Math 1201 Date:___________________

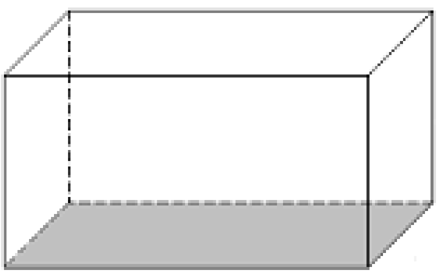
1.4 Surface Area of Right Pyramids and Right Cones

Understanding how to calculate surface area can be helpful in many real world applications. For example, surface area can be used to estimate the amount of paint needed to paint a house or to know how much wrapping is needed to cover a container.

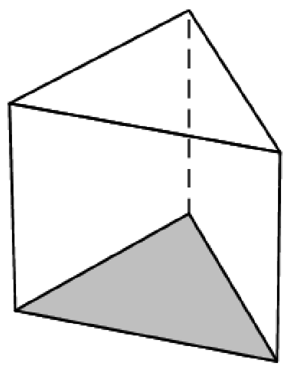
Throughout this unit you will be encouraged to draw diagrams to help them visualize the 3-D objects that are described.

In Grade 8, students used nets to calculate the surface area of a:

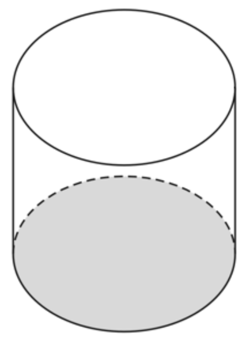
Right Rectangular Prism



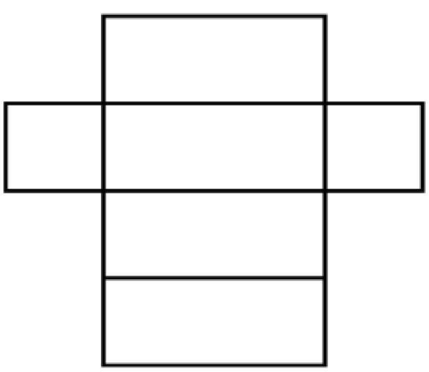
Right Triangular Prism

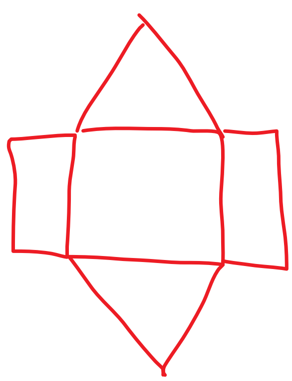


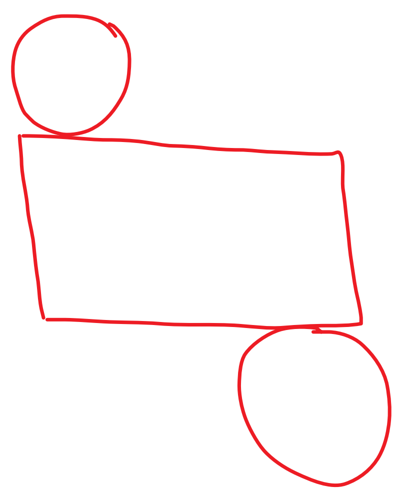
Right Cylinder



Draw the corresponding nets for each prism. The first one has been done for you.





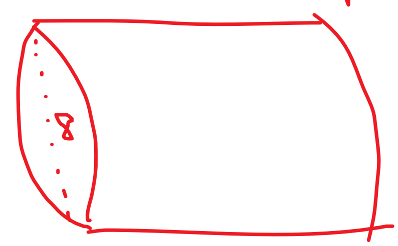


Example 1:
A water tank is the shape of a right circular cylinder 30 ft. long and 8 ft. in diameter. How many square feet of sheet metal was used in its construction?

