

Part I: Multiple Choice. Place the correct answer in the corresponding blank at the end of this section.

1. What is the common difference in the arithmetic sequence 1, 7, 13, 19, ... ?

(A) -6

(B) 6

(C) 7

(D) 12

2. Which of the given formulas for the general term of the sequence -25, -15, -5, 5, 15, ... is correct?

(A)  $t_n = 10n - 15$

(B)  $t_n = 10n - 35$

(C)  $t_n = -10n - 35$

(D)  $t_n = -10n - 15$

$$t_n = t_1 + (n-1)d$$

$$t_n = -25 + (n-1)10$$

$$t_n = -25 + 10n - 10$$

$$t_n = 10n - 35$$

or

$$y = mx + b$$

$$t_n = 10n - 35$$

3. What is the sum of the series  $(-4) + (-3) + (-2) \dots + (3)$  ?

(A) -8

(B) -4

(C) 0

(D) 48

$$S_n = \frac{n}{2} [t_1 + t_n] \quad S_8 = \frac{8}{2} [-4 + 3]$$

$$n = 8$$

$$S_8 = -4$$

4. What is the common ratio for the geometric sequence  $2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \dots$  ?

(A) -4

(B)  $-\frac{1}{4}$

(C)  $\frac{1}{4}$

(D) 4

$$r = \frac{\frac{1}{2}}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

5. How many terms are in the sequence 2, 10, 50, 250, 1250, ..., 156 250 ?

- (A) 6  $r = \frac{10}{2} = 5$   
 (B) 7  $t_n = t_1 \cdot r^{n-1}$   
 (C) 8  $156250 = \frac{2 \cdot 5^{n-1}}{2}$   
 (D) 9  $78125 = 5^{n-1}$   
 $5^7 = 5^{n-1}$   
 $7 = n-1$   
 $n = 8$

6. What is the sum of the geometric series  $6 + 30 + 150 + \dots + 3750$  ?

- (A) 749  $r = \frac{30}{6} = 5$   
 (B) 938  $t_n = t_1 \cdot r^{n-1}$   
 (C) 4686  $3750 = \frac{6 \cdot 5^{n-1}}{6}$   
 (D) 4688  $625 = 5^{n-1}$   
 $5^4 = 5^{n-1}$   
 $n = 5$   
 $S_5 = \frac{6(5^5 - 1)}{5 - 1} = 4686$

7. In an arithmetic sequence,  $t_3 = m$  and  $t_4 = n$ . Which expression represents  $t_6$ ?

- (A)  $2m - n$   
 (B)  $2n - m$   
 (C)  $3n - m$   
 (D)  $3n - 2m$   
 $t_3 = m$   
 $t_4 = n$   
 $t_5 = n + n - m = 2n - m$   
 $t_6 = 2n - m + n - m = 3n - 2m$

8. The sum of an infinite geometric series is 152 and its common ratio is  $\frac{3}{4}$ . What is the first term of the series?

- (A)  $\frac{3}{4}$   
 (B) 38  $S_\infty = \frac{t_1}{1-r}$   
 (C) 114  $152 = \frac{t_1}{1-\frac{3}{4}}$   
 (D)  $\frac{608}{3}$   
 $152 = \frac{t_1}{1-\frac{3}{4}}$   
 $152 = \frac{t_1 \cdot \frac{1}{4}}{\frac{1}{4}}$   
 $t_1 = 38$

9. Which of the following best describes the series  $-34 - 17 - \frac{17}{2} - \frac{17}{4} - \dots$  ?

- (A) The series is divergent and has no sum.  
 (B) The series is convergent and has a sum of  $-68$ .  
~~(C) The series is convergent and has no sum.~~  
~~(D) The series is divergent and has a sum of  $-68$ .~~  
 $r = \frac{-17}{-34} = \frac{1}{2}$   
 $-1 < \frac{1}{2} < 1$

10. What are the first three terms of the sequence given by  $t_n = 4\left(\frac{1}{8}\right)^{n-1}$ ?

(A)  $\frac{1}{2}, \frac{1}{16}, \frac{1}{128}$

(B)  $4, \frac{1}{2}, \frac{1}{16}$

(C) 4, 16, 64

(D)  $4, \frac{4}{7}, \frac{4}{49}$

$4, \frac{1}{2}, \frac{1}{16}$

**Answers to multiple choice.**

1.\_\_\_\_ 2.\_\_\_\_ 3.\_\_\_\_ 4.\_\_\_\_ 5.\_\_\_\_

6.\_\_\_\_ 7.\_\_\_\_ 8.\_\_\_\_ 9.\_\_\_\_ 10.\_\_\_\_

**Part II: Constructed Response. Answer each question in the space provided. Show all workings.**

11. Algebraically determine the number of terms in the geometric series,  $\frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \dots + 2187$ , and find the sum of the series.

$r = \frac{\frac{1}{27}}{\frac{1}{81}} = \frac{1}{27} \cdot \frac{81}{1} = 3$

$t_1 = \frac{1}{81}$

$t_n = t_1 \cdot r^{n-1}$

$81 \cdot 2187 = \frac{1}{81} \cdot 3^{n-1} \cdot 81$

$177147 = 3^{n-1}$

$3^{11} = 3^{n-1}$

$n = 12$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$S_{12} = \frac{\frac{1}{81}(3^{12} - 1)}{3 - 1}$

$S_{12} = \frac{531440}{81} \cdot \frac{1}{2}$

$S_{12} = \frac{531440}{81} \cdot \frac{1}{2}$

$S_{12} = \frac{265720}{81}$

12. The first three terms of a geometric sequence are  $x - 1, 2x, 3x + 9, \dots$ . Algebraically determine the value of  $x$ .

$$r = \frac{2x}{x-1}, \quad r = \frac{3x+9}{2x}$$

$$\frac{2x}{x-1} = \frac{3x+9}{2x}$$

$$4x^2 = 3x^2 + 6x - 9$$

$$4x^2 - 3x^2 - 6x + 9 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$x = 3$$

13. The first three terms of an arithmetic sequence are  $x + 4, 5x + 1, 7x + 4, \dots$ . Algebraically determine the value of  $x$  and state the common difference.

$$\begin{aligned} d &= 5x + 1 - (x + 4) \\ &= 5x + 1 - x - 4 \\ &= 4x - 3 \end{aligned}$$

$$\begin{aligned} d &= 7x + 4 - (5x + 1) \\ &= 7x + 4 - 5x - 1 \\ &= 2x + 3 \end{aligned}$$

$$4x - 3 = 2x + 3$$

$$4x - 2x = 3 + 3$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

$$d = 2(3) + 3$$

$$d = 9$$

14. The monthly production of crude oil, in barrels, for the first four months for a test well at Hebron is given below. In theory, what is the expected lifetime production of the well, to the nearest barrel?

Month	# of Barrels
1	40 000
2	34 000
3	28 900
4	24 565

$$r = \frac{34000}{40000} = 0.85$$

$$r = \frac{28900}{34000} = 0.85$$

$$- 1 < 0.85 < 1$$

$\therefore$  convergent

$$S_{\infty} = \frac{a_1}{1-r}$$

$$S_{\infty} = \frac{40000}{1-0.85}$$

$$S_{\infty} = 266666.666666\dots$$

$$S_{\infty} = 266667$$