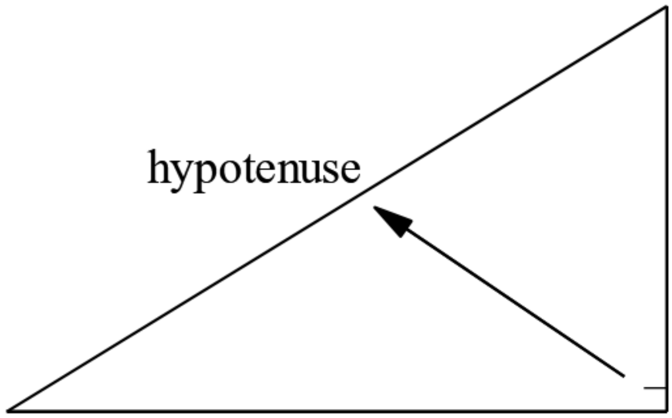
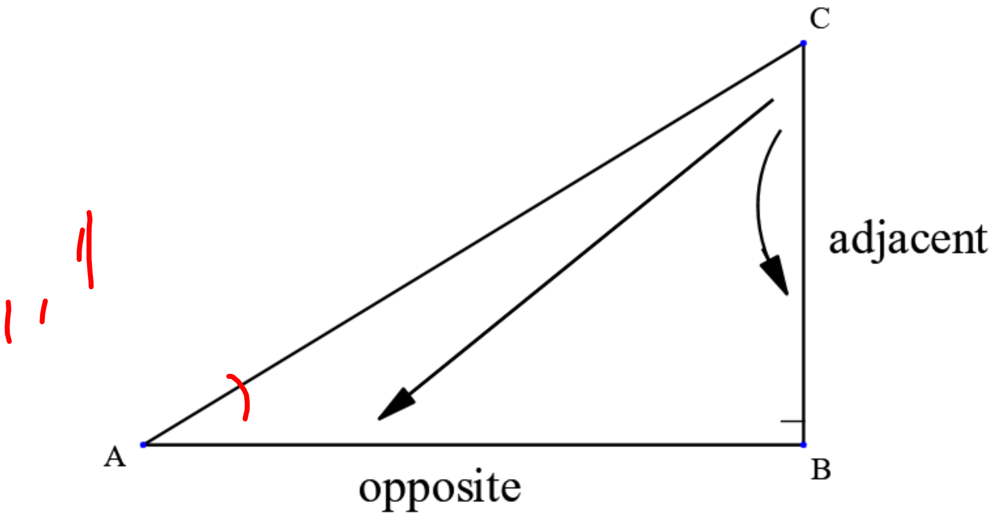
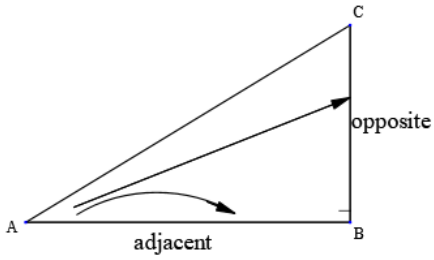
Before we begin, we need to learn a few terms.

The *hypotenuse* of a right triangle is the longest side of a right triangle. It is the opposite side of the right angle.



We also need to know the terms *opposite* and *adjacent sides*. The opposite and adjacent sides depend on the non-right angle we select in our right triangle. For example, consider ΔABC below. If we select $\angle A$, then the opposite and adjacent sides are determined relative to this angle:


C
adjacent
A
B
opposite

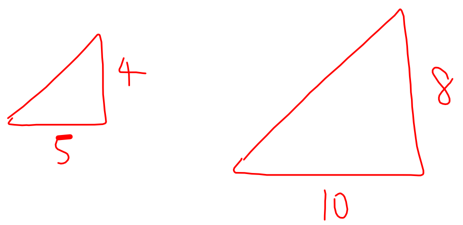


$$\tan A = \frac{\text{length of the opposite of an angle}}{\text{length of the adjacent side of an angle}}$$

Note:This property is only true for right triangles!

Note that $\tan D = \frac{2}{1} = 2$ and $\tan C = \frac{4}{2} = 2$. Therefore, we know that $\angle C = \angle D$. As well, when $\tan D = 2$, we know that the opposite side is 2 times as big as the adjacent side. Using trigonometry, we will learn how to find these angles.

Ex: Draw two triangles where $\tan X = \frac{4}{5} = 0.8$. Explain what is meant when we say that $\tan X = 0.8$



0.8 means the opposite side is 0.8 as big as the adjacent side.

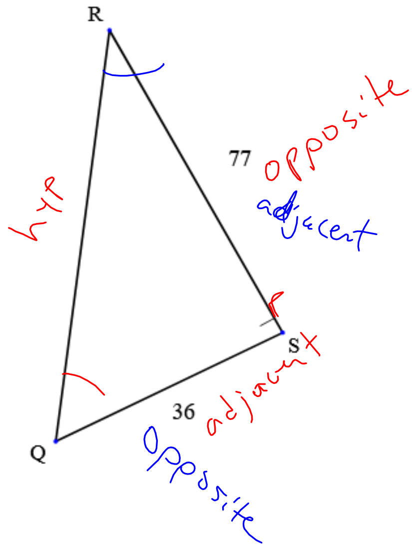
The Tangent Ratio

Ex: Determine $\tan Q$ and $\tan R$.

