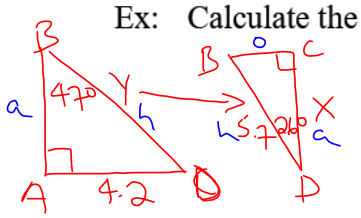
Section 2.7 Solving Problems Involving More than One Triangle

In this lesson, we will use trigonometry to solve problems modelled by more than one right triangle.

Ex: Calculate the length of CD to the nearest tenth of a centimetre.



SOH CAH TOA

$\cos 26° = \frac{X}{5.7}$

$X = 5.7 \cos 26°$

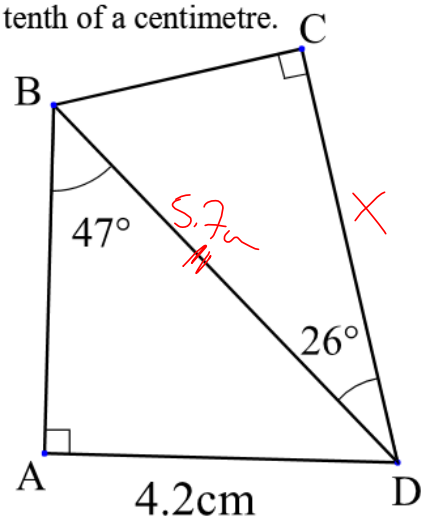
$X = 5.1\text{ cm}$

SOH CAH TOA

$\sin 47° = \frac{4.2}{Y}$

$Y = \frac{4.2}{\sin 47°}$

$Y = 5.7\text{ cm}$



Ex: Calculate the length of XY to the nearest tenth of a kilometre.

SOH CAH TOA

$\sin 20° = \frac{Y}{8.4}$

$Y = 8.4 \sin 20°$

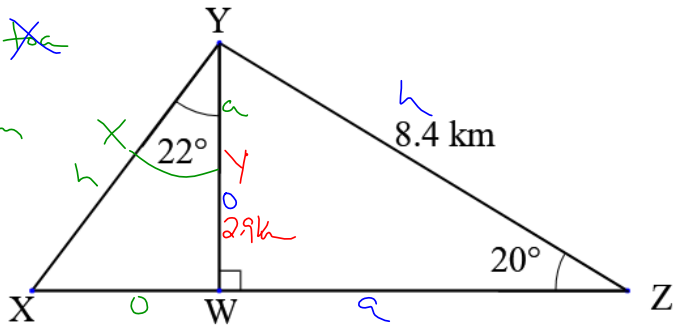
$Y = 2.9\text{ km}$

SOH CAH TOA

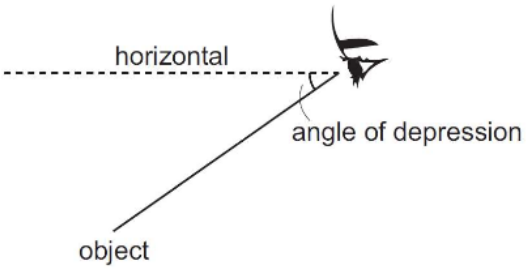
$\cos 22° = \frac{2.9\text{km}}{X}$

$X = \frac{2.9}{\cos 22°}$

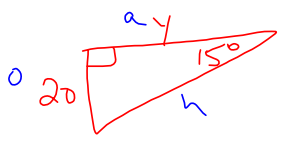
$X = 3.1\text{ km}$



Next, we look at a few problems involving the *angle of depression* – the angle between the horizontal line and the line of sight from an observer to an object below.



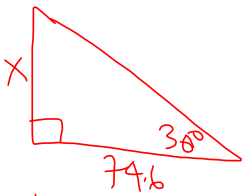
Ex: From the top of a 20m high building, a surveyor measured the angle of elevation of the top of another building and the angle of depression of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.



