Answer Key

MATH 1201
Sample Midterm

Part 1
Value: 30 Points

Instructions: Write the correct response for each multiple choice question on the answer sheet provided.

1. Which is the best referent for 1 yard?
   (A) height of a door knob from the floor
   (B) length of our school gymnasium
   (C) length of a spoon
   (D) width of your index finger

2. How many 15 cm lengths of a pipe can be cut from a pipe 1.20 m long?
   (A) 6
   (B) 8
   (C) 10
   (D) 18

   $15\text{cm} \times \frac{1\text{m}}{100\text{cm}} = \frac{15}{100}\text{m} = 0.15\text{m}$

   $1.20\text{m} \div 0.15\text{m} = 8$

3. What is 127 centimetres expressed in feet and inches?
   (A) 4 ft 2 in
   (B) 4 ft 6 in
   (C) 4 ft 10 in
   (D) 5 ft 2 in

   $127\text{cm} \times \frac{1\text{ in.}}{2.54\text{cm}} = 50\text{in}$

   Check each answer
   (A) $4\text{ft} \times \frac{12\text{in}}{1\text{ft}} = 48\text{in}$ and $4\text{ft}\,2\text{in} = 48\text{in} + 2\text{in} = 50\text{in}$
   ∴ (A) is correct

4. The volume of a right cylinder is 735 in.^{3}. What is the volume of a right cone with the same base and height?
   (A) 245 in.^{3}
   (B) 367 in.^{3}
   (C) 735 in.^{3}
   (D) 2205 in.^{3}

   $V_{cone} = \frac{V_{cylinder}}{3} = \frac{735}{3} = 245\text{in}^3$

5. The surface area of a cone with a radius of 8 cm is 502.65 cm^2. What is the slant height?
   (A) 3 cm
   (B) 12 cm
   (C) 20 cm
   (D) 141 cm

   $SA = \pi r^2 + \pi r s$
   $502.65 = \pi(8)^2 + \pi(8)s$
   $502.65 = 201.06 + 25.13s$
   $502.65 - 201.06 = 25.13s$
   $\frac{301.59}{25.13} = \frac{25.13\,s}{25.13}$
   $12\text{cm} = s$

6. How much air is needed to fill a volleyball with a diameter of 14 inches?
   (A) 205 in.^{3}
   (B) 808 in.^{3}
   (C) 1437 in.^{3}
   (D) 11 494 in.^{3}

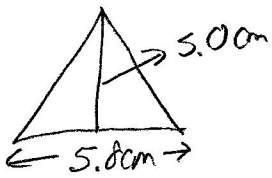
   $\rightarrow r = \frac{14\text{in}}{2} = 7\text{in.}$

   Find volume of a sphere
   $V = \frac{4\pi r^3}{3} = \frac{4\pi(7)^3}{3} = 1437\text{in}^3$

All sides are the same ∴ they have the same area

7. What is the surface area of the regular tetrahedron to the nearest square centimetre if $AB = 5.0$ cm and $CD = 5.8$ cm?

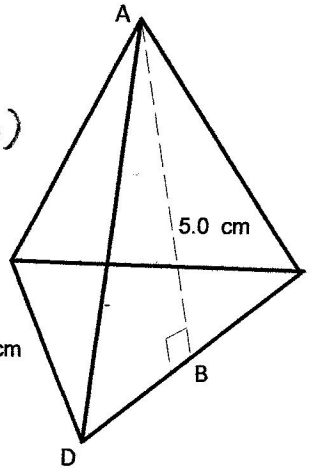
(A) 15
(B) 44
(C) 58
(D) 116



$A = \frac{bh}{2}$

$A = \frac{(5.8)(5.0)}{2}$

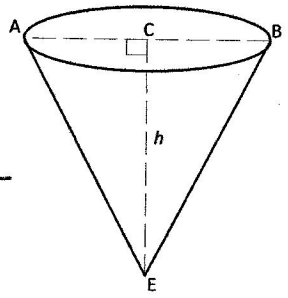
$A = 14.5 cm^2$



TSA = 4(A of one triangle)
= 4(14.5)
= 58 cm²

8. If diameter AB = 3.8 cm, and the height h is 4.66 cm, what is the volume of the right cone to the nearest tenth of a cubic centimetre?