$A_\circ = \pi r^2$ $\quad C = \pi d$

$A_\square = l \cdot w$ $\quad C = \pi(8) = 25.1 ft^2$

$r = \frac{d}{2} = \frac{8}{2} = 4$

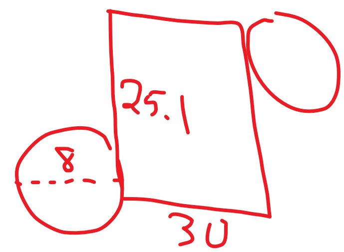


$A_\circ = 2[\pi(4)^2] = 100.5 ft^2$

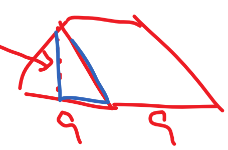
+

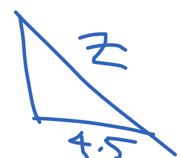
$A_\square = (30)(25.1) = 753 ft^2$

$853.5 ft^2$



Example 2:
A tent that has a square base with a side length of 9 ft. and a height of 7.5 ft. needs a canvas cover. Determine the amount of canvas needed to cover the tent. Hint: The floor of the tent is not made of canvas.





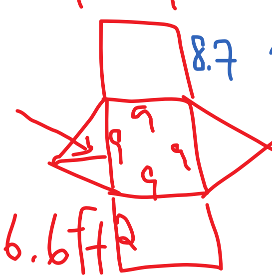
$A_\triangle = 2[\frac{1}{2} bh] = 2[\frac{1}{2}(9)(7.5)]$

$A_\triangle = 67.5 ft^2$

$7.5^2 + 4.5^2 = z^2$

$\sqrt{76.5} = \sqrt{z^2}$

$z = 8.7$



$A_\square = 2(l \cdot w) = 2(9)(8.7) = 156.6 ft^2$

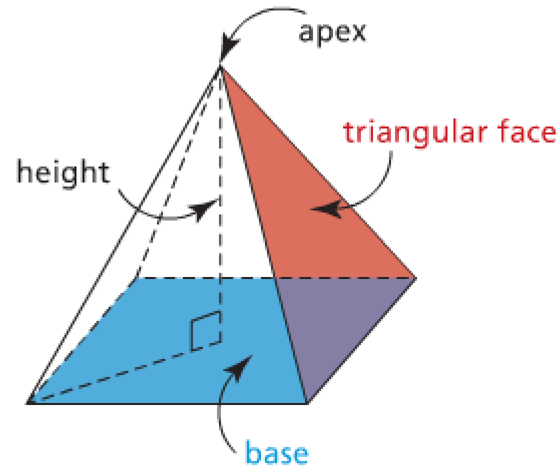
$A_T = 67.5 ft^2 + 156.6 ft^2 = 224.1 ft^2$

Pyramids

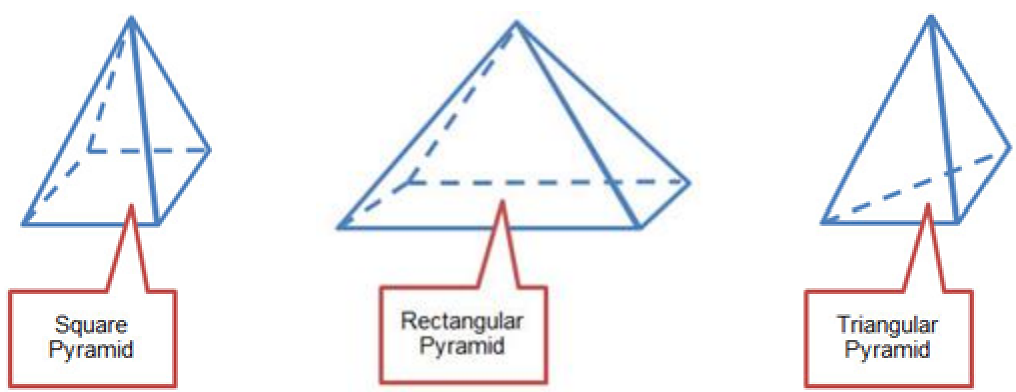
A pyramid is a 3-D figure with a polygon base. The shape of the base determines the name of the pyramid.

