

Part I: Multiple Choice. Place the correct answer in the corresponding blank at the end of this section.

1. What is $\frac{3\pi}{10}$ radians in degrees? $\frac{3\pi}{10} \cdot \frac{180^\circ}{\pi} = 54^\circ$
- (A) 30° (B) 54° (C) 108° (D) 600°
2. What is $\frac{5\pi}{3}$ radians in degrees? $\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$
- (A) 6° (B) 180° (C) 300° (D) 600°
3. What is 2.4 radians in degrees? $2.4 \times \frac{180^\circ}{\pi} = 138^\circ$
- (A) 24° (B) 138° (C) 275° (D) 432°
4. What is 240° in radians? $240^\circ \times \frac{\pi}{180^\circ} = \frac{4\pi}{3}$
- (A) $\frac{3\pi}{2}$ (B) 2π (C) 3π (D) $\frac{4\pi}{3}$
5. What is 230° in radians? $230^\circ \times \frac{\pi}{180^\circ} = 4.0$
- (A) 0.4 (B) 1.3 (C) 4.0 (D) 722.6
6. What is the domain of the function $y = \sin x$?
- (A) $\{x \mid x > 0, x \in R\}$ (B) $\{x \mid x \geq 0, x \in R\}$
 (C) $\{x \mid x \in R\}$ (D) $\{x \mid x \geq 1, x \in R\}$
7. What is the domain of the function $y = 2 \cos x - 3$?
- (A) $\{x \mid x \geq 2, x \in R\}$ (B) $\{x \mid x \geq -3, x \in R\}$
 (B) $\{x \mid x \geq 0, x \in R\}$ (D) $\{x \mid x \in R\}$

8. What is the range of $y = \cos x$?

(A) $\{y \mid y \in \mathbb{R}\}$

(C) $\{y \mid -1 \geq y \geq 1, y \in \mathbb{R}\}$

(B) $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

(D) $\{y \mid y \geq 0\}$

9. What is the range of $y = \sin x$?

(A) $\{y \mid y \in \mathbb{R}\}$

(C) $\{y \mid -1 \geq y \geq 1, y \in \mathbb{R}\}$

(B) $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

(D) $\{y \mid y \geq 0\}$

10. Which is an x -int of $y = \cos x$?

(A) 0°

(B) 90°

(C) 180°

(D) 360°

11. Which is an x -int of $y = \sin x$?

(A) 0°

(B) 90°

(C) 270°

(D) 300°

12. How can the graph of $y = \cos x$ be translated so that we get the graph of $y = \sin x$?

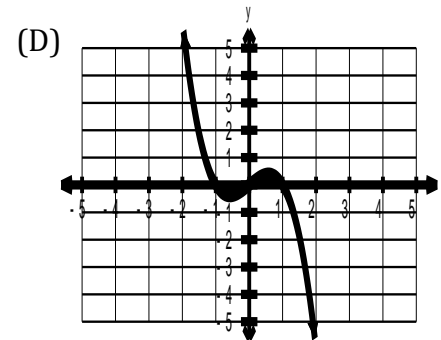
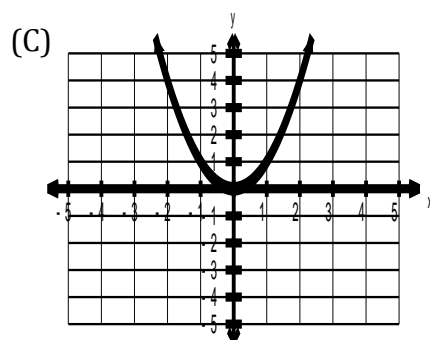
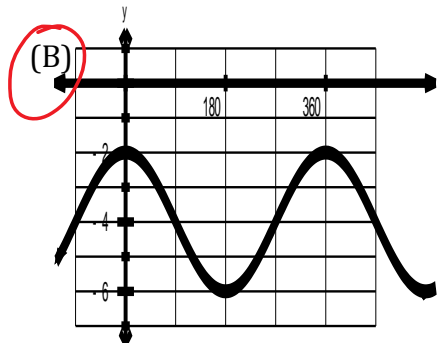
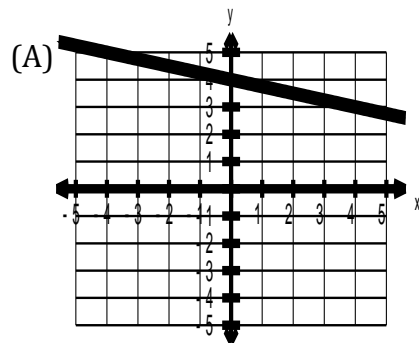
(A) 90° to the left