(A) 4.39 cm³
(B) 11.8 cm³
(C) 17.6 cm³
(D) 33.5 cm³



$r = 3.8 cm \div 2 = 1.9 cm$

$h = 4.666 cm$

$V = \frac{\pi r^2 h}{3} = \frac{\pi (1.9)^2 (4.666)}{3}$

$V = 17.6 cm^3$

9. A sphere has a surface area of 10.4 m². What is the diameter of the sphere?

(A) 0.83
(B) 0.91
(C) 1.66
(D) 1.82

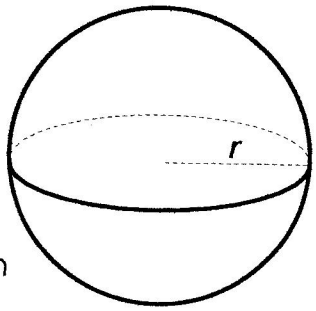
$SA = 4\pi r^2$

$10.4 = 4\pi r^2$

$\frac{10.4}{12.566} = \frac{12.566 r^2}{12.566}$

$0.8276 = r^2$

$\sqrt{0.8276} = r \rightarrow r = 0.91 m$



$d = 2 \times 0.91$
$= 1.82$

10. What is the GCF of 84 and 112?

(A) 2
(B) 7
(C) 28
(D) 336

84
4 21
2 2 3 7
$84 = 2^2 \times 3 \times 7$

112
4 28
2 2 4 7
2 2 2 7
$112 = 2^4 \times 7$

$GCF = 2^2 \times 7 = 28$
→ factors common to both
→ lowest exponents

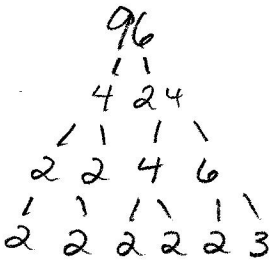
11. Which number is a perfect cube?

(A) 144
(B) 324
(C) 4096
(D) 6561

$\sqrt[3]{144} = 5.24...$ ✗
$\sqrt[3]{324} = 6.87...$ ✗
$\sqrt[3]{4096} = 16$ ✓

$96 = (2)(2)(2)\cdot 2\cdot 2\cdot 3$

$= 2^3 \cdot 12$



12. What is the simplified form of $\sqrt[3]{96}$?

(A) $2\sqrt[3]{6}$

(B) $2\sqrt[3]{12}$

(C) $4\sqrt[3]{24}$

(D) $16\sqrt[3]{6}$

$\sqrt[3]{96} = \sqrt[3]{2^3} \cdot \sqrt[3]{12}$

$= 2 \cdot \sqrt[3]{12}$

13. Which represents $(\sqrt[3]{2})^5$ written as a power? (3 denominator, 5 numerator)

(A) $2^{\frac{3}{5}}$

(B) $2^{\frac{5}{3}}$

(C) 2^2

(D) 2^{15}

$2^{\frac{5}{3}}$

14. Which represents $2\sqrt{5}$ written as an entire radical?

(A) $\sqrt{10}$

(B) $\sqrt{20}$

(C) $\sqrt{30}$

(D) $\sqrt{50}$

$= \sqrt{2^2} \cdot \sqrt{5}$

$= \sqrt{4} \cdot \sqrt{5}$

$= \sqrt{20}$

15. What is the index of $3\sqrt[2]{4^5}$?

(A) 2

(B) 3

(C) 4

(D) 5

16. Which is equivalent to $\sqrt[3]{5^2}$? (3 denominator, 2 numerator)

(A) $5^{\frac{2}{3}}$

(B) $5^{\frac{3}{2}}$

(C) $\frac{1}{5^{\frac{2}{3}}}$

(D) $\frac{1}{5^{\frac{3}{2}}}$

$5^{\frac{2}{3}}$

17. Which is equivalent to $\left(-\frac{1}{8}\right)^{-3}$? $= \left(-\frac{8}{1}\right)^{3}$ or $(-8)^3$

(A) $(-8)^3$

(B) $\left(-\frac{1}{8}\right)^{3}$

(C) $\left(\frac{1}{8}\right)^{3}$

(D) 8^3

18. Which is equivalent to $\left(\frac{2}{3}\right)^{4}\left(\frac{2}{3}\right)^{-2}$? $= \left(\frac{2}{3}\right)^{4+-2} = \left(\frac{2}{3}\right)^{2}$