Apex – the point all triangular faces meet at.

Height – the perpendicular distance from the apex to the center of the base.

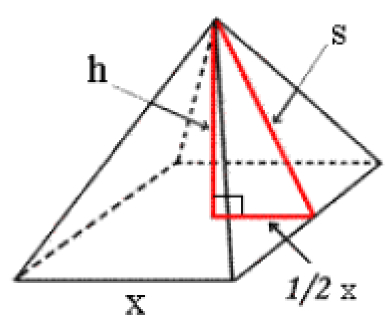


In this course, we will investigate a right pyramid with a triangular base, square base, and a rectangular base. When the base of a right pyramid is a regular polygon, the triangular faces are congruent or a regular pyramid.



You will have to differentiate between the height of a right pyramid and its slant height.

The **height** refers to the perpendicular height from the apex to the base, h, and the **slant height** is the altitude of the triangular face, s.

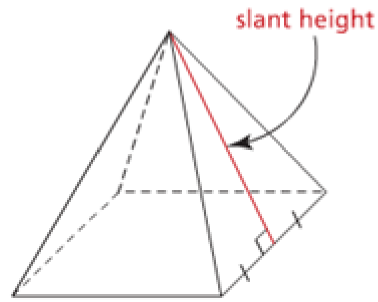


A common error is relating the slant height to the edge of the pyramid. Pythagorean Theorem will be used to calculate the slant height given the perpendicular height and the appropriate base dimension, x, or given the edge length from the apex to the base and the appropriate base dimension.

To find the surface area of a right pyramid you need to:

1. Find the area of each surface of the pyramid

2. Add up all these areas.

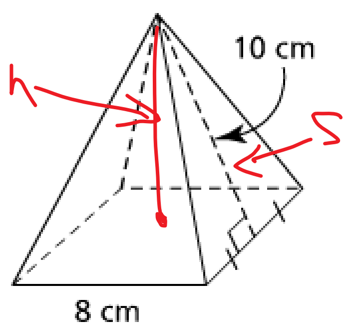


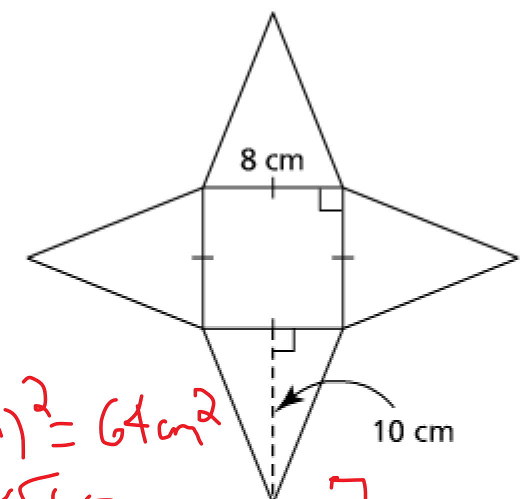
Right Pyramid with Square Base

A right square pyramid will have four equal triangular sides and a square base. You will be adding up five faces to determine the surface area. Remember, the area of a triangle is $A = \frac{1}{2}bh$ and the area of a square is $A = s^2$.

Example 3:

The right square pyramid has a slant height of 10 cm and a base side length of 8 cm. Calculate the surface area.



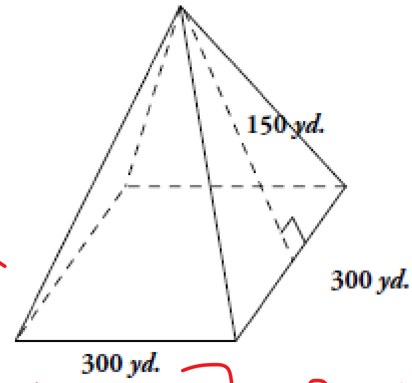


Base: $A = s^2 = (8\text{ cm})^2 = 64\text{ cm}^2$

Sides: $4\left[\frac{1}{2}b\cdot h\right] = 4\left[\frac{1}{2}(8\text{ cm})(10\text{ cm})\right] = 160\text{ cm}^2$