(C) 45° to the left

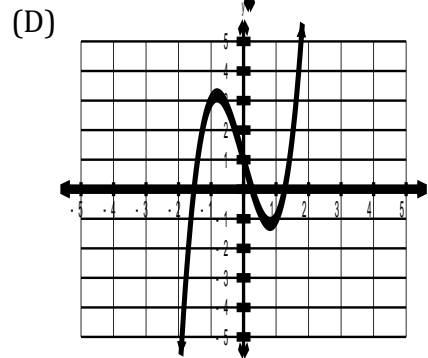
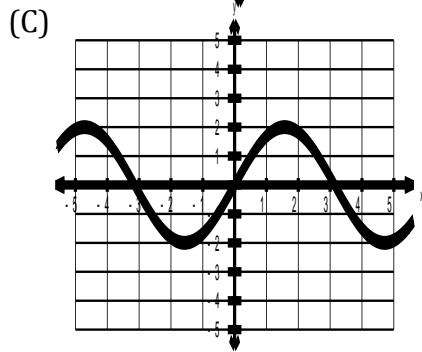
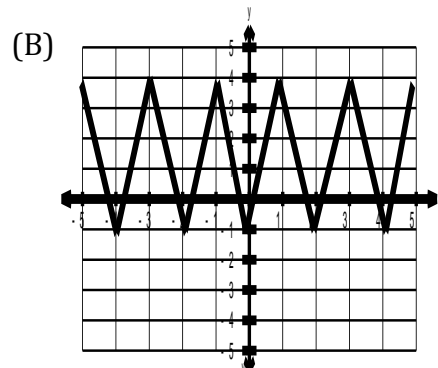
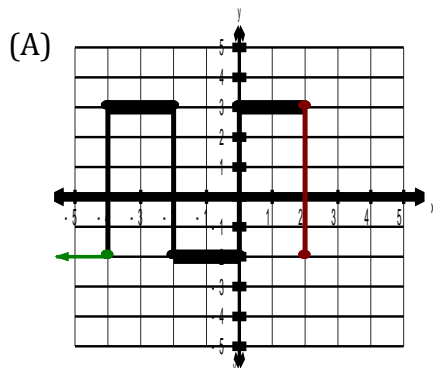
(B) 90° to the right

(D) 45° to the right

13. Which graph is periodic?



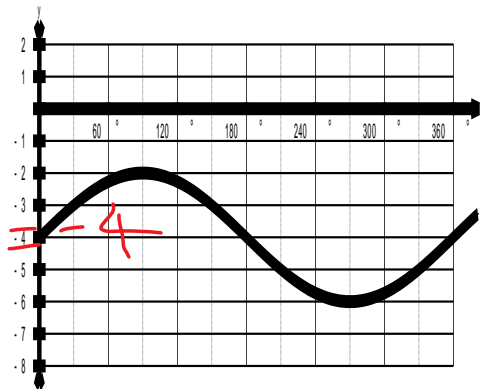
14. Which graph is sinusoidal?