(A) $\left(\frac{4}{9}\right)^{2}$

(B) $\left(\frac{2}{3}\right)^{2}$

(C) $\left(\frac{2}{3}\right)^{-8}$

(D) $\left(\frac{4}{9}\right)^{-8}$

19. Simplify: $(2x^2)^3(3x^{-3})^0$ $= 2^3(x^2)^3(1)$

(A) $8x^6$
(B) $2x^6$
(C) $8x^5$
(D) $2x^5$

$= 8x^6$

20. Simplify: $\frac{18x^3y^2}{6x^4y^1}$ $= 3x^{3-4}y^{2-1} = 3x^{-1}y = \frac{3y}{x}$

A) $\frac{3y}{x}$
B) $3xy$
C) $\frac{12y}{x}$
D) $12xy$

21. A student did not receive full marks for her solution to the question below. In which step did she make the **first** error?

Simplify: $\frac{(a^{-2}b^{7})^{-5}}{(a^{2}b^{-3})^{3}}$

$= \frac{a^{-2\times-5}\, b^{7\times-5}}{a^{2\times3}\, b^{-3\times3}}$

$= \frac{a^{10}b^{-35}}{a^{6}b^{-9}}$

Solution:

Step 1: $\frac{a^{-7}b^{6}}{a^{5}b^{0}}$ X

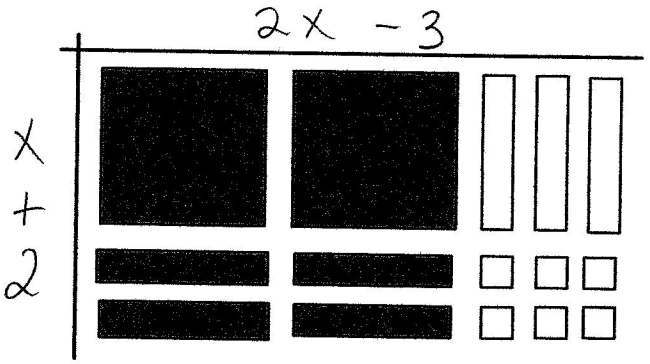
Step 2: $a^{-7-5}\, b^{2-0}$

Step 3: $a^{-12}b^{2}$

Step 4: $\frac{b^{2}}{-a^{12}}$

(A) 1
(B) 2
(C) 3
(D) 4

22. Which binomial product is modelled?



Note: □ = negative ■ = positive

(A) $(-2x+3)(-x+2)$
(B) $(-2x+3)(x+2)$
(C) $(2x-3)(x+2)$
(D) $(2x-3)(x-2)$

23. Expand and simplify: $(3x+1)(4-x)$

(A) $-3x^2+11x+4$
(B) $-3x^2+13x+4$
(C) $3x^2+11x-4$
(D) $3x^2+13x-4$

$= 12x - 3x^2 + 4 - x$] switch order

$= -3x^2 + 12x - x + 4$

$= -3x^2 + 11x + 4$

	x	-2
x	x^2	-2x
+5	5x	-10

24. Which model represents $(x + 5)(x - 2)$?

(A)

	x	-	2
x	2x		-2x
+			
5	5x		-10

(B)

	x	-	2
x	2x		-2x
+			
5	5x		3

(C)

	x	-	2
x	x^2		-2x
+			
5	5x		-10

(D)