$SA = 64\text{ cm}^2 + 160\text{ cm}^2 = 224\text{ cm}^2$

Example 4:
Determine the surface area of the object shown:



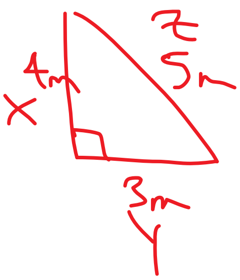
Base: $A_{\square} = s^2 = (300\,yd)^2$
$= 90\,000\,yd^2$

Sides: $A = 4\left[\frac{bh}{2}\right] = 4\left[\frac{(300\,yd)(150\,yd)}{2}\right] = 90\,000\,yd^2$

$SA = 90\,000\,yd^2 + 90\,000\,yd^2 = 180\,000\,yd^2$

Lateral Area
The lateral area is the total surface area of a pyramid without including the base.

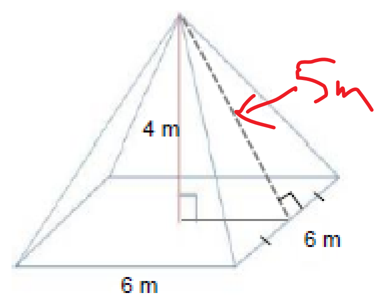
Example 5:
Determine the lateral area of the right square pyramid to the nearest square cm.



Pythagorean Triple

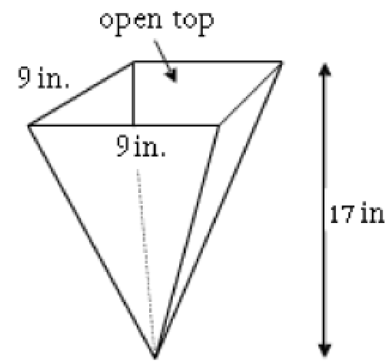
or

$x^2 + y^2 = z^2$
$(4)^2 + (3)^2 = z^2$
$16 + 9 = z^2$
$\sqrt{25} = \sqrt{z^2}$
$z = 5$



$LA = 4\left[\frac{bh}{2}\right] = 4\left[\frac{(6m)(5m)}{2}\right] = 60m^2$

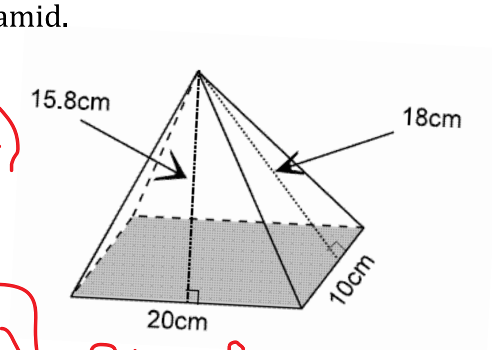
Example 6:
John hangs 6 flower pots around his house. The flower pots need to be painted. How much paint, in ~~square feet,~~ would John need to paint the outside of the flower pots?



Right Pyramid with Rectangular Base

Although similar to calculating the area of ~~prism~~ pyramid with a square base, there is one major difference. Since the base is rectangular, the slant heights will be different for two sides. You will have to compensate for this when calculating. There will still be five faces total, but this time you'll have two pairs of equivalent faces and a rectangular base. Remember, the area of a rectangle is $A = lw$.

Example 7:
Find the surface area of the following right rectangular pyramid.



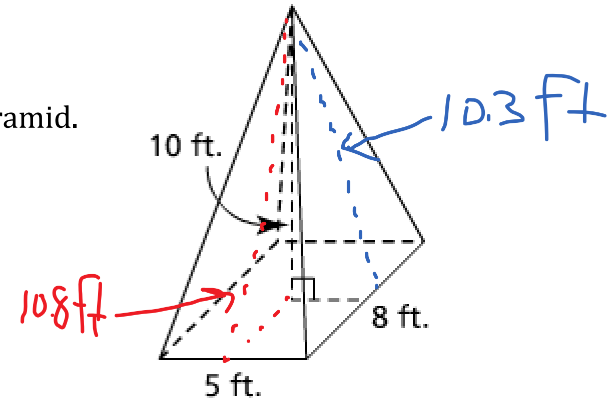
Base: $A_{\square} = lw = (20cm)(10cm)$
$= 200cm^2$

