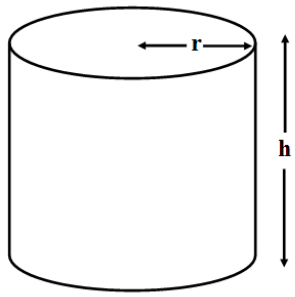
Math 1201 Date:__________________

1.5 Volumes of Right Pyramids and Right Cones

Here we will investigate the relationship between the volume of a right cone and a right cylinder and between the volume of a right pyramid and a right prism. Since volume is in three dimensions, any measurement will be in units^3.

Right Cylinder
A **cylinder** is a closed solid that has two Parallel, circular bases connected by a curved surface.

The volume formula of a right cylinder, $V = \pi r^2 h$ was explored in Grade 8.

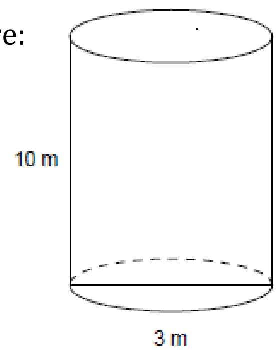


Example 1:
Find the volume of the following cylinder to the nearest cubic metre:

$V = \pi r^2 h$

$V = \pi (1.5)^2 (10)$

$V = 71 m^3$

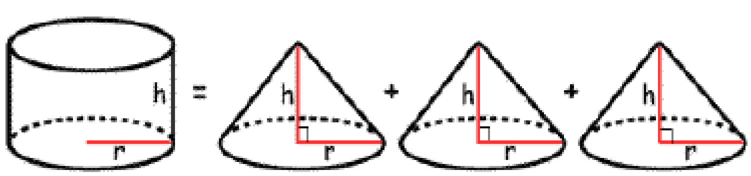


$d = 3m$
$r = 1.5m$

The volume of a right cone with the same base and the same height as a cylinder can be shown to have $\frac{1}{3}$rd the volume. The following clip demonstrates this concept:

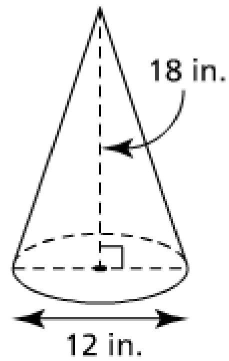
https://www.youtube.com/watch?v=GWk6ney-c24

You can see that the water from the three cones fills the cylinder entirely. This means it takes the volume of three cones to equal one cylinder. Looking at this in reverse, each cone is one-third the volume of a cylinder.



Hence, the **volume of a right cone** is represented by: $V = \frac{1}{3}\pi r^2$h

Example 2:
Determine the volume of the cone to the nearest cubic inch.



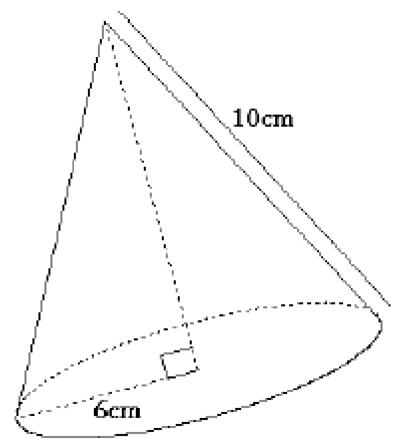
$d = 12\text{ in}$, $r = 6\text{ in}$

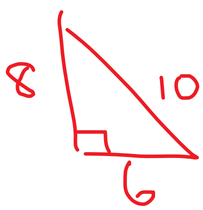
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (6\text{ in})^2 (18\text{ in})$

$V = 678\text{ in}^3$

Example 3:
Determine the volume of the following to the nearest cubic centimeter.





$h^2 + 6^2 = 10^2$

$h^2 = 100 - 36$

$\sqrt{h^2} = \sqrt{64}$

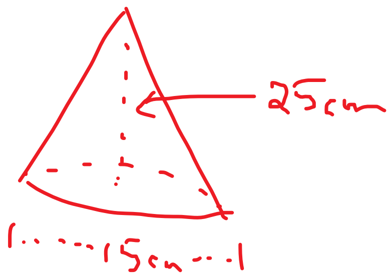
$h = 8$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (6\text{cm})^2 (8\text{ cm})$

$V = 302\text{cm}^3$

Example 4:
A cake decorating bag is in the shape of a cone. To the nearest cubic centimetre, how much frosting will fit into the bag if the diameter is 15 cm and the height is 25 cm?



$r = 7.5cm$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (7.5)^2 (25)$$

$$V = 1473 cm^3$$

Example 5:
A cone and a cylinder have the same height and the same base radius. If the volume of the cylinder is 81 cm^3 , what is the volume of the cone in cm^3? Explain.

$$81 cm^3 \times \frac{1}{3} = 27 cm^3$$

Volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same height and radius.

Example 6:
A cone has a volume of 30 cm^3 and a base area of 15 cm^2. What is the height of the cone to the nearest centimeter?

$A = \pi r^2$

$\frac{15}{\pi} = \frac{\pi r^2}{\pi}$

$4.8 = r^2$

$\sqrt{4.8} = \sqrt{r^2}$

$r = 2.2\text{cm}$

$V = \frac{1}{3}\pi r^2 h$

$30 = \frac{1}{3}\pi(2.2\text{cm})^2 h$

$\frac{30}{5.1} = \frac{5.1h}{5.1}$

$h = 6\text{ cm}$

Example 7:
A cylinder has a volume of 132.6 cm^{3}and a height of 8.5 cm. What is the diameter of the cylinder to the nearest tenth of a centimeter?

