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Part I: Multiple Choice. Write the correct answer in the space provided at the end of this section.

1. Which describes the graph of $y = 3(x + 4)^2 + 1$?

- (A) Vertex $(-4,1)$ and opens up.
- (B) Vertex $(-4,1)$ and opens down.
- (C) Vertex $(4,-1)$ and opens up.
- (D) Vertex $(4, -1)$ and opens down.

$a = 3 > 0$ opens up.
vertex $(-4, 1)$

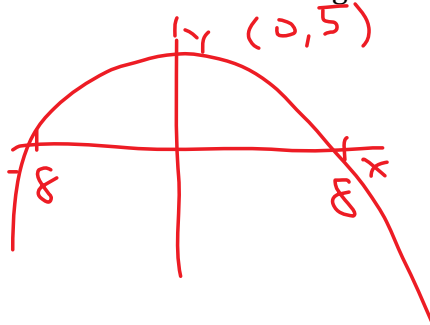
2. What is the range of the quadratic function $y = -2x^2 + 12x - 14$?

- (A) $\{y|y \leq -4, y \in R\}$
- (B) $\{y|y \geq -4, y \in R\}$
- (C) $\{y|y \leq 4, y \in R\}$
- (D) $\{y|y \geq 4, y \in R\}$

Final g .
 $p = -\frac{b}{2a} = \frac{-12}{2(-2)} = 3$
 $q = -2(3)^2 + 12(3) - 14 = 4$
 $a < 0$
so $y|y \leq 4$

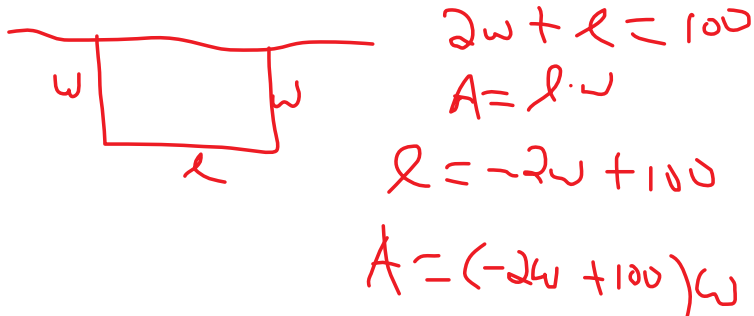
3. A quadratic function has a maximum height of 5 and x -intercepts at -8 and 8 . What is the vertex?

- (A) $(-5,0)$
- (B) $(0,-5)$
- (C) $(0,5)$
- (D) $(5,0)$



4. A lifeguard has 100 m of rope to enclose a rectangular swimming area. Which equation represents the maximum area of the enclosure if the lifeguard uses the beach as one side and the rope for the other three sides?

- (A) $A = w(50 - w)$
- (B) $A = w(50 - 2w)$
- (C) $A = w(100 - w)$
- (D) $A = w(100 - 2w)$



5. Which of the following quadratic functions has the most narrow parabola when compared to $y = x^2$?

(A) $y = \frac{4}{3}(x - 5)^2 - 2$

(B) $y = \frac{1}{2}(x - 5)^2 - 2$

(C) $y = \frac{2}{7}(x - 5)^2 - 2$

(D) $y = \frac{1}{5}(x - 5)^2 - 2$

The "larger" the a value,
the more narrow the graph.

6. Which function has axis of symmetry $x = 2$?

(A) $y = 3x^2 - 6x + 4$

~~(B) $y = 3x^2 + 6x + 4$~~

(C) $y = 3x^2 - 12x + 4$

~~(D) $y = 3x^2 + 12x + 4$~~

$p = -\frac{b}{2a}$
 $p = \frac{-(-12)}{2(3)} = 2$

7. The point $(2, 4)$ is on the graph of the quadratic equation $f(x) = -x^2 + bx + 12$. What is the value of b ?

(A) $-\frac{13}{2}$

(B) -6

(C) -2

(D) $\frac{3}{2}$

$4 = -(2)^2 + b(2) + 12 \rightarrow b = -2$
 $4 = -4 + 2b + 12$
 $4 + 4 - 12 = 2b$
 $-4 = \frac{2b}{2}$