$\tan 15° = \frac{20}{Y}$

$Y = \frac{20}{\tan 15°}$

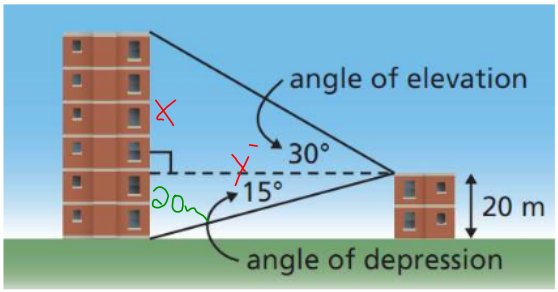
$Y = 74.6\text{m}$



$\tan 30° = \frac{X}{74.6}$

$X = 74.6 \tan 30°$

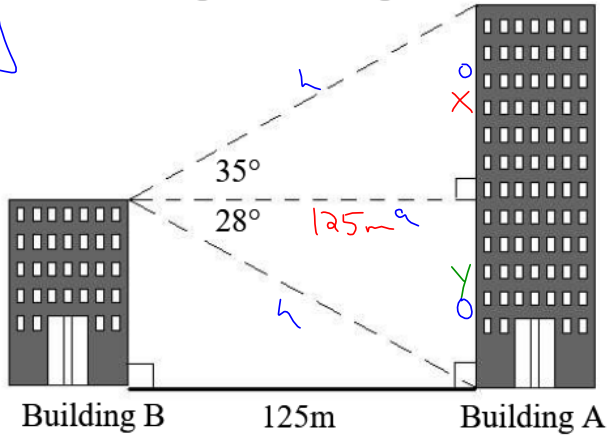
$X = 43.1\text{m}$



Building height = 43.1m + 20m
= 63.1m

Ex: An engineer needs to determine the height of Building A. Unfortunately, he does not have access to the blueprints, so he decides to stand on Building B, which is 125m away, and measures the angles shown in the diagram. Determine the height of Building A to the nearest metre.

SOH CAH TOA



$\tan 35° = \frac{x}{125}$

$x = 125 \tan 35°$

$x = 88m$

$\tan 28° = \frac{y}{125}$

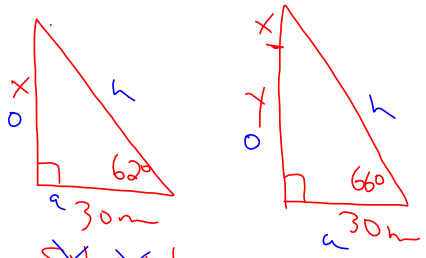
$y = 125 \tan 28°$

$y = 66m$

Height of building $= 88m + 66m$

$= 154m$

Ex: Determine the height of the flagpole.