	x	-	2
x	x^2		2x
+			
5	5x		10

25. What is the greatest common factor of $16x^2y^3$, $4x^3y^2$, and $-24x^3y^3$?

(A) $4x^2y^2$

(B) $4x^3y^3$

(C) $8x^2y^2$

(D) $8x^3y^3$

$4x^2y^2$

26. Which type of factoring must be used on the expression $x^2 - 5x + 6$?

(A) GCF

(B) Difference of Squares

(C) Quadratic

(D) GCF and Quadratic

27. Factor: $9x^2 - 25y^2 = (3x - 5y)(3x + 5y)$ Difference of Squares

(A) $(3x - 5y)(3x - 5y)$

(B) $(3x - 5y)(3x + 5y)$

(C) $(3x + 5y)(3x + 5y)$

(D) $(5y + 3x)(5y - 3x)$

28. Factor: $2a^2 + 11a + 12 = (a + \frac{3}{2})(a + \frac{8}{2})$

(A) $(2a + 3)(a + 4)$

(B) $(2a + 4)(a + 3)$

(C) $(2a + 6)(a + 4)$

(D) $(2a + 8)(a + 3)$

$(2a+3)(a+4)$

Prod. of a x c	Sum of 11
$(2)(12) = 24$	
$(8)(3) = 24$	$8 + 3 = 11$

29. Factor completely: $x^2 - 6x + 5$

(A) $(x-1)(x-5)$
(B) $(x-2)(x-3)$
(C) $(x-1)(x+5)$
(D) $(x+6)(x-1)$

prod $a \times c$	sum -6
$1 \times 5 = 5$	
$(-5)(-1) = 5$	$-5 + -1 = -6$

$x^2 - 6x + 5 = (x - \frac{5}{1})(x - \frac{1}{1})$
$= (x-5)(x-1)$

30. What is the missing value if the given polynomial is a perfect square trinomial?

$4x^2 + [?] + 9$
$12x$

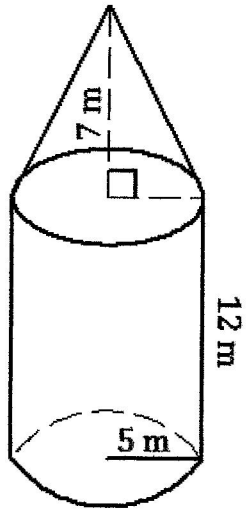
(A) $9x$
(B) $12x$
(C) $20x$
(D) $40x$

$2\sqrt{a}\sqrt{c}$
$= 2\sqrt{4}\sqrt{9}$
$= 2(2)(3)$
$= 12$

Part 2
Value: 30 Points

Instructions: Answer each question in the space provided. ALL workings must be shown to obtain full marks.

1. Determine the surface area of the following object, to the nearest square metre.
[Value: 4 Points]



Cone $SA = \pi r^2 + \pi r s$
$SA = \pi(5)^2 + \pi(5)(8.6)$
$= 213.6 m^2$

Slant Height

$S^2 = 5^2 + 7^2$
$S^2 = 74$
$S = \sqrt{74}$
$S = 8.6m$

Cylinder $SA = 2\pi r^2 + 2\pi rh$
$= 2\pi(5)^2 + 2\pi(5)(12)$
$= 534.1 m^2$

Overlap
$A = \pi r^2$
$= \pi(5)^2$
$= 78.5 m^2$

Total Overlap $= 2(78.5)$
$= 157 m^2$

$TSA = 213.6 + 534.1 - 157$
$= \boxed{590.7 m^2}$

2. (A) The volume of a sphere is 248.5 cm^3. What is the radius? **[Value: 3 Points]**

$V = \frac{4\pi r^3}{3}$

$248.5 = \frac{12.57r^3}{3}$

$\frac{745.5}{12.57} = \frac{12.57r^3}{12.57}$

$59.31 = r^3$

$\sqrt[3]{59.31} = r$

$r = 3.9\,cm$

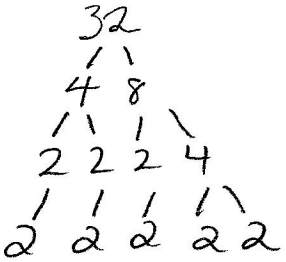
(B) What is the surface area of the sphere? **[Value: 1 Point]**