8. Which value of c makes $x^2 - 7x + c$ a perfect square?

(A) $\frac{7}{2}$

(B) $\frac{49}{4}$

(C) 7

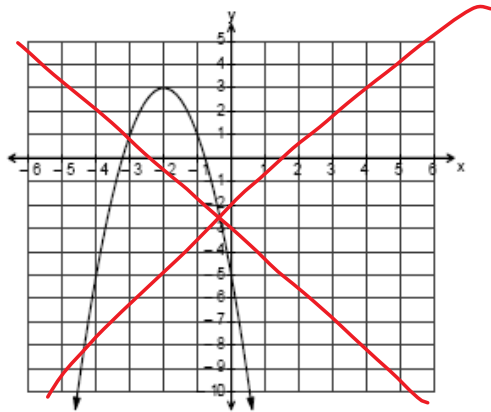
(D) 49

$c = \left(\frac{b}{2}\right)^2$
 $c = \left(\frac{-7}{2}\right)^2 = \frac{49}{4}$

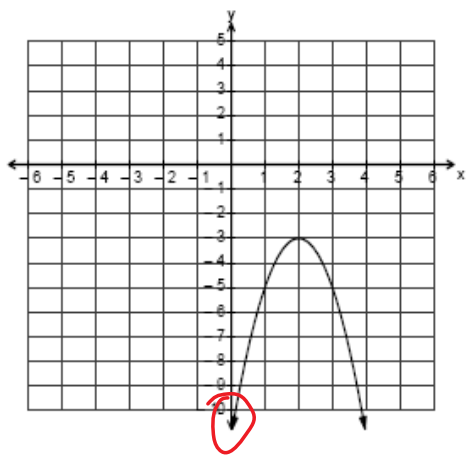
9. Which graph represents $y = -2(x - 2)^2 - 3$?

Vertex $(2, -3)$
 $y = -2(0 - 2)^2 - 3$
 $y = -2(4) - 3$
 $y = -8 - 3$
 $y = -11$

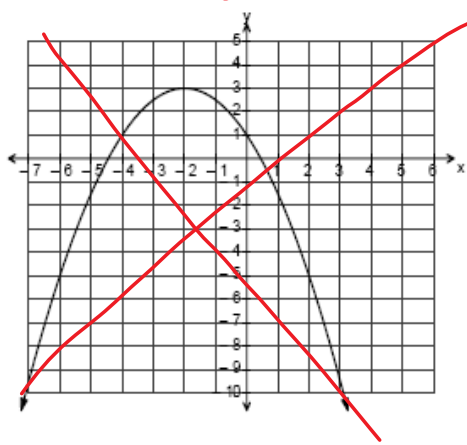
(A)



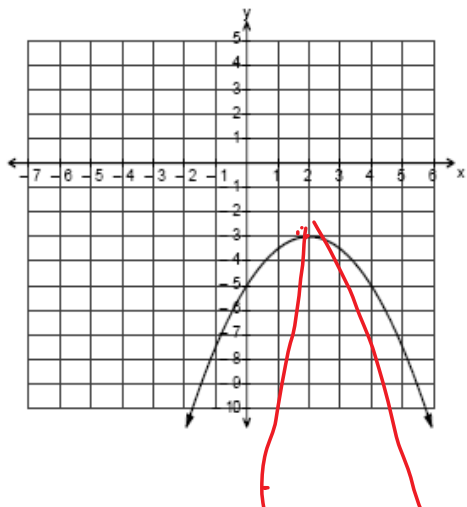
(B)



(C)



(D)



10. What is the value of a in the function $y = ax^2 - 48x + 12$, if the axis of symmetry for the graph of the function is $x = 6$?

- (A) -4
- (B) -1
- (C) 1
- (D) 4

Handwritten work for question 10:

$$p = -\frac{b}{2a}$$

$$\frac{12a}{12} = \frac{48}{12}$$

$$a = 4$$

Another handwritten calculation shows $\frac{6}{1} = \frac{-(-48)}{2a}$ with a red 'X' over the denominator $2a$.

