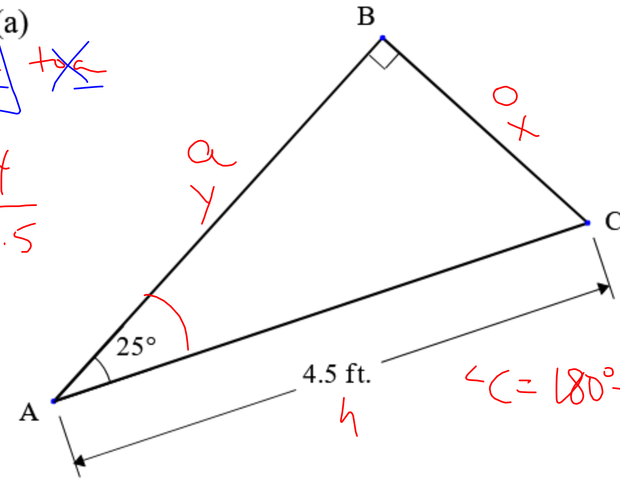
Section 2.6 Applying the Trigonometric Ratios

In this lesson, we will use the primary trigonometric ratios to solve problems modelled by right triangles.

Ex: *Solve* the following right triangles, giving your answer to the nearest tenth. (*Solving a triangle* means to give the measures of all the missing sides and lengths.)

(a)



s~~oh~~ $\boxed{cah}$ t~~oa~~

$\cos 25 = \dfrac{y}{4.5}$

$y = 4.5\cos 25°$

$y = 4.1\text{ft}$

$\boxed{soh}$ c~~ah~~ t~~oa~~

$\sin 25° = \dfrac{x}{4.5}$

$x = 4.5\sin 25°$

$x = 1.9\text{ft}$

$\angle C = 180° - (25° + 90°) = 65°$

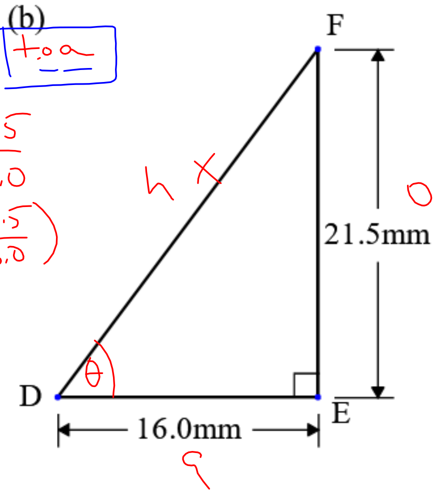
(b)

$\cancel{s}oh$ $c\cancel{a}h$ $\boxed{toa}$

$\tan\theta = \frac{21.5}{16.0}$

$\theta = \tan^{-1}\left(\frac{21.5}{16.0}\right)$

$\theta = 53^\circ$



$\boxed{soh}$ $\cancel{cah}$ $\cancel{toa}$

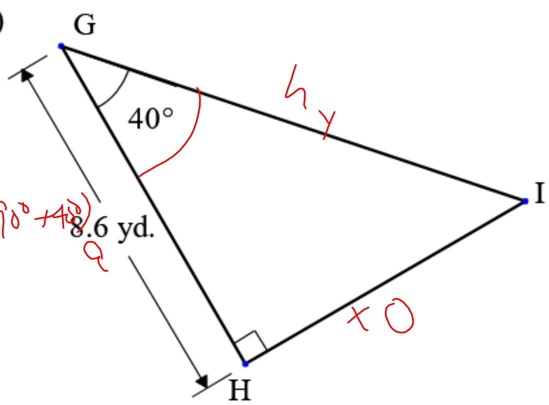
$\sin 53^\circ = \frac{21.5}{x}$

$x = \frac{21.5}{\sin 53^\circ}$

$x = 27\text{ mm}$

$\angle F = 180^\circ - (90^\circ + 53^\circ)$
$= 37^\circ$

(c)



