Math 1201   Date:_____
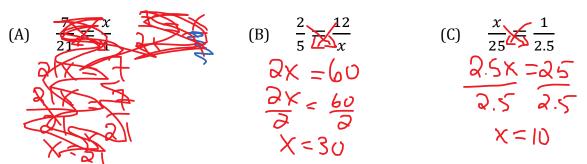
1.1C Conversions Between Measurement of Units

Proportional Reasoning

Students applied proportional reasoning to problems involving ratios in Grade 8 and similar polygons in Grade 9.

A proportion is a statement where two ratios are equal. For example,

$$\frac{100 \text{ cm}}{1 \text{ m}} = \frac{350 \text{ cm}}{3.5 \text{ m}}$$

Example 1:
Complete each proportion:

(A) $\frac{7}{21} = \frac{x}{?}$

(B) $\frac{2}{5} = \frac{12}{x}$

$2x = 60$

$\frac{2x}{2} = \frac{60}{2}$

$x = 30$

(C) $\frac{x}{25} = \frac{1}{2.5}$

$2.5x = 25$

$\frac{2.5x}{2.5} = \frac{25}{2.5}$

$x = 10$

A portion of an SI unit is written in decimal form. A decimal system is used for several reasons. It can immediately be seen which of the two values is greater and it is easier to do calculations involving decimal numbers. For example, 26.4 cm, 5.3 m or 32.7 km.

A portion of an imperial unit, however, is generally written in fractional form. Some examples would be 1/8 inch or 1/4 mile. Many rulers, for example, are marked with imperial units showing one inch divided into eighths or sixteenths.
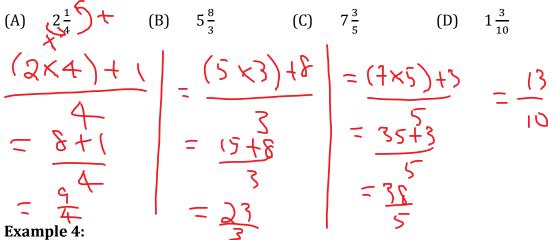

Working With Fractions
Students sometimes have difficulty working with fractions. Although you have worked with operations involving fractions in intermediate grades 7, 8 and 9, we will review some of these concepts before we continue.
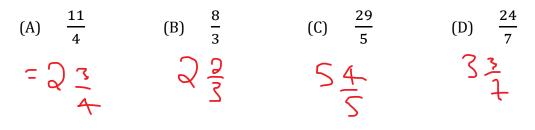
Example 2:
Answer the following:

(A) $\dfrac{1}{2} \times \dfrac{1}{3}$

$= \dfrac{1 \times 1}{2 \times 3}$

$= \dfrac{1}{6}$

(B) $\dfrac{2}{3} \times \dfrac{3}{7}$

$= \dfrac{2 \times 3}{3 \times 7}$

$= \dfrac{6}{21}$

(C) $\dfrac{5}{1} \times \dfrac{4}{11}$

$= \dfrac{5 \times 4}{1 \times 11}$

$= \dfrac{20}{11}$

(D) $\dfrac{2}{9} \times \dfrac{7}{1}$

$= \dfrac{14}{9}$

Example 3:
Change from mixed numeral to improper fraction:

(A) $2\frac{1}{4}$

$\dfrac{(2 \times 4) + 1}{4}$

$= \dfrac{8 + 1}{4}$

$= \dfrac{9}{4}$

(B) $5\frac{8}{3}$

$= \dfrac{(5 \times 3) + 8}{3}$

$= \dfrac{15 + 8}{3}$

$= \dfrac{23}{3}$

(C) $7\frac{3}{5}$

$= \dfrac{(7 \times 5) + 3}{5}$

$= \dfrac{35 + 3}{5}$

$= \dfrac{38}{5}$

(D) $1\frac{3}{10}$

$= \dfrac{13}{10}$

Example 4:
Change from improper fraction to mixed numeral:

(A) $\dfrac{11}{4}$

$= 2\frac{3}{4}$

(B) $\dfrac{8}{3}$

$2\frac{2}{3}$

(C) $\dfrac{29}{5}$

$5\frac{4}{5}$
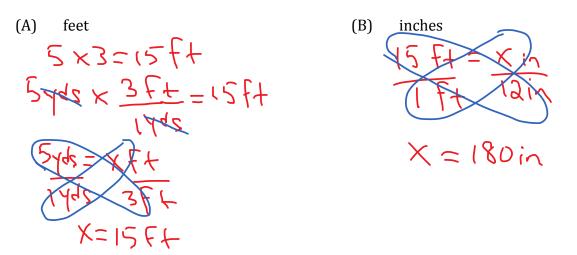
(D) $\dfrac{24}{7}$

$3\frac{3}{7}$

Converting Within Imperial Units
This involves only conversions between measurements inside the Imperial System. In this course, we are only responsible for inches, yards, feet and miles. The following conversions **ARE** provided on any assignments, tests, midyear and final exams.