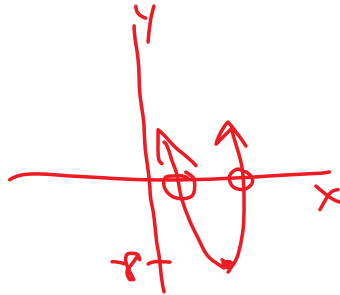
11. What is the domain of the quadratic function $f(x) = -3x^2 + 12x - 1$?

- (A) $\{x|x \in R\}$
- (B) $\{x|x \geq -1, x \in R\}$
- (C) $\{x|x \geq 2, x \in R\}$
- (D) $\{x|x \geq 11, x \in R\}$

Handwritten note: ← always.

12. A quadratic function has a minimum value at -8 and $a > 0$. How many x -intercepts are there?

- (A) 0
- (B) 1
- (C) 2
- (D) 3



13. What is the vertex form of $y = 3x^2 - 12x + 1$?

- (A) $y = 3(x - 2)^2 + 13$
- (B) $y = 3(x - 2)^2 - 11$
- (C) $y = 3(x + 2)^2 + 13$
- (D) $y = 3(x + 2)^2 + 11$

Handwritten work for question 13:

$$y = 3(x^2 - 4x + 4) + 1 - 12$$

$$y = 3(x - 2)^2 - 11$$

14. The quadratic function, $f(x) = -2x^2 + 8x - 5$, has an axis of symmetry of $x = 2$. $\leftarrow P$
What is the maximum value?

(A) -29

(B) -5

(C) 2

(D) 3

$$g = f(2) = -2(2)^2 + 8(2) - 5 \\ = 3$$

15. Jimmy sells makes and sells hacky sacks. The material for each hacky sack costs \$3.00 and he has been selling about 10 per day for \$8.00 each. He has been thinking about raising the price, so he takes a survey and finds that for every \$2 increase in price he would lose three sales a day. What equation could be used to determine the hacky sack price that results in the greatest revenue?

(A) $y = (5 - 2n)(10 + 3n)$

(B) $y = (5 + 2n)(10 - 3n)$

(C) $y = (5 - 3n)(10 + 2n)$

(D) $y = (5 + 3n)(10 - 2n)$

$$P = (\text{price})(\# \text{ sold})$$

$$P = (5 + 2n)(10 - 3n)$$

$$n = \# \text{ of } \$1 \text{ increments.}$$

Answers to multiple choice.

1. ___

2. ___

3. ___

4. ___

5. ___

6. ___

7. ___

8. ___

9. ___

10. ___

11. ___

12. ___

13. ___

14. ___

15. ___

Part II: Constructed Response. Answer each question in the space provided. Show all workings.

4

16. A toy rocket is launched into the air and reaches a maximum height of 80 m after a time of 4 seconds. If the rocket lands after 8 seconds, determine the quadratic function that describes the flight path of the rocket. Use the function to determine the height of the rocket at 6.5 seconds.

$$y = a(x-p)^2 + q$$

$$0 = a(0-4)^2 + 80$$

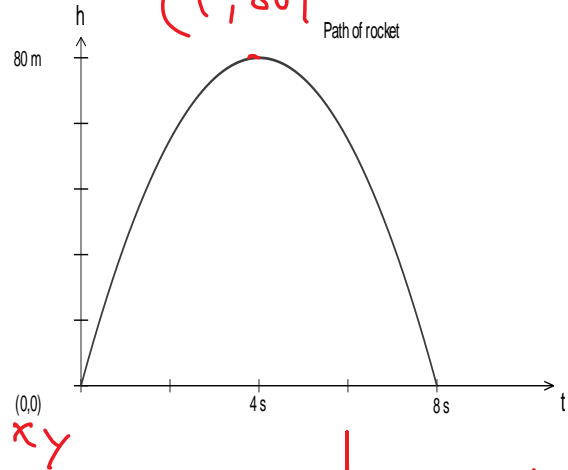
$$\frac{-80}{16} = \frac{16a}{16}$$

$$a = -5$$

$$y = -5(x-4)^2 + 80$$

$$y = -5(6.5-4)^2 + 80$$

$$y = 48.75$$



The rocket is 48.75 m high @ 6.5 s.