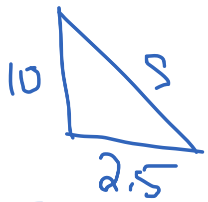
Side: $2\left[\frac{b \cdot h}{2}\right] = \cancel{2}\left[\frac{(20cm)(15.8cm)}{\cancel{2}}\right] = 316cm^2$

Side: $2\left[\frac{(10cm)(18cm)}{2}\right] = 180cm^2$

$SA = 200cm^2 + 316cm^2 + 180cm^2 = 696cm^2$

Example 8:
Determine the surface area of the right rectangular pyramid.



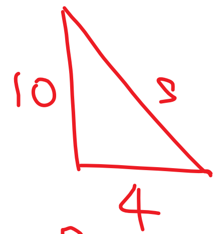


$S^2 = 10^2 + 2.5^2$

$S^2 = 106.25$

$\sqrt{S^2} = \sqrt{106.25}$

$S = 10.3$



$S^2 = 10^2 + 4^2$

$S^2 = 116$

$\sqrt{S^2} = \sqrt{116}$

$S = 10.8$

Base: $A = l \cdot w = (5ft)(8ft)$
$= 40ft^2$

$A = 2\left[\frac{b \cdot h}{2}\right] =$

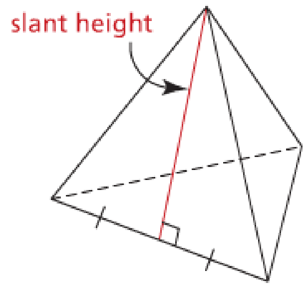
$= 2\left[\frac{(8ft)(10.3ft)}{2}\right]$

$= 82.4ft^2$

$A = 2\left[\frac{(5ft)(10.8ft)}{2}\right]$

$A = 54ft^2$

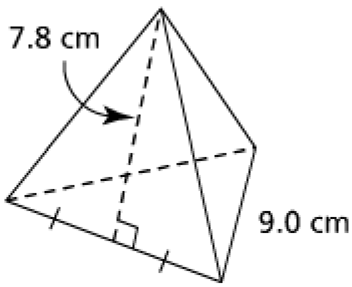
$SA = 82.4ft^2 + 54ft^2 + 40ft^2$
$= 176ft$

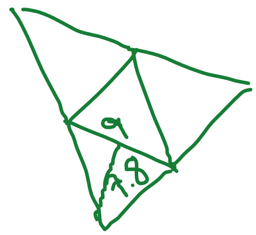


Right Pyramid with Triangular Base
This object is also called a tetrahedron. To find the surface area, simply find the area of each side and add them up.

Example 9:
Calculate the surface area of the following right triangular pyramids or tetrahedron.

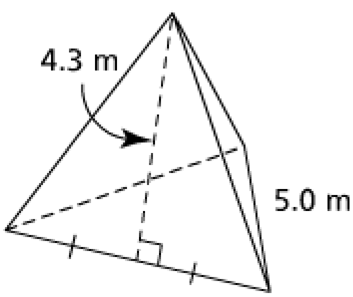
(A)





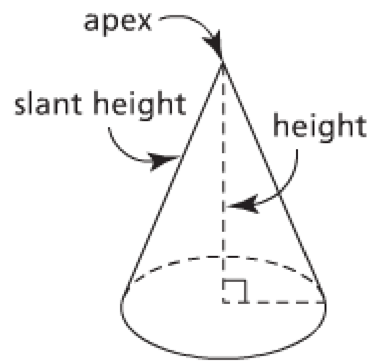
$$SA = 4\left[\frac{b \cdot h}{2}\right] = 4\left[\frac{(9\,cm)(7.8\,cm)}{2}\right] = 140.4\,cm^2$$

(B)



$$SA = 4\left[\frac{b \cdot h}{2}\right]$$
$$= 4\left[\frac{(5\,m)(4.3\,m)}{2}\right]$$
$$= 43\,m^2$$

Right Cones
The surface area of a cone consists of the area of the circular base and the curved surface. As with pyramids, there is a difference between the height and slant height of a cone.