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ yard} = 3 \text{ feet}$$

$$1 \text{ mile} = 1760 \text{ yards}$$

Example 1:
Convert 5 yards to:

(A)   feet

$$5 \times 3 = 15 ft$$

$$5 yds \times \frac{3 ft}{1 yds} = 15 ft$$

$$\frac{5 yds = x ft}{1 yds \quad 3 ft}$$

$$X = 15 ft$$

(B)   inches

$$\frac{15 ft = x in}{1 ft \quad 12 in}$$

$$X = 180 in$$

Expressing Answers in Feet and Inches
When a length in feet has decimal points or when it is written as an improper fraction, we always change the decimal part or fraction part to a value in inches.

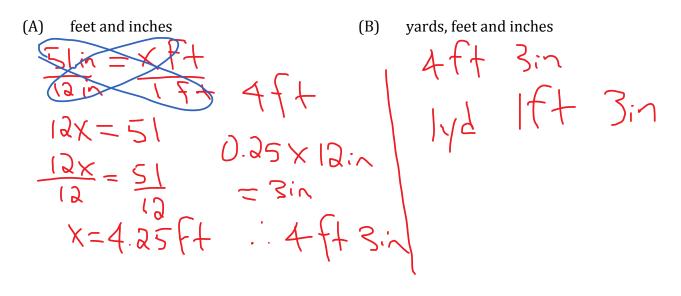For example, we would write 8.5 feet as 8 feet 6 inches (0.5 of a foot is 6 inches).

To convert the decimal or fraction part of a value in feet to inches, multiply the decimal or fraction by 12, since there are 12 inches in a foot.

Example 2:
What is 13.6 ft in feet and inches?

$$13 ft \qquad 0.6 \times 12 = 7.2 \sim 7$$

$$13 ft \ 7 in$$

Example 3:
Convert 51 inches to

(A)   feet and inches

$$\frac{51 in}{12 in} = \frac{x \, ft}{1 ft}$$

$$12x = 51$$

$$\frac{12x}{12} = \frac{51}{12}$$

$$x = 4.25 \, ft$$

4 ft

$0.25 \times 12 \, in$
$= 3 in$

$\therefore 4 \, ft \; 3 in$

(B)   yards, feet and inches

4 ft 3 in

1 yd 1 ft 3 in

Example 4:
Rick builds a fence out of 38 inch pieces of railing. If there are 18 pieces of railing used to enclose a rectangular garden, what is the perimeter of the garden in yards?

$$38 \, in \times 18 = 684 \, in$$

$$\frac{684 in}{12 in} = \frac{x \, ft}{1 ft}$$

$$12x = 684$$

$$\frac{12x}{12} = \frac{684}{12}$$

$$x = 57 \, ft$$

$$\frac{57 ft}{3 ft} = \frac{x \, yd}{1 yd}$$

$$3x = 57$$

$$\frac{3x}{3} = \frac{57}{3}$$

$$x = 19 \, yd$$

Example 5:
A carpenter is placing trim around a rectangular window measuring 41 in by 27 in. If the trim costs $1.92/ft, what is the approximate cost?

$$\text{total trim: } 41 \text{ in} + 27 \text{ in} = 68 \text{ in} \times 2 = 136 \text{ in}$$

$$\frac{136 \text{ in}}{12 \text{ in}} = \frac{x \text{ ft}}{1 \text{ ft}}$$

$$\frac{12x}{12} = \frac{136}{12}$$

$$11.3 \text{ ft} \sim 12 \text{ ft}$$

$$12 \text{ ft} \times \frac{\$1.92}{\text{ft}} = \$23.04$$
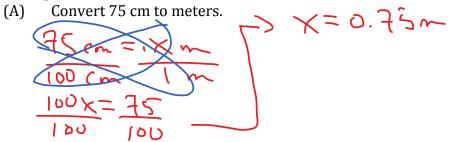
Converting Within SI Units
Some of the conversion factors we will use will be given on a formula sheet that can be used during assessments. There are some others however that are not given, and thus must be remembered.

