

y-int: $(0, -8)$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4)$

$x = -2, x = 4$



39. Solve the given equation. State the solution(s) in **exact** form. [3 marks]

$6x^2 = -4x + 3$

$6x^2+4x-3=0$ (a, b, c)

$x=\frac{-4\pm\sqrt{4^2-4(6)(-3)}}{2(6)}$

$x=\frac{-4\pm\sqrt{16+72}}{12}$

$x=\frac{-4\pm\sqrt{88}}{12}$

$x=\frac{-4\pm\sqrt{4}\sqrt{22}}{12}$

$x=\frac{-4\pm2\sqrt{22}}{12}$

$x=\frac{-2\pm\sqrt{22}}{6}$

40. On another planet, the path of a rock that is thrown is given by $h = -t^2 + 4t + 6$, where h is height in metres and t is time in seconds. At what time(s) would the height of the rock be 9 m? [3 marks]

$9=-t^2+4t+6$

$t^2-4t-6+9=0$

$t^2-4t+3=0$

$(t-1)(t-3)=0$

$t=1, t=3$

The rock is 9m high at 1s and 3s.

41. Avalon Supermarket sells a box of 48 granola bars for $7.99 and a box of 8 bars for $1.99. What is the least expensive way to buy 70 granola bars? Justify your reasoning. [3 marks]

① 2 boxes of 48 = 2($7.99) = $15.98 for 96 bars

② 9 boxes of 8 = 9($1.99) = $17.91 for 72 bars

③ 1 box of 48 + 3 boxes of 8 = $7.99 + 3($1.99) = $13.96 for 72 bars

Option ③ is least expensive.

Mathematics 2201 Common Assessment – June 2013
Answer Sheet

Name: _____

Mathematics Teacher: _____

1.	A	B	C	D	15.	A	B	C	D
2.	A	B	C	D	16.	A	B	C	D
3.	A	B	C	D	17.	A	B	C	D
4.	A	B	C	D	18.	A	B	C	D
5.	A	B	C	D	19.	A	B	C	D
6.	A	B	C	D	20.	A	B	C	D
7.	A	B	C	D	21.	A	B	C	D
8.	A	B	C	D	22.	A	B	C	D
9.	A	B	C	D	23.	A	B	C	D
10.	A	B	C	D	24.	A	B	C	D
11.	A	B	C	D	25.	A	B	C	D
12.	A	B	C	D	26.	A	B	C	D
13.	A	B	C	D	27.	A	B	C	D
14.	A	B	C	D	28.	A	B	C	D