15. What is the equation of the midline for the graph shown?

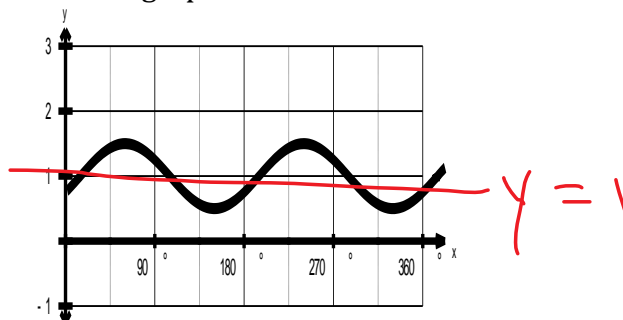
- (A) $y = -2$
- (B) $y = -3$
- (C) $y = -4$
- (D) $y = -6$

$$\frac{\text{max} + \text{min}}{2} = \frac{-2 + (-6)}{2} = -4$$



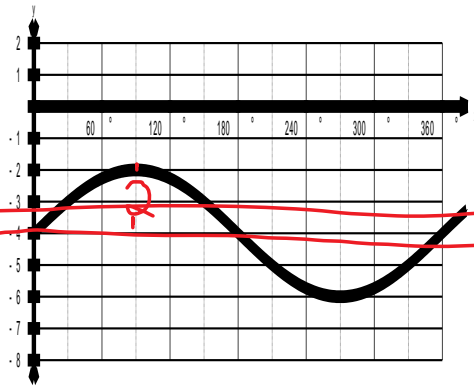
16. What is the equation of the midline for the graph shown?

- (A) $y = 1.5$
- (B) $y = 1$
- (C) $y = 0.75$
- (D) $y = 0.5$



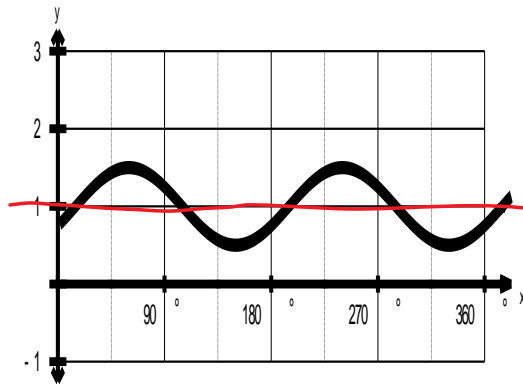
17. What is the amplitude for the graph shown?

- (A) -4
- (B) -2
- (C) 2
- (D) 4



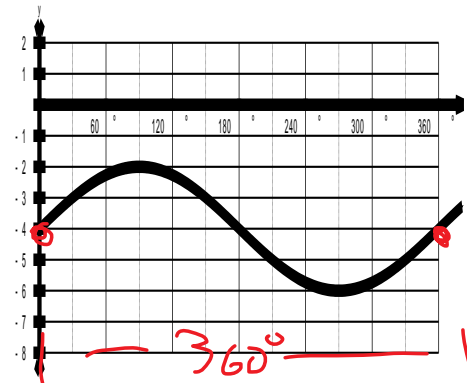
18. What is the amplitude for the graph shown?

- (A) -0.5
- (B) 0.75
- (C) 0.5
- (D) 1



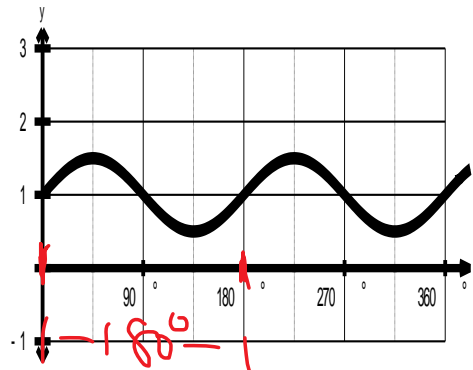
19. What is the period for the graph shown?