$SA = 4\pi r^2$

$= 4\pi(3.9)^2$

$SA = 191\,cm^2$

3. (A) What is the most simplified form of $\sqrt[3]{32}$? **[Value: 2 Points]**



$32 = (2)(2)(2)\cdot 2\cdot 2$

$= 2^3 \cdot 4$

$\sqrt[3]{32} = \sqrt[3]{2^3}\cdot\sqrt[3]{4}$

$= 2\sqrt[3]{4}$

(B) Write the radical $\sqrt[2]{4^3}$ as a power. **[Value: 2 Points]**

$4^{\frac{3}{2}}$

4. Carissa did not receive full marks for her solution below.

$\frac{(p^{-3}\,q^{2})^{-4}}{(2p^{2}q^{-3})^{3}}$ **Step 1**

$= \frac{p^{12}q^{-8}}{2p^{6}q^{-9}}$ **Step 2**

$= \frac{p^{12-6}q^{-8-9}}{2}$ **Step 3**

$= \frac{p^{6}q^{-17}}{2}$ **Step 4**

$= \frac{p^{6}}{2q^{17}}$ **Step 5**

(A) Identify which step she made her FIRST mistake in, and explain what the mistake was. **[Value: 1 Point]**

step 2 → forgot to raise 2 to the exponent 3

(B) Show the correct solution. **[Value: 3 Points]**

$\frac{p^{-3x-4}\,q^{2x-4}}{2^{3}p^{2x3}q^{-3x3}}$

$= \frac{p^{+12}q^{-8}}{8p^{6}q^{-9}}$

$= \frac{p^{12-6}q^{-8--9}}{8}$

$= \frac{p^{6}q^{1}}{8} = \frac{p^{6}q}{8}$

5. Simplify completely. Write your answer using positive exponents. **[Value: 4 Points]**

$$\left(\frac{36x^4y^3}{4x^8y^{-1}}\right)^{\frac{1}{2}}$$

$= (9x^{4-8}y^{3--1})^{\frac{1}{2}}$

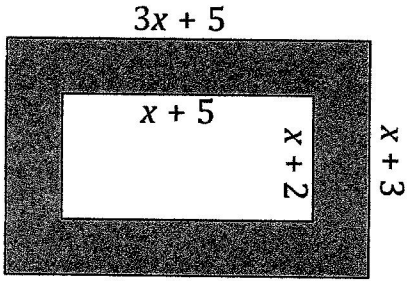
$= (9x^{-4}y^4)^{\frac{1}{2}}$

$= 9^{\frac{1}{2}}(x^{-4})^{\frac{1}{2}}(y^4)^{\frac{1}{2}}$

$= 3x^{-\frac{4}{2}}y^{\frac{4}{2}}$

$= 3x^{-2}y^2 = \boxed{\frac{3y^2}{x^2}}$

6. Determine the area of the shaded region, in simplest form. **[Value: 4 Points]**



$A_{shaded} = [(3x+5)(x+3)] - [(x+5)(x+2)]$

$= [3x^2+9x+5x+15] - [x^2+2x+5x+10]$

$= [3x^2+14x+15] - [x^2+7x+10]$

$= 3x^2+14x+15-x^2-7x-10$

$= 3x^2-x^2+14x-7x+15-10$

$= \boxed{2x^2+7x+5}$

7. Multiply the following: $(2x-1)(x^2-3x+4)$. **[Value: 3 Points]**

$(2x-1)(x^2-3x+4)$

$= 2x^3-6x^2+8x-x^2+3x-4$

$= 2x^3-6x^2-x^2+8x+3x-4$

$= \boxed{2x^3-7x^2+11x-4}$

8. Factor the following completely: $12x^2+20x+8$. **[Value: 3 Points]**

$12x^2+20x+8$ GCF = 4

$= 4(3x^2+5x+2)$

Quadratic

$a=3 \quad b=5 \quad c=2$

prod. of axc	sum of b
$(3)(2)=6$	5
$(2)(3)=6$	$2+3=5$

numbers are 2 and 3

$= 4\left(x+\frac{2}{3}\right)\left(x+\frac{3}{3}\right)$

$= \boxed{4(3x+2)(x+1)}$