will be constant, regardless of how big or small the triangle is. For example, consider the following right triangles with a 40° angle:

$c^2 = a^2 + b^2$

$c^2 = (14.301)^2 + (12.0)^2$

$c^2 = 348.5186$

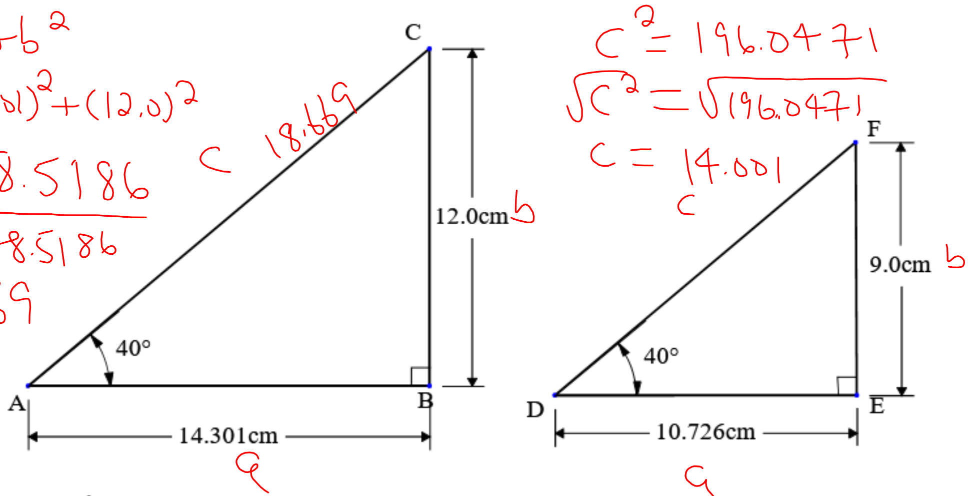
$\sqrt{c^2} = \sqrt{348.5186}$

$c = 18.669$

$c^2 = 196.0471$

$\sqrt{c^2} = \sqrt{196.0471}$

$c = 14.001$



Note that:

- for $\angle A$, $\dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{12.0}{14.301} \approx 0.8391$;
- for $\angle D$, $\dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{9.0}{10.726} \approx 0.8391$

This result tells us that $\tan 40° \approx 0.8391$, which can be verified on our calculators.

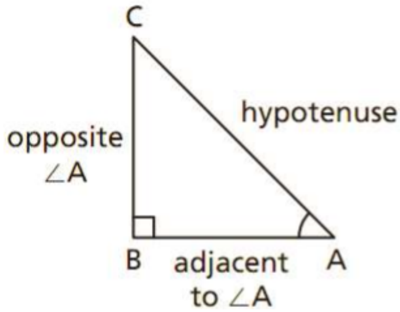
Using the above triangles, determine the hypotenuse for both (using the Pythagorean Theorem) and complete the following chart:

	$\frac{\text{opposite side}}{\text{hypotenuse}}$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$
$\angle A$	$\frac{12.0}{18.669} = 0.6428$	✓
$\angle D$	$\frac{9.0}{14.001} = 0.6428$	✓

Note that these ratios are also constant! The only way to change the ratio is to change the angle. Mathematicians give these ratios special names as well.

If $\angle A$ is an acute angle in a right triangle, then:

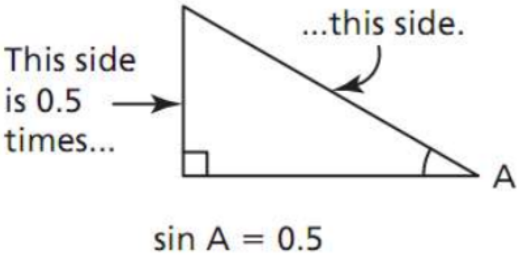
- $\text{sine}\angle A = \sin \angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$
- $\text{cosine}\angle A = \cos \angle A = \frac{\text{adjacent side}}{\text{hypotenuse}}$