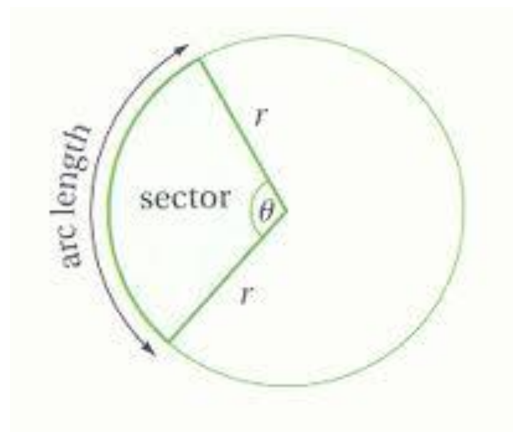


The surface area ~~of a~~ is comprised of the area of the circular base, $A = \pi r^2$, plus the lateral area around the cone.

To find the area of the lateral area of a cone we need some definitions:

Sector - A "pie-slice" part of a circle. The area between two radii and the connecting arc of a circle.

Arclength - The arc length is the measure of the distance along the curved line making up the arc.

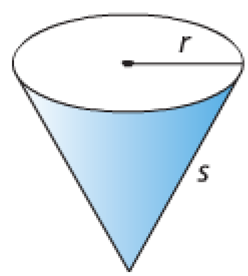


The following video clip shows how the area of a cone is derived:

https://www.youtube.com/watch?v=K2ghejiUDXg

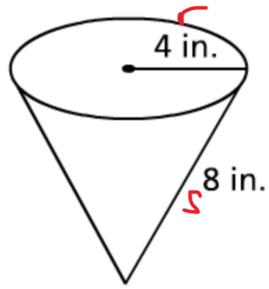
Surface Area of a Right Cone = lateral area + base area

$$SA = \pi r s + \pi r^2$$



Example 10:
Calculate the surface area of the following right cones:

(A)



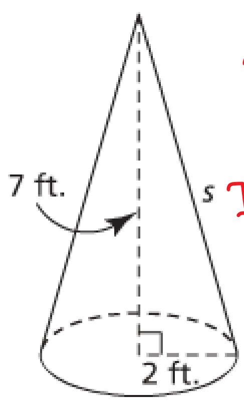
$SA = \pi r s + \pi r^2$

$SA = \pi(4in)(8in) + \pi(4in)^2$

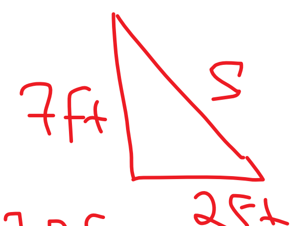
$SA = 100.5 in^2 + 50.3 in^2$

$SA = 150.8 in^2 \sim 151 in^2$

(B)



7.3ft



$S^2 = (7ft)^2 + (2ft)^2$

$S^2 = 49 + 4$

$\sqrt{S^2} = \sqrt{53}$

$S = 7.3ft$

$SA = \pi r s + \pi r^2$

$= \pi(2ft)(7.3ft) + \pi(2ft)^2$

$= 58.4 ft^2$

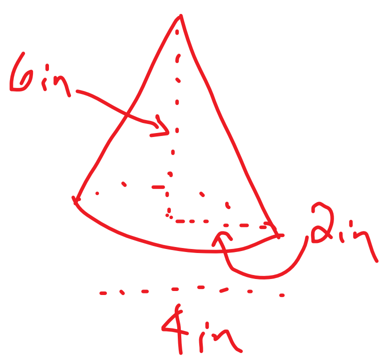
$0.4 \times 12 = 5 in$

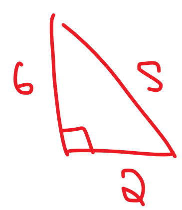
$= 58 ft\ 5 in$

Example 11:
Mary has made about 10 conical party hats out of cardboard. How much cardboard was used in total if each hat has a radius of 14 cm and a slant height of 25 cm?

$$LA = 10\pi r s$$
$$= 10\pi(14cm)(25cm)$$
$$= 10996\,cm^2$$

Example 12:
Tyler works at a local ice cream parlor making waffle cones. If a finished cone is 6 in. high and has a base diameter of 4 in., what is the surface area of the cone not including the area of the base?