- (A) 90°
- (B) 180°
- (C) 270°
- (D) 360°

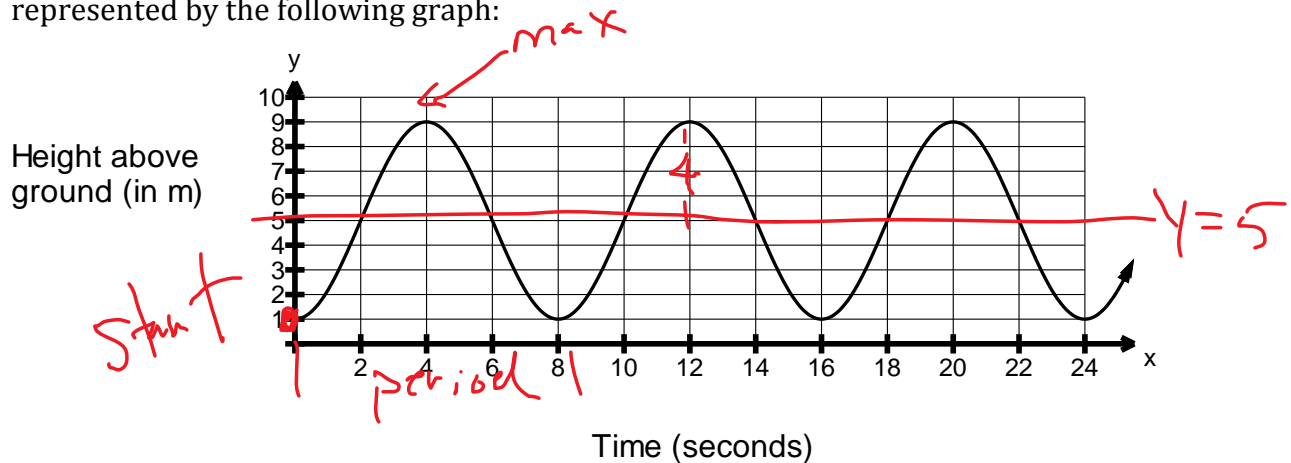


20. What is the period for the graph shown?

- (A) 45°
- (B) 90°
- (C) 135°
- (D) 180°



While riding a Ferris wheel, Lily's height above the ground in terms of time can be represented by the following graph:



Use the graph to answer #21-#25

21. At what height does Lily board the Ferris Wheel?

- (A) 1 m (B) 4 m (C) 5 m (D) 9 m

22. What is the maximum height above the ground?

- (A) 1 m (B) 4 m (C) 5 m (D) 9 m

23. What is the height of the axle of the Ferris Wheel? *Same as midline: 5*

- (A) 1 m (B) 4 m (C) 5 m (D) 9 m

24. What is the radius of the Ferris Wheel? *Same as amplitude: 4*

- (A) 1 m (B) 4 m (C) 5 m (D) 9 m

25. What length of time does it take for the Ferris Wheel to complete 1 rotation? *Same as period*

- (A) 2 seconds (B) 4 seconds (C) 6 seconds (D) 8 seconds

26. What is the amplitude of $y = 3\sin^4(x - 30^\circ) + 2$?

- (A) 2 (B) 3 (C) 4 (D) 30°

27. What is the amplitude of $y = 0.5\cos(3x - 1.2) - 4$?

- (A) 0.5 (B) 1.2 (C) 3 (D) 4

28. What is the equation of the midline of $y = 4 \sin 2(x - 2.4) + 5$?
 (A) $y = 2$ (B) $y = 2.4$ (C) $y = 4$ (D) $y = 5$
29. What is the equation of the midline of $y = 0.75 \cos 0.5(x + 15^\circ) - 2$?
 (A) $y = -2$ (B) $y = 0.5$ (C) 0.75 (D) $y = 15$
30. What is the period of $y = 3 \sin 4(x - 2) + 1$?
 (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 4π (D) 8π
period = $\frac{2\pi}{4} = \frac{\pi}{2}$
31. What is the period of $y = 4 \sin 0.5x - 1$?
 (A) 90° (B) 180° (C) 360° (D) 720°
period = $\frac{360^\circ}{0.5} = 720^\circ$
32. What is the period of $y = 2 \sin 3\left(x - \frac{\pi}{4}\right)$?
 (A) $\frac{\pi}{4}$ (B) $\frac{2\pi}{3}$ (C) π (D) 6π
period = $\frac{2\pi}{3}$
33. What is the phase shift of $y = 4 \sin 2(x - 2.4) + 5$?
 (A) 2.4 radians to the right (B) 2.4 radians to the left
 (C) 5 radians to the right (D) 5 radians to the left
opposite to sign
34. What is the phase shift of $y = 0.75 \cos 0.5(x + 15^\circ) - 2$?
 (A) 15° to the right (B) 15° to the left
 (C) 2 to the right (D) 2 to the left
opposite to sign
35. What is the maximum value of $y = 3 \sin 4(x - 30^\circ) + 2$?
 (A) 1 (B) 5 (C) 6 (D) 7
max = d + a = 2 + 3 = 5
36. What is the minimum value of $y = 0.75 \cos 0.5(x + 15^\circ) - 2$?
 (A) -2.75 (B) -2.5 (C) -2 (D) -1.25
min = d - a = -2 - 0.75 = -2.75