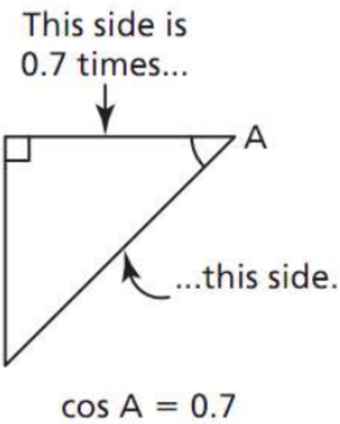
Tangent, sine, and cosine are called the *primary trigonometric ratios*. The word *trigonometry*, the student of triangles, is Greek for three angle measure.

If we use our calculators, we see that $\sin 40° \approx 0.6428$ and $\cos 40° \approx 0.7660$, which are the ratios that appear in our table above.

It is important to remember that sine and cosine are ratios of sides in right triangles. For example, if we know that $\sin A = 0.5$, then the opposite side of A is 0.5 times as long as the hypotenuse regardless of how long each side of the triangle is:



Similarly, if we are told that $\cos A = 0.7$, then the side adjacent to A is 0.7 times as long as the hypotenuse:



What happens to $\sin A$ as $\angle A$ approaches 0°? What happens to $\cos A$?

As $\angle A$ approaches 0°, $\sin A$ approaches 0.

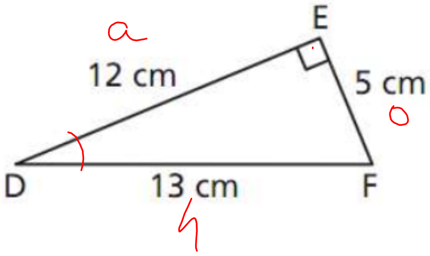
As $\angle A$ approaches 0°, $\cos A$ approaches 1.

Unit 2, Section 4, Lesson 3 Page 4
The Sine and Cosine Ratios

Ex: Determine the value of $\sin D$ and $\cos D$ in the triangle below to the nearest hundredth.

$\sin D = \frac{o}{h} = \frac{5}{13} = 0.3846$

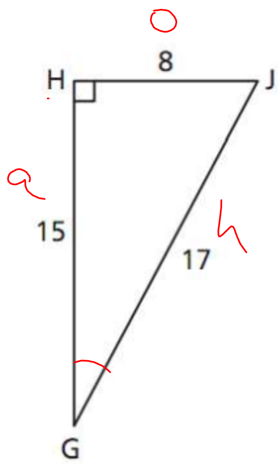
$\cos D = \frac{a}{h} = \frac{12}{13} = 0.9231$



Ex: Determine the value of $\sin G$ and $\cos G$ to the nearest hundredth.

$\sin G = \frac{8}{17} = 0.4706$

$\cos G = \frac{15}{17} = 0.8824$



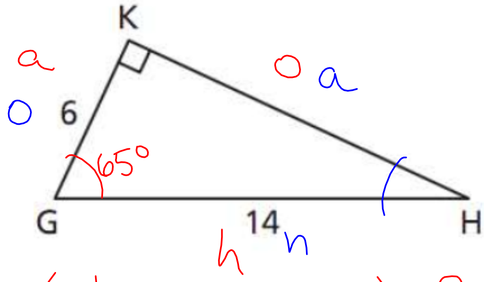
Unit 2, Section 4, Lesson 3 Page 5
The Sine and Cosine Ratios

Ex: Determine the missing angles to the nearest tenth of a degree in the triangles below:

(a) ~~soh~~ $\boxed{cah}$ ~~toa~~

$\cos G = \frac{6}{14} = 0.4286$

$G = \cos^{-1}\left(\frac{6}{14}\right) = 65°$



$\angle H = 180° - (65° + 90) = 25°$

$\boxed{soh}$ ~~cah~~ ~~toa~~

$\sin H = \frac{6}{14}$

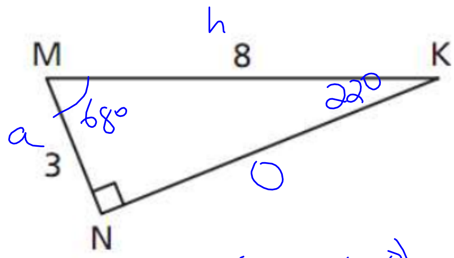
$H = \sin^{-1}\left(\frac{6}{14}\right) = 25°$

(b) ~~soh~~ $\boxed{cah}$ ~~toa~~

$\cos M = \frac{3}{8}$

$M = \cos^{-1}\left(\frac{3}{8}\right)$

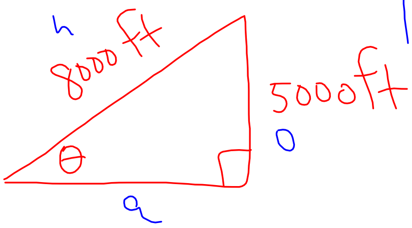
$M = 68°$



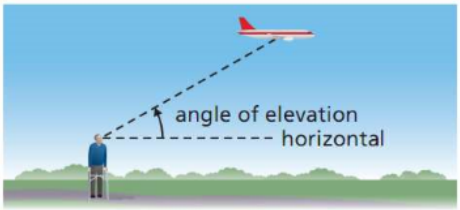
$180° - (90° + 68°) = 22°$

Ex: A water bomber is flying at an altitude of 5000 ft. The plane's radar shows that it is 8000 ft. from the target. What is the *angle of elevation* of the plane measured from the target site, to the nearest degree?

$\boxed{\text{Soh}}$ ~~cah~~ ~~toa~~



The **angle of elevation** of an object above the horizontal is the angle between the horizontal and the line of sight from an observer.

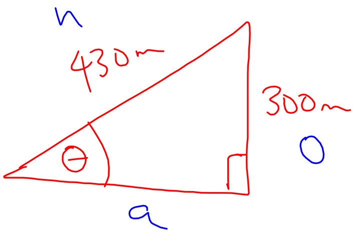


$\sin\theta = \frac{5000}{8000}$

$\theta = \sin^{-1}\left(\frac{5}{8}\right)$

$\theta = 39°$

Ex: An observer is sitting on a dock watching a float plane in Vancouver harbour. At a certain time, the plane is 300 m above the water and 430 m from the observer. Determine the angle of elevation of the plane measured from the observer, to the nearest degree.



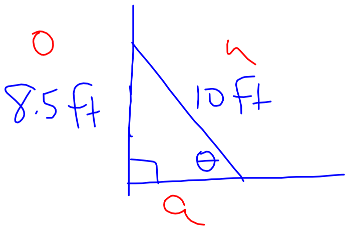
Soh ~~cah~~ ~~toa~~

$\sin\theta = \frac{300}{430}$

$\theta = \sin^{-1}\left(\frac{300}{430}\right)$

$\theta = 44°$

Ex: A 10 ft. ladder reaches 8.5 ft. up a wall. Determine the angle of elevation of the ladder.

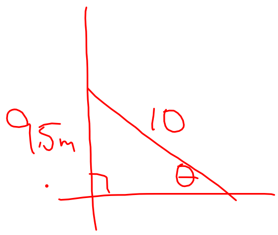


$\sin\theta = \frac{8.5}{10}$

$\theta = \sin^{-1}\left(\frac{8.5}{10}\right)$

$\theta = 58°$

Ex: The ladder is adjusted so that is now reaches 9.5 ft. up the same wall. What is the increase in the angle of elevation?



$\sin\theta = \frac{9.5}{10}$

$\theta = \sin^{-1}\left(\frac{9.5}{10}\right)$

$\theta = 72°$

$72° - 58° = 14°$

HW: p95 #4, 5, 6, 7, 8, 9a, d, 10, 12, 13, 14, 17