$\tan Q = \frac{opp}{adj}$

$\tan Q = \frac{77}{36} = 2.1$

$\tan R = \frac{36}{77} = 0.47$



Ex: Determine $\tan \theta$.

Pythagorean theorem:

$a^2 + b^2 = c^2$

$11^2 + b^2 = 61^2$

$121 + b^2 = 3721$

$b^2 = 3721 - 121$

$b^2 = 3600$

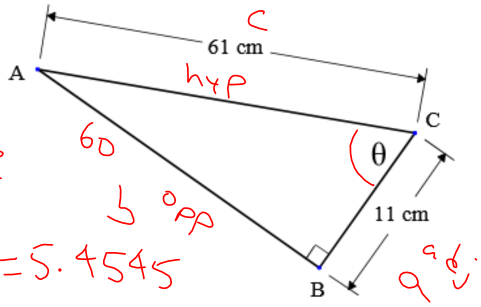
$\sqrt{b^2} = \sqrt{3600}$

$b = 60$

$\tan \theta = \frac{opp}{adj}$

$\tan \theta = \frac{60}{11} = 5.4545$

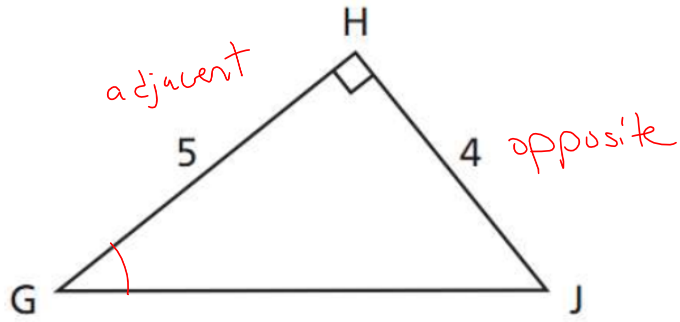
* Always round to <u>4</u> decimal places.



Note: θ is the Greek letter *theta*. Greek letters are often used in trigonometry to indicate angles.

Next, we look at how to use the tangent ratio to determine missing angles in a triangle.

Ex: Determine the measures of $\angle G$ and $\angle J$ to the nearest tenth of a degree.



First, let's find $\angle G$. Note that:

$$\tan \angle G = \tfrac{4}{5}$$

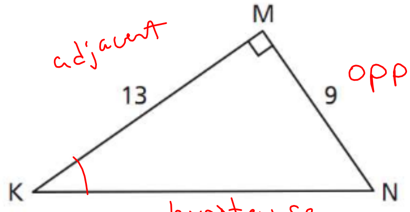
To find $\angle G$, we need to use the *inverse tangent* function ($\tan^{-1}$):

$$\angle G = \tan^{-1}\left(\tfrac{4}{5}\right) = 38.7°$$

To find $\angle J$ we do not need to use trigonometry (although we can!). Recall that the angles of a triangle add to 180°:

$$\angle G + \angle J + \angle H = 180°$$
$$38.7° + \angle J + 90° = 180°$$
$$\angle J = 180° - 90° - 38.7°$$
$$\angle J = \boxed{51.3°}$$

Ex: Determine the measure of $\angle K$ and $\angle N$ to the nearest tenth of a degree.



$\tan K = \frac{\text{opposite}}{\text{adjacent}}$

$\tan K = \frac{9}{13}$

$\tan K = 0.6923$

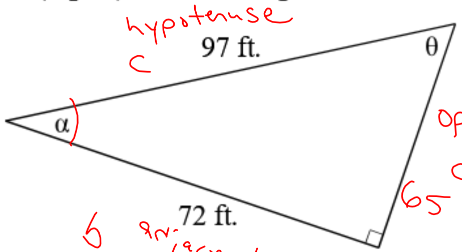
$K = \tan^{-1}(0.6923)$

$K = 34.7°$

$N = 180° - (90° + 34.7°)$

$N = 55.3°$

Ex: Determine the values of θ and α (alpha) in the triangle below.



$a^2 + b^2 = c^2$

$a^2 + 72^2 = 97^2$

$a^2 + 5184 = 9409$

$a^2 = 9409 - 5184$

$a^2 = 4225$

$\sqrt{a^2} = \sqrt{4225}$

$a = 65$

$\tan \alpha = \frac{opp}{adj}$

$\tan \alpha = \frac{65}{72}$

$\tan \alpha = 0.9028$

$\alpha = \tan^{-1}(0.9028)$

$\alpha = 42°$

$\theta = 180° - (90° + 42°)$

$\theta = 48°$

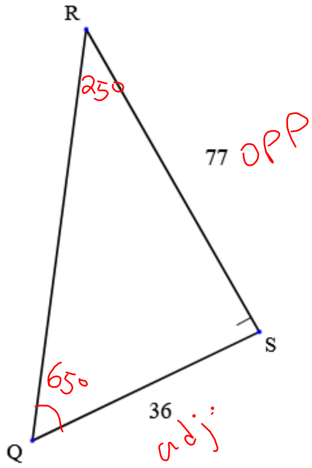
Ex: Determine the missing angles.