The following are **NOT** provided but are useful to know:

$$1 \text{ m} = 1000 \text{ mm}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ cm} = 10 \text{ mm}$$

Example 6:

(A)   Convert 75 cm to meters.

$$\frac{75 \text{ cm}}{100 \text{ cm}} = \frac{x \text{ m}}{1 \text{ m}}$$

$\Rightarrow x = 0.75 \text{ m}$

$$\frac{100x}{100} = \frac{75}{100}$$

(B)   Convert 8.5 m to millimeters.

$$\frac{8.5 \text{ m}}{1 \text{ m}} = \frac{x \text{ mm}}{1000 \text{ mm}}$$

$x = 8500 \text{ mm}$

(C)   Convert 6.2 km to meters.

$$\frac{6.2 \text{ km}}{1 \text{ km}} = \frac{x \text{ m}}{1000 \text{ m}}$$

$x = 6200 \text{ m}$

(D)   Convert 28.2 cm to mm

$$\frac{28.2 \text{ cm}}{1 \text{ cm}} = \frac{x \text{ mm}}{10 \text{ mm}}$$

$x = 282 \text{ mm}$

Conversion Using Unit Analysis

Also called Dimensional Analysis, Unit Analysis is a problem-solving method that uses the fact that any number or expression can be multiplied by one without changing its value. It is a very useful technique in mathematics.

For example, when converting 200 meters to kilometers,

$$200 \cancel{m} \times \frac{1 \text{ km}}{1000 \cancel{m}} = \frac{200 \text{ km}}{1000} = 0.2 \text{ km}$$

It is important for students to notice that the units have changed but the actual distance has not. The distance of 200 m is the same as 0.200 km.

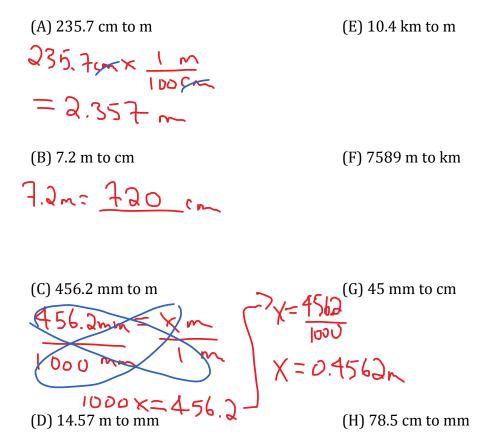**Example 7:**
(A)   Convert 3.4 km to meters using unit analysis.

$$3.4 \cancel{km} \times \frac{1000 \text{ m}}{1 \cancel{km}} = 3400 \text{ m}$$

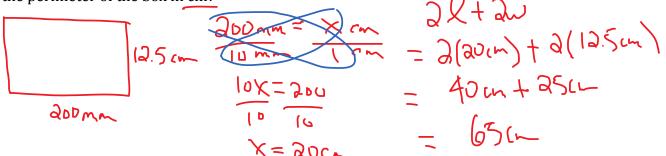(B)   Convert 418 meters to kilometers using unit analysis.

$$418 \cancel{m} \times \frac{1 \text{ km}}{1000 \cancel{m}} = 0.418 \text{ km}$$

(C)   Convert 34 yd to feet using unit analysis.

$$34 \cancel{yd} \times \frac{3 \text{ ft}}{1 \cancel{yd}} = 102 \text{ ft}$$

Example 8:
Convert the following:

(A) 235.7 cm to m

$235.7 \text{cm} \times \dfrac{1 \text{ m}}{100 \text{ cm}}$

$= 2.357 \text{ m}$

(E) 10.4 km to m

(B) 7.2 m to cm

$7.2 \text{m} = \underline{720} \text{ cm}$

(F) 7589 m to km

(C) 456.2 mm to m

$\dfrac{456.2 \text{ mm}}{1000 \text{ mm}} = \dfrac{x \text{ m}}{1 \text{ m}}$

$1000 x = 456.2$

$x = \dfrac{456.2}{1000}$

$x = 0.4562 \text{ m}$

(G) 45 mm to cm

(D) 14.57 m to mm

(H) 78.5 cm to mm

(I) A rectangular box is built from sides having length 200 mm and width 12.5 cm. What is the perimeter of the box in cm?

12.5 cm

200 mm

$\dfrac{200 \text{ mm}}{10 \text{ mm}} = \dfrac{x \text{ cm}}{1 \text{ cm}}$

$10x = 200$

$\dfrac{10}{10} \quad \dfrac{200}{10}$

$x = 20 \text{ cm}$

$2\ell + 2w$

$= 2(20 \text{cm}) + 2(12.5 \text{cm})$

$= 40 \text{cm} + 25 \text{cm}$

$= 65 \text{cm}$

Textbook Questions: page 11 - 12 #3, 7, 8, 9, 10, 12, 14, 15