$\angle I = 180^\circ - (90^\circ + 40^\circ)$

$\angle I = 50^\circ$

$s\cancel{o}h$ $c\cancel{a}h$ $\boxed{toa}$

$\tan 40^\circ = \frac{x}{8.6}$

$x = 8.6\tan 40^\circ$

$x = 7.2\text{ yd}$

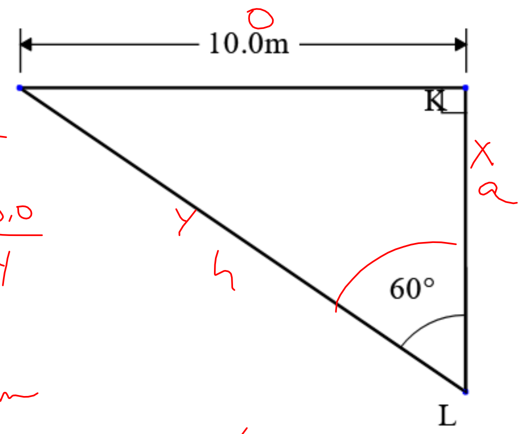
$s\cancel{o}h$ $\boxed{cah}$ $\cancel{toa}$

$\cos 40^\circ = \frac{8.6}{y}$

$y = \frac{8.6}{\cos 40^\circ}$

$y = 11.2\text{ yd}$

(d)



$\boxed{\text{SOH}}$ $\cancel{\text{cah}}$ $\cancel{\text{toa}}$

$\sin 60° = \frac{10.0}{y}$

$y = \frac{10.0}{\sin 60°}$

$y = 11.5\text{ m}$

$\cancel{\text{soh}}$ $\cancel{\text{cah}}$ $\boxed{\text{TOA}}$

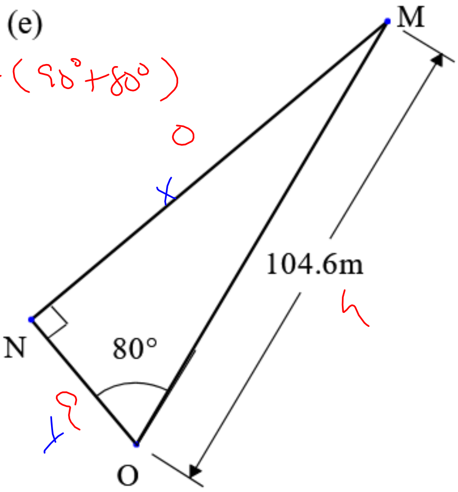
$\tan 60° = \frac{10.0}{x}$

$x = \frac{10.0}{\tan 60°}$

$x = 5.8\text{ m}$

$\angle J = 180° - (60° + 90°)$
$= 30°$

(e)



$\angle M = 180° - (90° + 80°)$
$= 10°$

$\boxed{\text{SOH}}$ $\cancel{\text{cah}}$ $\cancel{\text{toa}}$

$\sin 80° = \frac{x}{104.6}$

$x = 104.6 \sin 80°$

$x = 103\text{ m}$

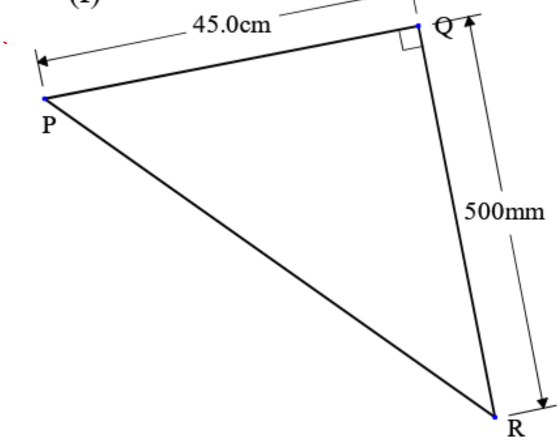
$\cancel{\text{soh}}$ $\boxed{\text{CAH}}$ $\cancel{\text{toa}}$

$\cos 80° = \frac{y}{104.6}$

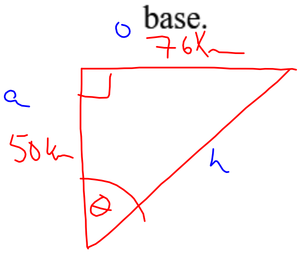
$y = 104.6 \cos 80°$

$y = 18.2\text{ m}$

(f)



Ex: A helicopter leaves its base and flies 76km due west in order to pick up an injured person. The helicopter then flies 50km due south to the hospital. Finally, the helicopter flies directly back to its base. Determine how far the helicopter flew for the total trip, and determine the angle between the path it took due south and the return flight to the base.