37. What is the maximum value of $y = 4 \cos 0.5(x-1) + 7$? $d+a = 7 + 4 = 11$
 (A) 3 (B) 4 (C) 8 (D) 11

38. What is the minimum value of $y = 1.5 \sin 5\left(x - \frac{\pi}{6}\right) - 4$ $d-a = -4 - 1.5 = -5.5$
 (A) -5.5 (B) -2.5 (C) 1 (D) 6.5

39. What is the range of $y = 2 \sin 4\left(x - \frac{\pi}{4}\right) - 1$?
 $\text{max: } d+a = -1+2 = 1$
 $\text{min: } d-a = -1-2 = -3$
 $\therefore \{y \mid -3 \leq y \leq 1, y \in \mathbb{R}\}$

(A) $\{y \mid -5 \leq y \leq 3, y \in \mathbb{R}\}$ (B) $\{y \mid -3 \leq y \leq 1, y \in \mathbb{R}\}$
 (C) $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$ (D) $\{y \in \mathbb{R}\}$

40. What is the range of $y = 2.5 \cos 3(x-2.4) + 3.5$?
 $\text{max: } d+a = 3.5+2.5 = 6$
 $\text{min: } d-a = 3.5-2.5 = 1$
 $\therefore \{y \mid 1 \leq y \leq 6, y \in \mathbb{R}\}$

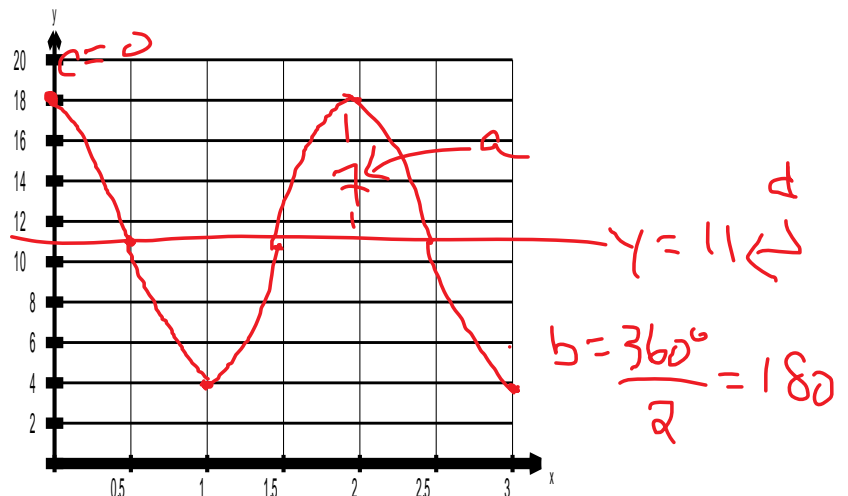
(A) $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$ (B) $\{y \mid 1 \leq y \leq 6, y \in \mathbb{R}\}$
 (C) $\{y \mid 0.5 \leq y \leq 6.5, y \in \mathbb{R}\}$ (D) $\{y \in \mathbb{R}\}$

Part II: Constructed Response. Answer each question in the space provided. Show all workings.

41. The table below shows the height of a bicycle pedal over time:

Time (s)	Height (in.)
0	18
0.5	11
1.0	4
1.5	11
2.0	18
2.5	11
3.0	4

(a) Graph the function on the grid below:



(b) What is the height of the axle of the bicycle pedal?

11 in

(c) What is the length of time taken to complete one rotation?

25

(d) Suppose another cyclist pedals the same bike at a rate of 1 revolution per second. How would the graph change?