$\tan 62° = \frac{x}{30}$

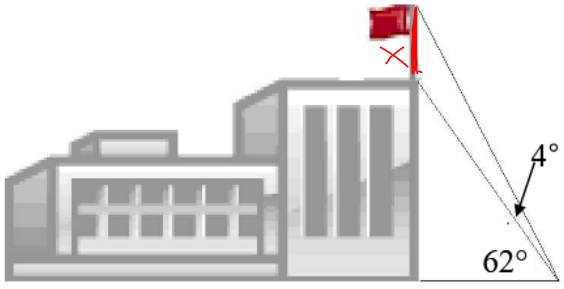
$x = 30\tan 62°$

$x = 56.4\text{m}$

$\tan 66° = \frac{y}{30}$

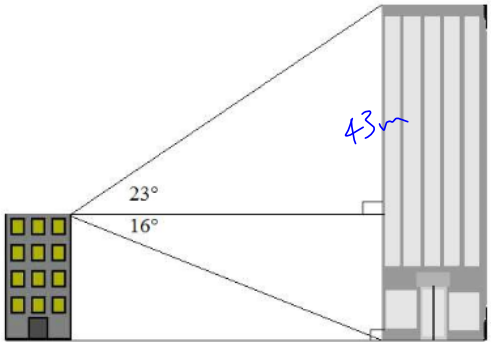
$y = 30\tan 66°$

$y = 67.4\text{m}$

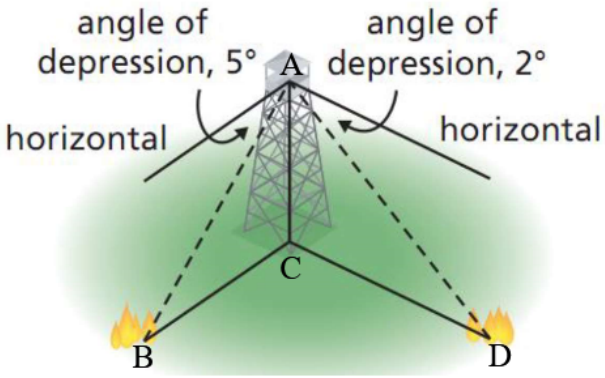


Flagpole $= 67.4 - 56.4 = 11.0\text{m}$

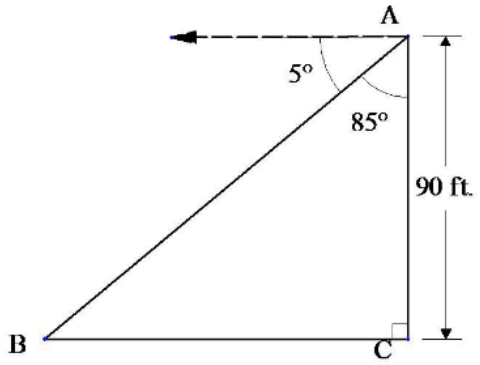
Ex: The taller building is 43m higher than the shorter building. Determine the height of both buildings.



Ex: From the top of a 90-ft. observation tower, a fire ranger observes one fire due west of the tower at an angle of depression of 5°, and another fire due south of the tower at an angle of depression of 2°. How far apart are the fires to the nearest foot?



First, note that all three triangles are right. We need to find the length of BD. If we find BC and CD, we can use the Pythagorean Theorem to find BD. First, we look at ΔABC:

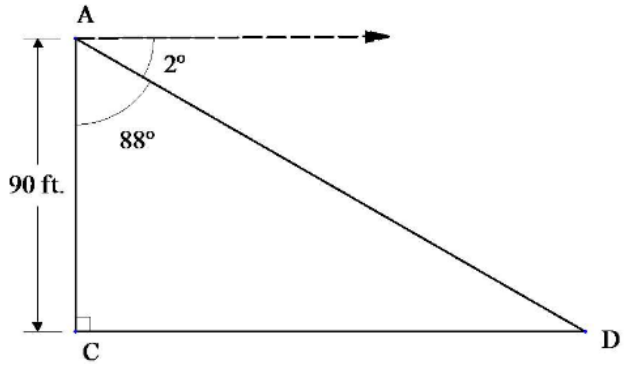


We can use trigonometry to find BC:

$$\tan 85° = \frac{BC}{90}$$

$$BC = 90 \times \tan 85° = 1028.7047...$$

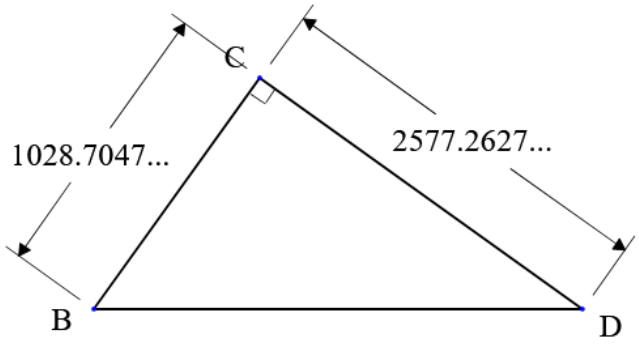
Next, we look at ΔADC:



Again, we use trigonometry to find CD:

$$\tan 88° = \frac{CD}{90}$$
$$CD = 90 \times \tan 88° = 2577.2627....$$

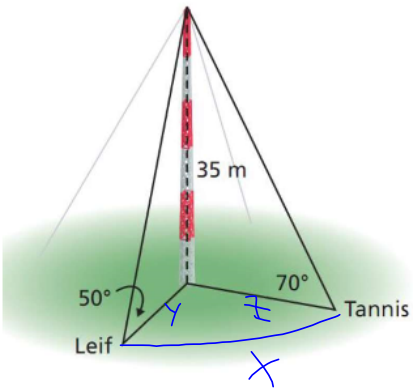
Finally, we can use the Pythagorean Theorem to find BD:

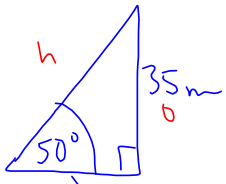


$$1028.7047....^2 + 2577.2627....^2 = BD^2$$
$$BD = \sqrt{1028.7047....^2 + 2577.2627....^2} = 2774.9804...$$

The distance between the fires is approximately 2775 ft.

Ex: A communications tower is 35 m tall. From a point due north of the tower, Tannis measures the angle of elevation of the top of the tower as 70°. Her brother Leif, who is due east of the tower, measures the angle of elevation of the top of the tower as 50°. How far apart are the students to the nearest metre?