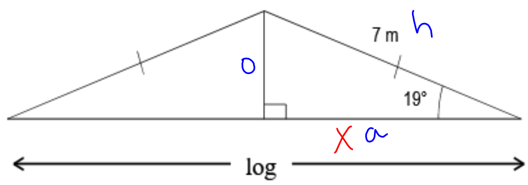
~~SOH~~ ~~CAH~~ $\boxed{TOA}$

$\tan\theta = \frac{76}{50}$

$\theta = \tan^{-1}\left(\frac{76}{50}\right)$

$\theta = 57^{\circ}$

Ex: Mountaineer Joe is building a log cabin. He has two logs, 7 m in length each, to make the peak of the roof. He is going to angle them each at 19°. In order to do this, he needs a single log to each from one side of the roof straight across to the other side. What is the length of the log Joe needs to complete this task?



$s\cancel{o}h\ \boxed{cah}\ t\cancel{oa}$

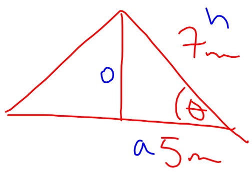
$\cos 19° = \frac{x}{7}$

$x = 7\cos 19°$

$x = 6.6m$

$2x = 2(6.6m) = 13.2m$

Ex: If the longest log Joe can get is 10 m in length, what is the new angle of elevation to the peak of the roof?



$s\cancel{o}h\ \boxed{cah}\ t\cancel{oa}$

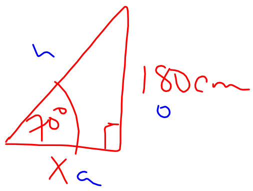
$\cos\theta = \frac{5}{7}$

$\theta = \cos^{-1}\left(\frac{5}{7}\right)$

$\theta = 44°$

Ex: Martha is 180cm tall and her daughter Heidi is 90cm tall. Martha goes outside at 9:00am when the sun is at an angle of elevation of 70°. Later in the day, Heidi goes outside when the angle of elevation of the sun is 35°. Determine who casts the longer shadow and by how much.

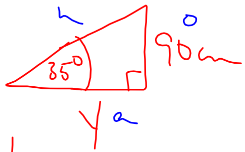
~~SOh~~ ~~CAh~~ $\boxed{\text{TOA}}$



$$\tan 70° = \frac{180}{x}$$

$$x = \frac{180}{\tan 70°}$$

$$x = 66\text{cm}$$



$$\tan 35° = \frac{90}{y}$$

$$y = \frac{90}{\tan 35°}$$

$$y = 129\text{cm}$$

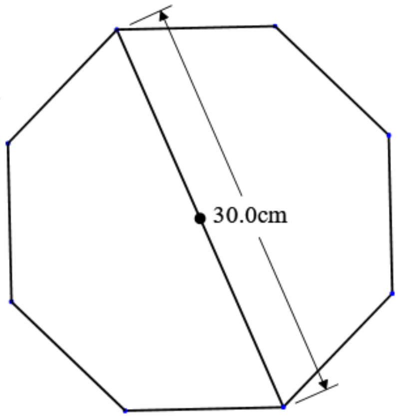
Ex: A guy wire is 83m long and is attached to the top of a 75m pole. Assuming the pole makes a right angle with the ground, what angle does the guy wire make with the pole?

Ex: Ron is flying a kite. He lets out 30m of string at an angle of elevation of 71°. If Ron is 1.7m tall, what is the height of the kite?

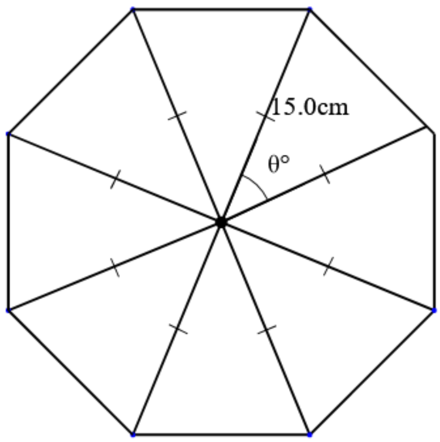
Ex: A kite is attached to a tent peg using 55m of string. Initially, it is flying at a height of 40m. After a few minutes, the wind dies down, reducing the height of the kite to 35m. Determine the decrease in the angle of elevation.