Smaller period (15)

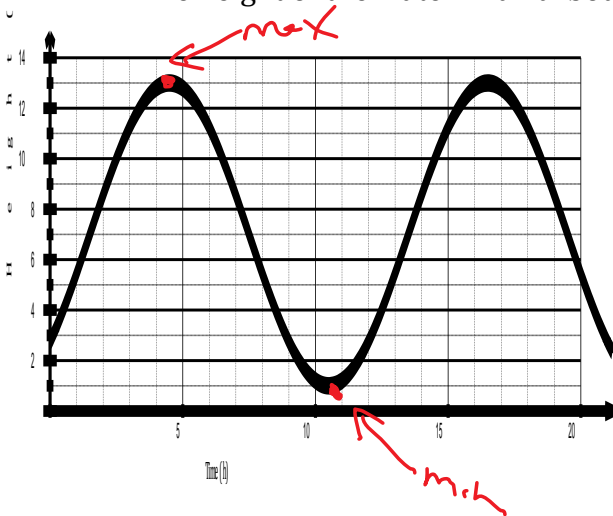
(e) Write the equation of the function in the form $y = a \cos bx + d$

$a = 7$ $c = 0$

$b = 180$ $d = 11$

$y = 7 \cos 180x + 11$

42. The height of the water in a harbour since 12 PM is shown in the graph below:



(a) What is the maximum height of the water?

13 ft

(b) What is the minimum height of the water?

1 ft

(c) What is the length of time between low tide and high tide?

6 h

(d) John wants to go collect mussels (the tide must be relatively low for this). He thinks that 6:00 PM would be a good time to go. Do you agree or disagree with John? Justify.

No. Tide is high.

43. The height of water in a wave pool oscillated between a maximum of 13 ft and a minimum of 5 ft. The wave generator pumps 6 waves per minute.

(a) What is the equation of the midline?

$$\frac{\text{max} + \text{min}}{2} = \frac{13 + 5}{2} = 9 \quad y = 9$$

(b) What is the amplitude?

$$13 - 9 = 4$$

(c) What is the period?

$$\frac{360^\circ}{6} = 60$$

(d) Write the equation of the function in the form $y = a \sin bx + d$

$$y = 4 \sin 60x + 9$$

44. The equation $y = 70.5 + 19.5 \sin \frac{\pi}{6}(t - 4)$ models the average monthly temperature for Phoenix, Arizona, in degrees Fahrenheit. In this equation, t denotes the number of months, with $t = 1$ representing January. What is the average monthly temperature for July?

$$y = 70.5 + 19.5 \sin \left[\frac{\pi}{6}(7 - 4) \right] = 90^\circ \text{ F}$$

(change calculator to radian mode)

45. A horse on a carousel goes up and down as the carousel goes round and round. The height of the horse in inches, h , as a function of time (s) is given by

$$h = 9 \sin \frac{2\pi}{7}t + 46 \quad \text{change calculator to radians.}$$

(a) Determine the height of the horse after 8 seconds.

$$h = 9 \sin \left[\frac{2\pi}{7}(8) \right] + 46 = 53 \text{ inches}$$

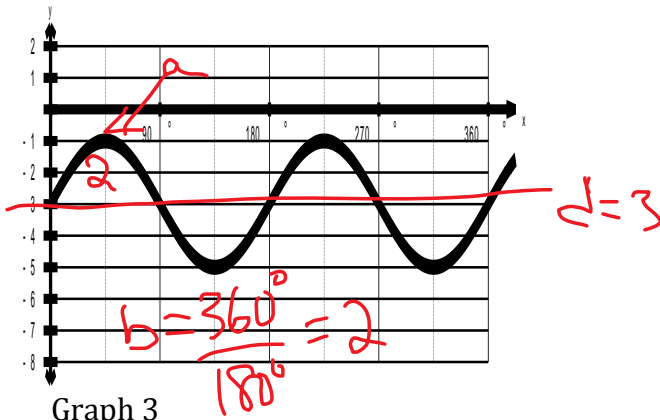
(b) The children have been complaining that the carousel is too slow. How would the equation change the conductor makes the carousel go around faster?