$V = \pi r^2 h$

$132.6 = \pi r^2 (8.5)$

$\frac{132.6}{26.7} = \frac{26.7r^2}{26.7}$

$5.0 = r^2$

$\sqrt{5.0} = \sqrt{r^2}$

$r = 2.2$

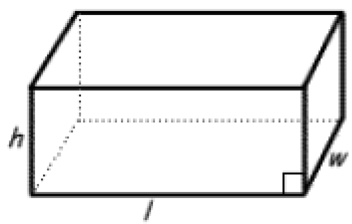
$d = 2r$

$d = 2(2.2\text{cm})$

$d = 4.4\text{cm}$

Right Pyramids
Prisms are named according to the shape of their base such as a triangular prism or rectangular prism.

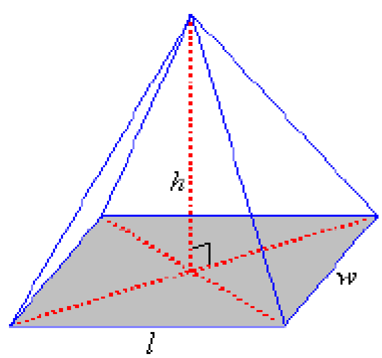
The volume of a right rectangular prism was developed in Grade 8 using the formula V = (base area) × height. Students will continue to use this formula for any right prism. For example:



Volume of a right rectangular prism is given by:

V = (base area) × height.
V = (Area of rectangle) × h
V = $l \times w \times h$

Just like the relationship between a cone and cylinder of the same base area and height, the volume of a right pyramid is found by calculating one third of the volume of its related right prism.



Volume of a right rectangular pyramid is given by:

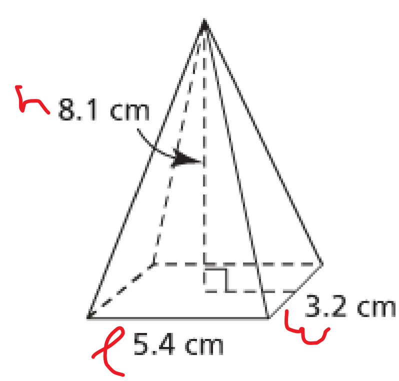
$V = \frac{1}{3}$(base area) × height
$V = \frac{1}{3} lwh$

Example 8:
Find the volume of the following pyramid to the nearest tenth of a cubic centimeter:

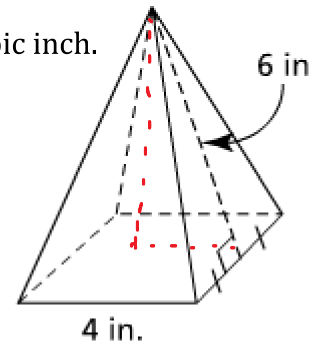
$V = \frac{1}{3} l \cdot w \cdot h$

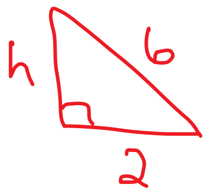
$V = \frac{1}{3}(5.4\text{cm})(3.2\text{cm})(8.1\text{cm})$

$V = 46.7\text{cm}^3$



Example 9:
Calculate the volume of the right square pyramid to the nearest cubic inch.





$h^2 + 2^2 = 6^2$

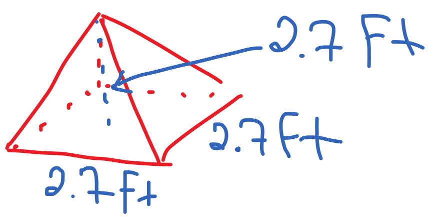
$h^2 = 36 - 4$

$\sqrt{h^2} = \sqrt{32}$

$h = 5.7\text{in}$

$V = \frac{1}{3}(4\text{in})(4\text{in})(5.7\text{in}) = 30\text{in}^3$

Example 10:
Find the volume, to the nearest cubic foot, of a square based pyramid where the length of each base side and the height measures 2.7 ft.



$V = \frac{1}{3} l \cdot w \cdot h$

$V = \frac{1}{3}(2.7\text{Ft})^3$

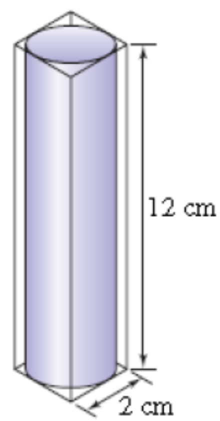
$V = 7\text{Ft}^3$

Example 11:
A cord of firewood is 128 cubic feet. Jan has three storage bins for firewood that each measure 2 ft. by 3 ft. by 4 ft. Does she have enough storage space to hold a full cord of firewood? Explain.

Bins: $V_T = 3 \cdot l \cdot w \cdot h$
$= 3(2ft)(3ft)(4ft)$
$= 72ft^3$

No, she only has enough room for $72ft^3$.

Example 12:
A closed cylindrical can is packed in a square based box. What is the volume of the empty space between the can and the box to the nearest cubic cm?



$V_{Box} = l \cdot w \cdot h$
$= (2cm)(2cm)(12cm)$
$= 48cm^3$

$V_{cyl} = \pi r^2 h$
$= \pi(1cm)^2(12cm)$
$= 38cm^3$

$48cm^3 - 38cm^3 = 10cm^3$

Textbook Questions: page 42, 43 #4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 18