$\tan Q = \frac{opp}{adj} = \frac{77}{36} = 2.1389$

$Q = \tan^{-1}(2.1389)$

$Q = 65°$

$R = 180° - (65° + 90°) = 25°$



Ex: Determine the missing angles.

$a^2 + b^2 = c^2$

$a^2 + (55)^2 = (73)^2$

$a^2 + 3025 = 5329$

$a^2 = 5329 - 3025$

$a^2 = 2304$

$\sqrt{a^2} = \sqrt{2304}$

$a = 48$

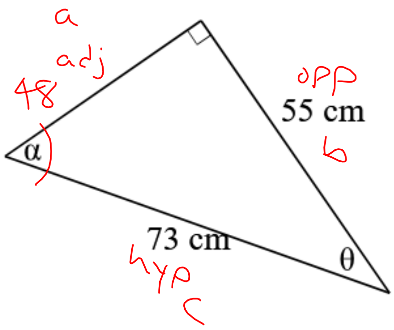
$\tan \alpha = \frac{opp}{adj}$

$\tan \alpha = \frac{55}{48}$

$\tan \alpha = 1.1458$

$\alpha = \tan^{-1}(1.1458)$

$\alpha = 49°$



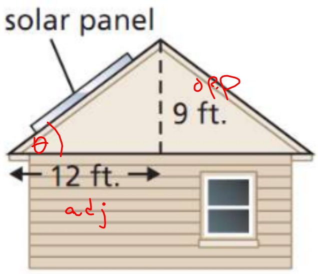
$\theta = 180° - (90° + 49°)$

$\theta = 41°$

Problem Solving using the Tangent Ratio

Let's look at a few word problems.

Ex: South-facing solar panels on a roof work best when the *angle of inclination* of the roof is equal to the latitude of the house. The house shown is in Fort Smith, NWT, which has a latitude of approximately 60°. Determine whether this design is best for Fort Smith. Justify your answer.



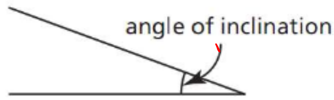
$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{9}{12} = 0.75$

$\theta = \tan^{-1}(0.75) = 37^\circ$

No, not best design for Fort Smith.

The **angle of inclination** of a line or line segment is the acute angle it makes with a horizontal line.



Ex: Cox's Cove has a latitude of 49.6°. Determine if the roof below is the best design for a solar panel.

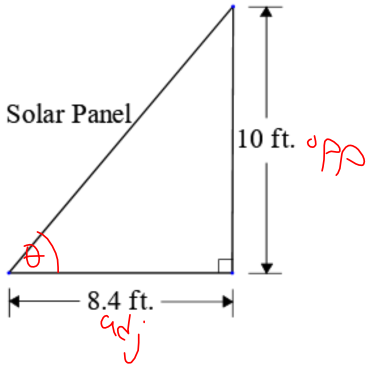
$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{10}{8.4} = 1.1905$

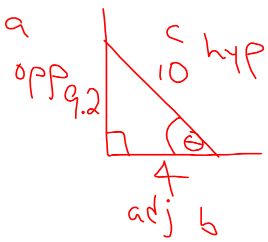
$\theta = \tan^{-1}(1.1905)$

$\theta = 50^{0}$

Roof is best design for Cox's Cove.



Ex: A 10 ft. ladder leans against the side of a building with its base 4 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?



$a^2 + b^2 = c^2$

$a^2 + 4^2 = 10^2$

$a^2 + 16 = 100$

$a^2 = 100 - 16$

$a^2 = 84$

$\sqrt{a^2} = \sqrt{84}$

$a = 9.2$

$\tan\theta = \frac{opp}{adj}$

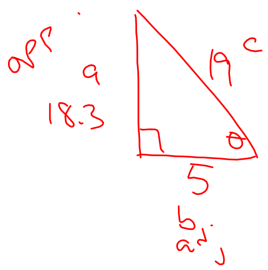
$\tan\theta = \frac{9.2}{4}$

$\tan\theta = 2.3$

$\theta = \tan^{-1}(2.3)$

$\theta = 67^{0}$

Ex: A support cable is anchored to the ground 5m from the base of a telephone pole. The cable is 19m long. It is attached to the top of the pole. What angle, to the nearest degree, does the cable make with the pole?



$a^2 + b^2 = c^2$

$a^2 + 5^2 = 19^2$

$a^2 + 25 = 361$

$a^2 = 361 - 25$

$a^2 = 336$

$\sqrt{a^2} = \sqrt{336}$

$a = 18.3$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{18.3}{5}$

$\tan\theta = 3.66$

$\theta = \tan^{-1}(3.66)$

$\theta = 75^\circ$

Note: We can only use the tangent ratio in right triangles!!!!!

HW: p77 #3, 4, 5, 6a,d,e, 8, 10, 11, 12, 13, 14, 15, 17