Math 3201 Exponential Constructed Response Problems Name: _______________

1. A radioactive isotope decays at the rate described by the function $A(t) = 600\left(\frac{1}{2}\right)^{\frac{t}{1500}}$, where t is the time in years and $A(t)$ is the amount of isotope remaining in grams.

(A) Determine the initial mass of the isotope.

600g

(B) How long will it take for the isotope to be reduced to half of its original amount?

1500 years

(C) What is the mass of the isotope after 3000 years?

150g

$$A(t) = 600\left(\frac{1}{2}\right)^{\frac{3000}{1500}}$$
$$= 600(1/2)^2$$
$$= 150g$$

2. The initial concentration of bacteria in a Petri dish was 400 mg/mm². The growth of the bacteria population can be modeled by the function $P(t) = 400(3)^{\frac{t}{15}}$, where t represents time in minutes. How long does it take the population to reach 10 800?

$t = ?$

$P(t) = 10\,800$

$$\frac{10800}{400} = \frac{\cancel{400}(3)^{\frac{t}{15}}}{\cancel{400}} \rightarrow t = 45$$
$$27 = 3^{\frac{t}{15}}$$
$$3^3 = 3^{\frac{t}{15}}$$
$$15 \cdot 3 = \frac{t}{\cancel{15}} \cdot \cancel{15}$$

or $\frac{3}{1} = \frac{t}{15}$

$t = 45$

3. Sharon purchased a house for \$92 500, with the foresight that the market would double every 10 years. Using the function, $A(t) = 92\,500(2)^{\frac{t}{10}}$, determine the time it would take for Sharon's home to be worth \$370 000.

$t = ?$

$A(t) = 370000$

$$\frac{370\,000}{92500} = \frac{\cancel{92500}(2)^{\frac{t}{10}}}{\cancel{92500}}$$

$$4 = 2^{\frac{t}{10}}$$

$$2^2 = 2^{\frac{t}{10}}$$

$$\frac{2}{1} = \frac{t}{10}$$

$$t = 20$$

4. Rick has been prescribed a medication that remains in his bloodstream for a specified period of time after each ~~does~~ dose. The directions on the bottle indicate that the concentration of the prescription has a half-life of 4 hours. If the situation is modeled by the function $A(t) = 40\left(\frac{1}{2}\right)^{\frac{t}{4}}$, where t is time in hours and *A(t)* is amount in mg/cm^3, how many hours will it take for the medication to reduce to a concentration of 5 mg/cm^3?

$t = ?$

$A(t) = 5$

$$\frac{5}{40} = \frac{40\left(\frac{1}{2}\right)^{\frac{t}{4}}}{40}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\frac{3}{1} = \frac{t}{4}$$

$$t = 12$$

5. The population of a specific type of bacteria growing in a Petri dish is modelled by the function $P(t) = 700(5)^{\frac{t}{6}}$, where P(t) represents then number of bacteria and t represents the time, in hours, after the initial count.

Determine the value of t when $P(t) = 87\,500$. What does your answer mean in this context?

$t = ?$

$$\frac{87500}{700} = \frac{700(5)^{\frac{t}{6}}}{700}$$

$$125 = 5^{\frac{t}{6}}$$

$$5^3 = 5^{\frac{t}{6}}$$

$$\frac{3}{1} = \frac{t}{6}$$

$$t = 18$$

6. Algebraically solve for x: $9^{2x-1} = \sqrt[4]{27}$

$3^{2(2x-1)} = 3^{3\left(\frac{1}{4}\right)}$

$4(4x-2) = \frac{3}{\cancel{4}} \cdot \cancel{4}$

$16x-8=3$

$16x=3+8$

$16x=11$

$x=11/16$

7. Algebraically solve for x: $\left(\frac{1}{3}\right)^{x-2} = \sqrt[3]{9}$

$3^{-1(x-2)} = 3^{2\left(\frac{1}{3}\right)}$

$3^{-x+2} = 3^{\frac{2}{3}}$

$3(-x+2) = \frac{2}{\cancel{3}} \cdot \cancel{3}$

$-3x+6=2$

$-3x=2-6$

$-3x=-4$

$x=-4/-3$

$x=4/3$

8. Algebraically solve for x: $(2^{x-3})(8^{x}) = 32$

$(2^{x-3})(2^{3x}) = 2^5$

$2^{x-3+3x} = 2^5$

$2^{4x-3} = 2^5$

$4x-3=5$

$4x=5+3$

$\frac{4x}{4}=\frac{8}{4}$

$x=2$