The period would be smaller.

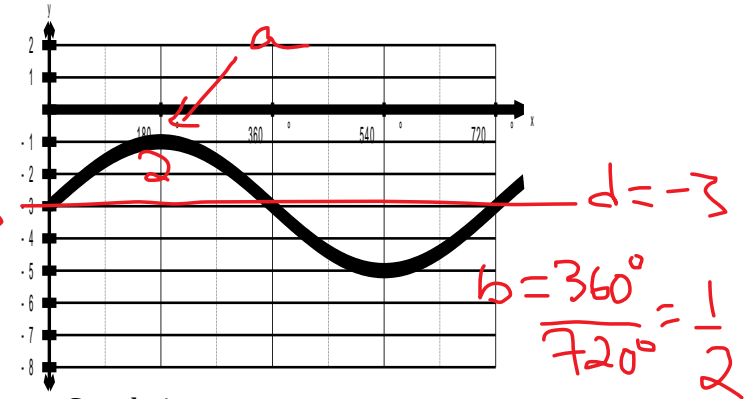
b would be larger.

46. Match the graph with its equation:

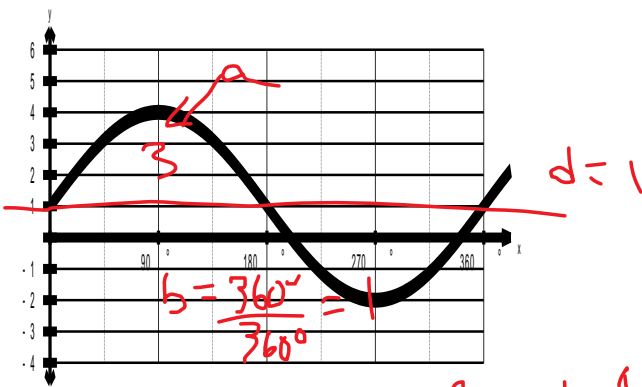
Graph 1



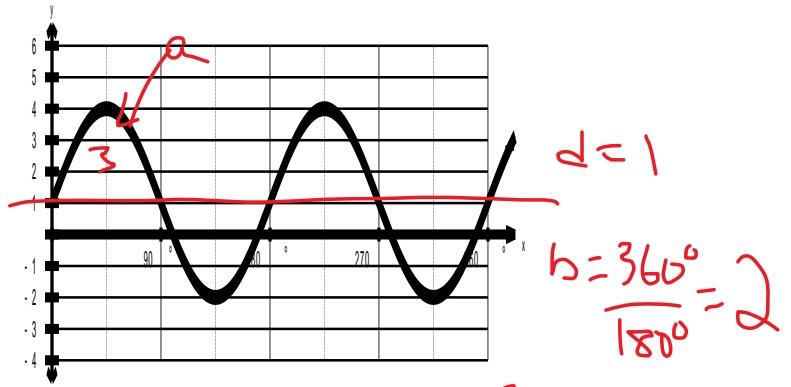
Graph 2



Graph 3



Graph 4



Equation 1: $y = 2\sin 2x - 3$ Graph 1

Equation 2: $y = 3\sin x + 1$ Graph 4

Equation 3: $y = 2\sin 0.5x - 3$ Graph 2

Equation 4: $y = 3\sin 0.5x + 1$ Graph 3

47. Bob collects data on the average monthly temperatures of his hometown. He uses his graphing calculator to perform a Sinusoidal Regression. The screenshot of his calculator is shown below (where y represents the temperature in degrees Fahrenheit and x represents the month with $t = 1$ representing January):

```
SinReg
y=a*sin(bx+c)+d
a=22.4279482
b=.5093726541
c=-2.099400715
d=54.65794153
```

(a) Write the equation of the curve of best fit.

$$y = 22.4 \sin(0.51x - 2.1) + 54.7$$

(b) Use your equation to determine the temperature in June.

$$y = 22.4 \sin[0.5(6) - 2.1] + 54.7$$

$$= 73^\circ \text{ F}$$