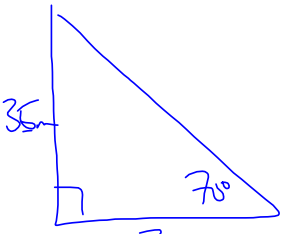




$\tan 50° = \frac{35}{Y}$

$Y = \frac{35}{\tan 50°}$

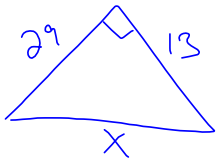
$Y = 29\text{m}$



$\tan 70° = \frac{35}{Z}$

$Z = \frac{35}{\tan 70°}$

$Z = 13\text{m}$



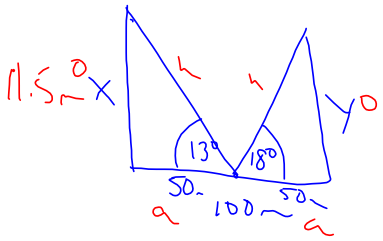
$X^2 = 29^2 + 13^2$

$X^2 = 1010$

$\sqrt{X^2} = \sqrt{1010}$

$X = 32\text{m}$

Ex: Jason is lying on the ground midway between two trees, 100 m apart. The angles of elevation of the tops of the trees are 13° and 18°. How much taller is one tree than the other? Give the answer to the nearest tenth of a metre.



SOH CAH TOA

$\tan 13° = \frac{X}{50}$

$X = 50\tan 13°$

$X = 11.5\text{m}$

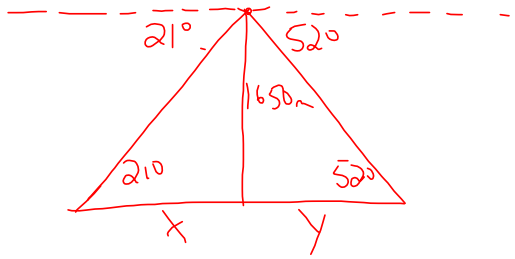
$\tan 18° = \frac{Y}{50}$

$Y = 50\tan 18°$

$Y = 16.2\text{m}$

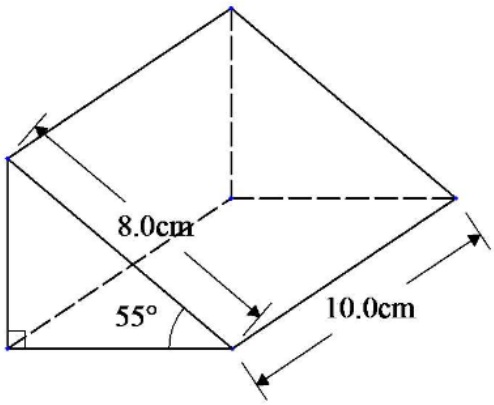
$16.2\text{m} - 11.5\text{m} = 4.7\text{m}$

Ex: From a small plane, V, the angle of depression of a sailboat is 21°. The angle of depression of a ferry on the other side of the plane is 52°. The plane is flying at an altitude of 1650 m. How far apart are the boats, to the nearest metre?

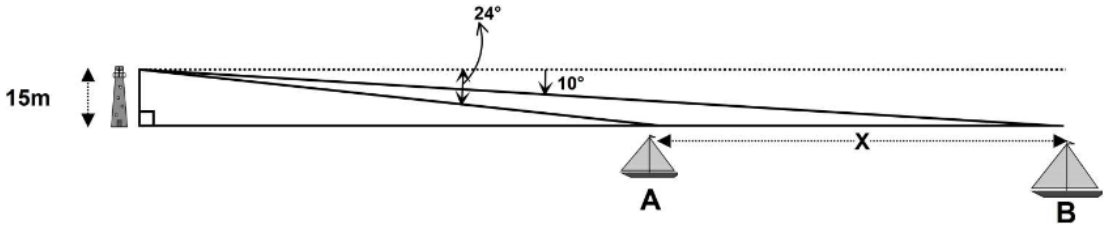


Ex: A lighthouse is located at the top of a cliff. From a point 150m offshore, the angle of elevation of the foot of the lighthouse is 25°, and the angle of elevation of the top of the lighthouse is 31°. Determine the height of the lighthouse to the nearest tenth of a metre.

Ex: Determine the surface area and volume of the following right triangular prism.



Ex: A lighthouse keeper spots two sailboats in distress. Sailboat A is observed at an angle of depression of 24° and sailboat B at an angle of depression of 10°. If the lighthouse is 15m above the water level, determine the distance, x in metres, between the two sailboats to the nearest metre.



HW: p118 #4, 5, 6, 8, 9, 11, 12, 14