Section 2.2 Using the Tangent Ratio to Calculate Lengths

In the last lesson, we defined the tangent ratio and used it to find angles in a right triangle. In this lesson, we will apply the tangent ratio to find the lengths of sides of right triangles.

Think About It – In ΔPQR, $\angle Q = 90°$, $\angle P = 34.5°$, and $PQ = 46.1cm$. Determine the length of RQ to the nearest tenth of a centimetre.

We can use the tangent ratio to determine the missing side lengths of a right triangle as long as we are given one acute angle and one leg.

Ex: Determine the indicated values:

(a) AB

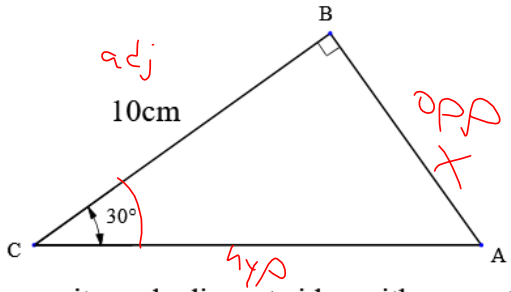
$\tan\theta = \frac{opp}{adj}$

$\tan 30° = \frac{x}{10}$

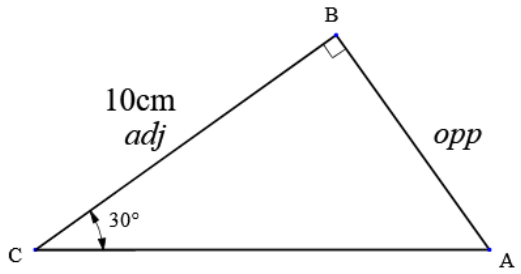
$10\tan 30° = \frac{x}{10} \cdot 10$

$10\tan 30° = x$

$5.8 = x$



If we label our opposite and adjacent sides with respect to $\angle C$:



we see that:

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

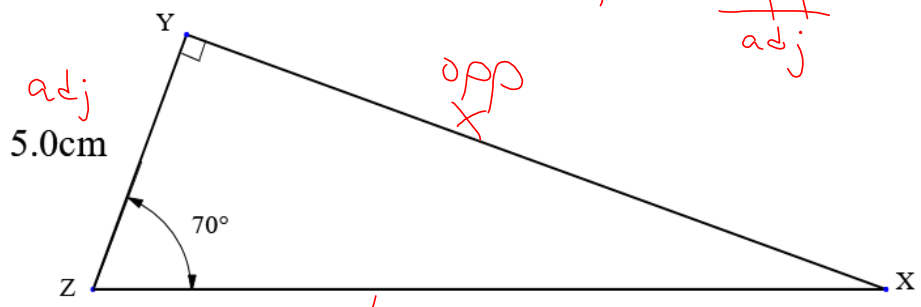
$$\tan 30° = \frac{AB}{10}$$

$$10(\tan 30°) = 10\left(\frac{AB}{10}\right)$$ ← Multiply both sides by 10 to eliminate the fraction.

→ $5.7735... = AB$ Your calculator can work this out!

Therefore, AB is approximately 5.8cm long.