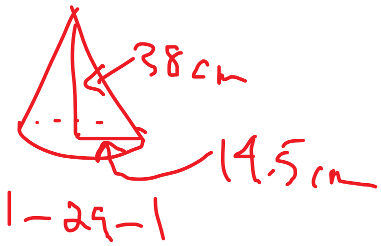


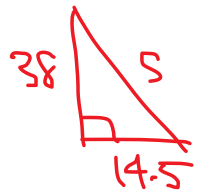
$$s^2 = 6^2 + 2^2$$
$$s^2 = 36 + 4$$
$$\sqrt{s^2} = \sqrt{40}$$
$$s = 6.3\,in$$

$$LA = \pi r s$$
$$= \pi(2in)(6.3in)$$
$$= 40\,in^2$$

Example 13: Homework

A right cone has a circular base with a diameter 29 cm and a height of 38 cm. Calculate the surface area of the cone to the nearest tenth of a square centimetre.





$s^2 = 38^2 + 14.5^2$

$\sqrt{s^2} = \sqrt{1654.25}$

$s = 40.7$

$SA = \pi r s + \pi r^2$

$SA = \pi(14.5)(40.7) + \pi(14.5)^2$

$SA = 2514.5\text{ cm}^2$

Example 14: lateral

The ~~surface~~ lateral area of a right cone is 125 in^2 and its radius is 4.7 in. What is the slant height of the right cone?

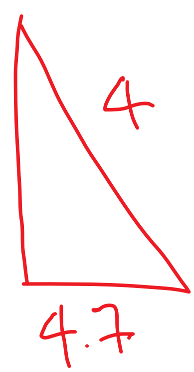
$LA = \pi r s$

$125 = \pi(4.7)s$

$125 = 14.8s$

$\frac{125}{14.8} = \frac{14.8s}{14.8}$

$s = 8.4\text{ in}$



Example 15:

A cylinder has a surface area of 412 cm^2. The height is three times greater than the radius. Approximate the height of the cylinder.

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r(3r)$

$SA = 2\pi r^2 + 6\pi r^2$

$SA = 8\pi r^2$

$\frac{412}{8\pi} = \frac{8\pi r^2}{8\pi}$

$h = 3r$

$16.4 = r^2$

$\sqrt{16.4} = \sqrt{r^2}$

$r = 4in$

$h = 3(4in) = 12in$

Example 16: Homework

A right pyramid has a surface area of 154 cm^2. A right cone has a base radius of 3 cm. The cone and pyramid have equal surface area. What is the height of the cone to the nearest tenth of a centimetre?

$SA = \pi r s + \pi r^2$

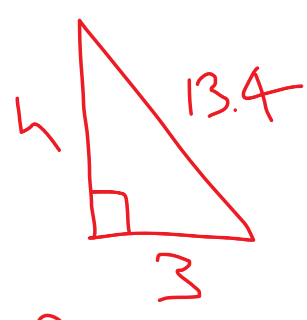
$154 = \pi(3) \cdot s + \pi(3)^2$

$154 = 9.4s + 28.3$

$154 - 28.3 = 9.4s$

$\frac{125.7}{9.4} = \frac{9.4s}{9.4}$

$s = 13.4cm$



$h^2 + 3^2 = 13.4^2$

$\sqrt{h^2} = \sqrt{170.6}$

$h = 13.1$

Textbook Questions: page 34, 35 #3, 5, 6b, 7b, 8, 9, 10, 13, 16