Math 2200 **Maximum/Minimum Problems**

1. Two numbers have a difference of 8. Find the numbers if their product is a minimum.

① $x - y = 8$

② $M = xy$

Solve ① for x

$x \; y + 8$

Sub ① into ②

$M = (y+8)y$

$M = y^2 + 8y$

$P \; \frac{-b}{2a} = \frac{-8}{2(1)} = -4$

$y = -4$

$x = y - 8$

$x = 4 + 8$

$x = 4$

2. The sum of two numbers is 12. If their product is a maximum, find the numbers.

① $x + y = 12$

② $M = xy$

Solve ① for x

$x = -y + 12$

Sub ① into ②

$M = (-y + 12)y$

$M = -y^2 + 12y$

$P \; \frac{b}{2a}$

$P = \frac{12}{2(-1)} = \frac{-12}{2} = 6$

$y = 6$

$x = -y + 12$

$x \; -6 + 12$

$x \; 6$

3. Two numbers differ by 20. Find the numbers if the sum of their squares is a minimum.

① $x \; y = 20$

② $M \; x^2 + y^2$

Solve ① for x

$x = y + 20$

Sub ① into ②

$M \; (y+20)^2 + y^2$

$M = (y+20)(y+20) + y^2$

$M = y^2 + 20y + 20y + 400 + y^2$

$M = 2y^2 - 40y + 400$

$P = \frac{-b}{2a} = \frac{40}{2(2)} = -10$

$y = 10$

$x \; y + 20$

$x = -10 + 20$

$x = 10$

4. On a forward somersault , Greg's height above the water is given by $h = -5t^2 + 6t + 3$, where t is time in seconds and h is height in meters.

(A) Find Greg's maximum height above the water. ← do this second, once x is found

$q = -5(0.6)^2 + 6(0.6) + 3$

$q = 4.8\,m$

(B) How long does it take him to reach that maximum height? ← do this first!

$P = \frac{-b}{2a} = \frac{-6}{2(-5)} = 0.6\,s$

(C) How high is the diving board?

$h = -5t^2 + 6t + 3$ ← y-intercept, height of diving board: 3m

(D) What is his height after 1.5 seconds?

$h = -5(1.5)^2 + 6(1.5) + 3 = 0.75\,m$

5. The power P watts supplied to a circuit by a 9 volt battery is given by the formula $P = 9I - 0.5I^2$ where I is the current in amperes.

(A) For what value of the current will the power be a maximum?

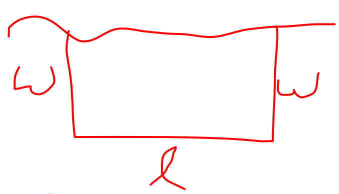
$P = -0.5I^2 + 9I$ ← typical form

$P = \frac{-b}{2a} = \frac{-9}{2(-0.5)} = 9$ amps

(B) What is the maximum power?

$q = -0.5(9)^2 + 9(9) = 40.5$ watts

6. A rectangular lot is bounded on one side by a river and on the other three sides by 80 m of fencing. Find the dimensions that will enclose the maximum area.



① $l + 2w = 80$

② $A = lw$

Solve ① for l

$l = -2w + 80$

Sub ① into ②

$A = (-2w + 80)w$

$A = -2w^2 + 80w$

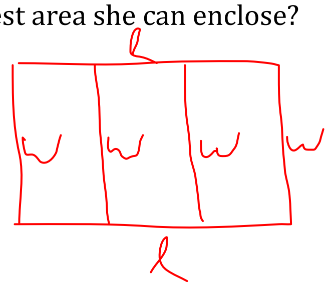
P $\frac{-b}{2a} = \frac{-80}{2(-2)} = 20\text{m}$

$w = 20\text{m}$

$l = -2(20) + 80 = 40\text{m}$

The lot is $20\text{m} \times 40\text{m}$

7. A lifeguard marks off a rectangular swimming area at a lake with 200 m of rope. She then divides the swimming area into three sections for beginner, intermediate and advanced swimmers. What is the greatest area she can enclose?



① $2l + 4w = 200$

② $A = lw$

Solve ① for l

$2l = -4w + 200$

$\frac{2l}{2} = \frac{-4w}{2} + \frac{200}{2}$

$l = -2w + 100$

Sub ① into ②

$A = (-2w + 100)w$

$A = -2w^2 + 100w$

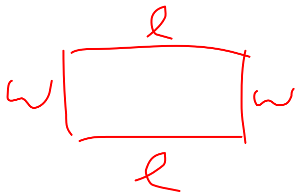
P $\frac{-b}{2a} = \frac{-100}{2(-2)} = 25$

$w = 25\text{m}$

$y = -2(25)^2 + 100(25) = 1250$

Largest area enclosed is 1250m^2

8. 80 m of fencing are available to enclose a rectangular play area. What dimensions will yield the maximum area? What is the maximum area?



① $2l+2w=80$

② $A=l\cdot w$

Solve ① for w:

$2w=-2l+80$

$\frac{2w}{2}=\frac{-2l+80}{2}$

$w=-l+40$

Sub ① into ②

$A=l(-l+40)$

$A=-l^2+40l$

$p=\frac{-b}{2a}=\frac{-40}{2(-1)}=20$

$\therefore l=20\text{m}$

$w=-20+40=20\text{m}$

$q=-(20)^2+40(20)=400\text{m}^2$

or $A=l\cdot w=(20\text{m})(20\text{m})=400\text{m}^2$

9. A producer of synfuel from coal estimates that the cost C dollars per barrel for a production run of x thousand barrels is given by $C = 9x^2 - 180x + 940$. How many thousand barrels should be produced each run to keep the cost per barrel at a minimum? What is the minimum cost per barrel of synfuel?

$p=\frac{-b}{2a}=\frac{-(-180)}{2(9)}=10$

$\therefore x=10$ thousand barrels

$q=9(10)^2-180(10)+940$

$q=40$

10 000 barrels should be produced to minimize the price of oil at \$40/barrel.

10. Two numbers have a difference of 24. Find the numbers if the result of adding their sum and their product is a minimum.

① $X - Y = 24$

Sum: $X + Y$

product: $X \cdot Y$

② $M = X + Y + XY$

Solve ① for X:

$X = Y + 24$

Sub ① into ②

$M = (Y+24) + Y + (Y+24)Y$

$M = 2Y + 24 + Y^2 + 24Y$

$M = Y^2 + 26Y + 24$

$P = \frac{-b}{2a} = \frac{-26}{2(1)} = -13$

$\therefore Y = -13$

$X = -13 + 24 = 11$

11. A local restaurant averages 200 customers per day who spend \$30 per meal. The manager estimates a loss of 10 customers per day for each \$3 increase in meal price. If the average cost to prepare each meal is \$12, write a quadratic function to model the daily profit and use it to determine the meal price that will maximize the profit.

$R = (18)(200)$

Let n be # \$1 increments

$R = (18 + 3n)(200 - 10n)$

$R = 3600 - 180n + 600n - 30n^2$

$R = -30n^2 + 420n + 3600$

$P = \frac{-b}{2a} = \frac{-420}{2(-30)} = 7$

$\therefore n = \$7$

$g = -30(7)^2 + 420(7) + 3600$

$g = 5070$

A meal price of \$37 will maximize profit at \$5070/day.