17. Determine the equation, in the form $y = a(x-p)^2 + q$, of the quadratic function that contains the points $(1, -9)$ and $(7, -9)$ and has a minimum value of -12 .

$$\frac{1+7}{2} = 4$$

$$-9 = a(1-4)^2 - 12$$

$$-9 + 12 = 9a$$

$$3 = 9a$$

$$\frac{3}{9} = \frac{9a}{9}$$

$$a = \frac{1}{3}$$

$$y = \frac{1}{3}(x-4)^2 - 12$$

3

18. A baseball is hit and follows a parabolic path described by the function $h(t) = -3t^2 + 12t + 1$, where t is time in seconds after the ball is hit and $h(t)$ is the height of the ball above ground in metres. Algebraically determine the maximum height reached by the ball and the time it takes the ball to reach its maximum height.

$$p = \frac{-b}{2a} = \frac{-12}{2(-3)} = 2$$

$$q = -3(2)^2 + 12(2) + 1 = 13$$

The ball is at a max. height of 13m @ 2s.

3

19. Convert the following function from standard form to vertex form by completing the square:

$$f(x) = -6x^2 - 48x + 29$$

$$y = -6(x^2 + 8x + 16) + 29 + 96$$

$$y = -6(x + 4)^2 + 125$$

- 4 20. If the points $(-2, 6)$ and $(1, -6)$ are on the graph of the quadratic function $f(x) = -3x^2 + bx + c$, determine the values of b and c .

$$\begin{aligned}
 6 &= -3(-2)^2 + b(-2) + c \\
 6 &= -12 - 2b + c \\
 \textcircled{1} \quad -2b + c &= 18 \\
 \hline
 -6 &= -3(1)^2 + b(1) + c \\
 -6 &= -3 + b + c \\
 \textcircled{2} \quad b + c &= -3 \\
 \hline
 \textcircled{1} \quad -2b + c &= 18 \\
 \textcircled{2} \quad b + c &= -3 \\
 \hline
 3b &= -21 \\
 b &= -7 \\
 \text{Sub } -7 \text{ into } \textcircled{1} \\
 -7 + c &= -3 \\
 c &= -3 + 7 \\
 c &= 4 \\
 \hline
 Y &= -3x^2 - 7x + 4
 \end{aligned}$$

- 4 21. A farmer uses 400 m of fencing to create a rectangular pig pen and to divide it into four regions of equal area as shown. Algebraically determine the function which gives the area of the pig pen as a function of its width, and state the dimensions that produce maximum area.

$$\begin{aligned}
 \textcircled{1} \quad 2l + 5w &= 400 \\
 \textcircled{2} \quad A &= l \cdot w
 \end{aligned}$$

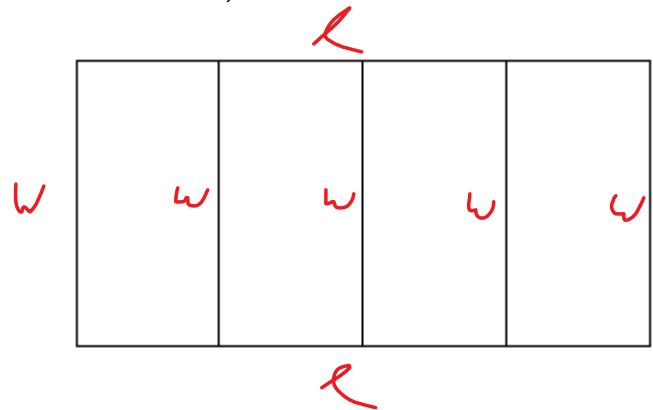
$$\frac{2l}{2} = \frac{-5w + 400}{2}$$

$$l = -2.5w + 200$$

$$A = (-2.5w + 200)w$$

$$A = -2.5w^2 + 200w$$

$$p = \frac{-b}{2a} = \frac{-200}{2(-2.5)} = 40$$



$$\text{So } w = 40\text{m}$$

$$l = -2.5(40) + 200$$

$$l = 100\text{m}$$

$$A = (40\text{m})(100\text{m}) = 4000\text{m}^2$$