(b) XY



$\tan\theta = \frac{opp}{adj}$

$\tan 70° = \frac{X}{5}$

$X = 5\tan 70°$

$X = 13.7$

(c) EF

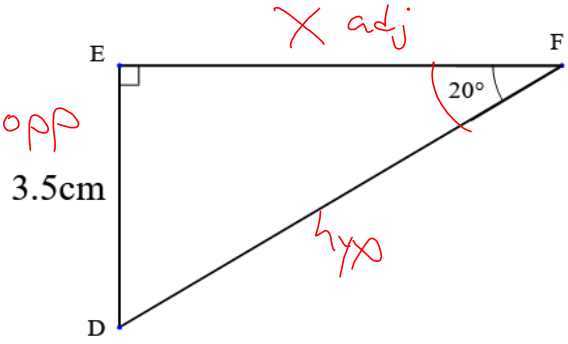
$\tan 20° = \frac{3.5}{X}$

$X \tan 20° = \frac{3.5}{X} \cdot X$

$X \tan 20° = 3.5$

$\frac{X \tan 20°}{\tan 20°} = \frac{3.5}{\tan 20°}$

$X = \frac{3.5}{\tan 20°} = 9.6$

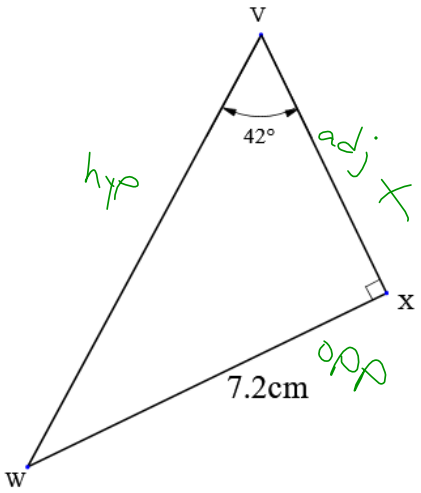


(d) VX

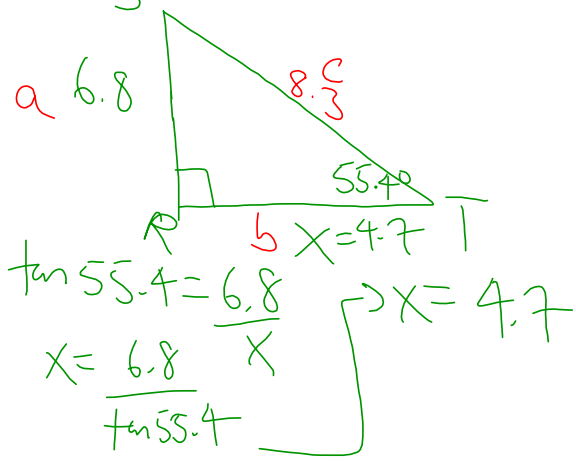
$\tan 42° = \frac{7.2}{X}$

$X = \frac{7.2}{\tan 42°}$

$X = 8.0$



Ex: In ΔRST, $\angle R = 90°$, $\angle T = 55.4°$, and $RS = 6.8$ ft. Determine both of the missing sides of the triangle.



$\tan 55.4 = \frac{6.8}{X}$

$X = \frac{6.8}{\tan 55.4}$

$X = 4.7$

$c^2 = a^2 + b^2$

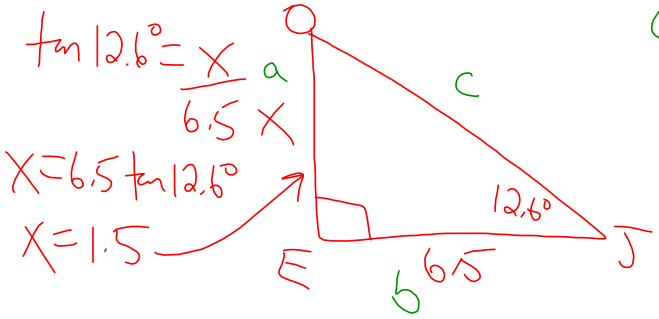
$c^2 = 6.8^2 + 4.7^2$

$c^2 = 68.3$

$\sqrt{c^2} = \sqrt{68.3}$

$c = 8.3$

Ex: In ΔJOE with a right angle at $\angle E$, determine the missing sides if $\angle J = 12.6°$ and $EJ = 6.5m$.



$c^2 = a^2 + b^2$

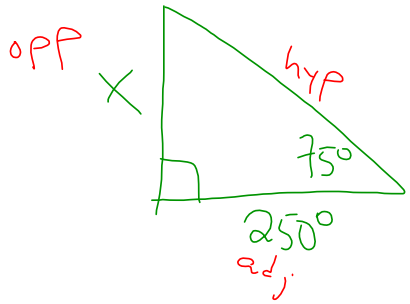
$c^2 = 1.5^2 + 6.5^2$

$c^2 = 44.5$

$\sqrt{c^2} = \sqrt{44.5}$

$c = 6.7m$

Ex: A searchlight beam shines vertically on a cloud. At a horizontal distance of 250m, the angle between the ground and the line of sight is 75°. Determine the height of the cloud to the nearest metre.



$\tan 75° = \frac{X}{250}$

$X = 250 \tan 75°$

$X = 933\text{m}$

Ex: The base of a ladder is 2.5m from a wall. If the ladder makes a 22° angle with the wall, determine how high up the wall the ladder reaches, and determine the length of the ladder.

Ex: A kite is tied to a tent peg that is driven into the ground. The string makes a 65° angle with the ground. If the kite is at a height of 52m, determine the length of the string.

Ex: In ΔABC, $\angle B = 90°$ and $\angle C = 13°$. If $AB = 10.3cm$, determine the area of the triangle to the nearest square centimetre.

HW: p82 #3, 4, 5, 6, 7, 8, 9, 11, 12, 14