HW: p111 #4, 6, 7, 8, 9, 11, 12

Ex: A small table has the shape of a regular octagon. The distance from one vertex to the opposite vertex is 30.0cm. There is a strip of wood veneer around the edge of the table. What is the length of this veneer, to the nearest centimetre?

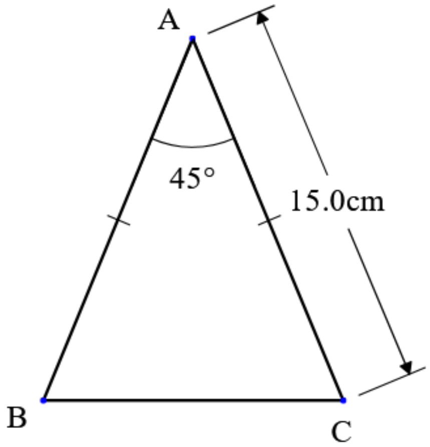


First, note that we can divide this octagon into 8 isosceles triangles with two legs equal to 15.0cm:

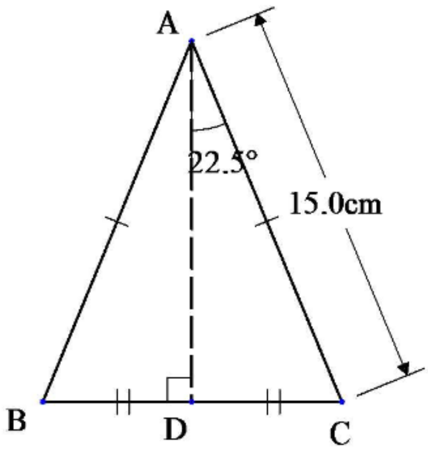


We can find θ by noting there are 8 of these angles in the middle, and a circle has 360°. Therefore $\theta = \frac{360°}{8} = 45°$.

Since we need to find the perimeter, we need to find the value of BC in the triangle below and then multiply it by 8 (since there are 8 of these lengths around the whole octagon:



Since this is an isosceles triangle, if we draw the line from A to the midpoint of BC (called a *median*), we divide BC in half, meet BC at a right angle, and divide $\angle A$ in half:



We can now find DC by using sine:

$$\sin 22.5° = \frac{DC}{15.0}$$
$$DC = 15 \times \sin 22.5°$$
$$DC = 5.74025...$$

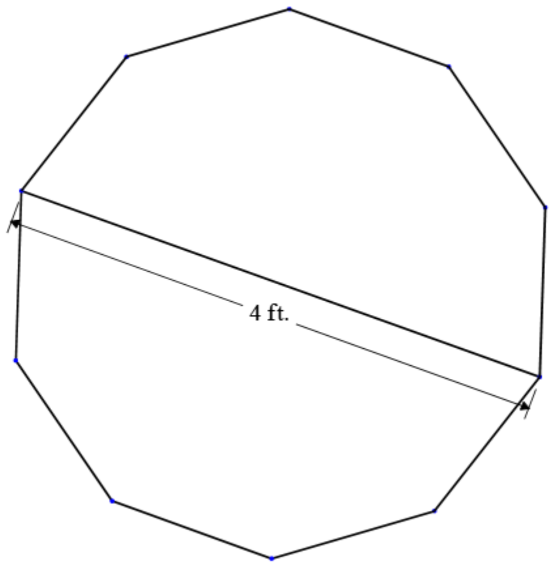
We now find BD:

$$BD = 2 \times DC$$
$$= 2 \times 5.74025...$$
$$\doteq 11.5cm$$

Therefore, the perimeter is $8(11.5) = 92cm$.

Ex: Find the area of the above octagon.

Ex: Find the perimeter and area of the following *decagon* (ten-sided figure).



HW: p111 #4, 6, 7, 8, 9, 11, 12