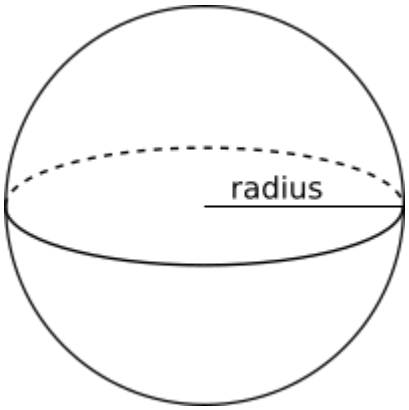
Math 1201 **Date:**_____

1.6 Surface Area and Volume of a Sphere

Sphere - a perfectly round geometrical object in three-dimensional space that is the surface of a completely round ball.



What are some real life examples of spheres?

- any ball (basketball, baseball)
- planet
- Sun

Surface Area of a Sphere

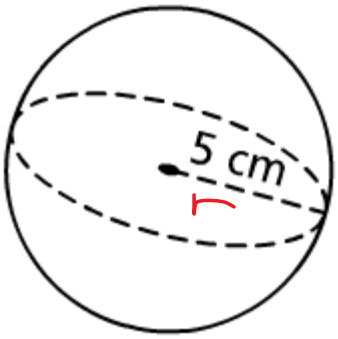
The following video shows how the surface area of a sphere is equal to four times the surface area of circle with equivalent circumference.

https://www.youtube.com/watch?v=FB-acn7d0zU

So the surface area of a sphere is: $SA = 4\pi r^2$

Example 1:

Calculate the surface area to the nearest square cm.



$SA = 4\pi r^2$

$SA = 4\pi(5\text{ cm})^2$

$SA = 314\text{ cm}^2$

Example 2:
A sphere has a surface area of 80 in^2. Determine the diameter to the nearest inch.

$SA = 80\ in^2$

$SA = 4\pi r^2$

$80 = 4\pi r^2$

$\frac{80}{4\pi} = \frac{4\pi r^2}{4\pi}$

$6.37 = r^2$

$\sqrt{r^2} = \sqrt{6.37}$

$r = 2.5\ in$

$d = 2r = 2(2.5\ in) = 5\ in$

Example 3:
An official basketball has a radius of 12.5 cm and usually has a leather covering. Approximately how much leather, in cm^2, is required to cover 12 official basketballs?

$SA = 12 \cdot 4\pi r^2$

$SA = 12 \cdot 4\pi (12.5\ cm)^2$

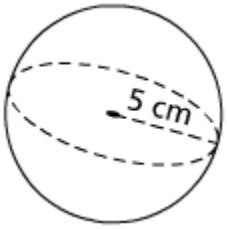
$SA = 23562\ cm^2$

Volume of a Sphere

The Volume of a Sphere is $V = \frac{4}{3}\pi r^3$. This is how much 3D space is contained inside the sphere.

Example 4:

Determine the volume of the following sphere to the nearest cubic centimeter.



$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (5\text{cm})^3$

$V = 524\text{ cm}^3$

Example 5:

The sun approximates a sphere with diameter 870 000 mi. What is the approximate volume of the sun?

$r = \frac{870000}{2} = 435000\text{ mi}$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (435000\text{ mi})^3$

$= 3.4 \times 10^{17}\text{ mi}^3$

Example 6:
A carnival clown has 75 m^3 of helium compressed in a tank. How many spherical balloons with a radius of 0.25 m can be filled with the helium from the tank?

$V_{Balloon} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(0.25\,m)^3$

$= 0.065\,m^3$

$\frac{75\,m^3}{0.065\,m^3} = 1153$ balloons

Example 7:
Eight basketballs are put in a container. The radius of each basketball is 10 cm. If the container is shaped like a square based pyramid, approximately how much space not occupied by basketballs will be left if each side of the base measures 40 cm and the height is 70 cm?

$V_{Pyr} = \frac{1}{3}l \cdot w \cdot h$

$= \frac{1}{3}(40\,cm)(40\,cm)(70\,cm)$

$= 37333\,cm^3$

$V_{Ball} = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(10\,cm)^3$

$V = 4189\,cm^3$

$8 \times 4189\,cm^3 = 33512\,cm^3$

$37333\,cm^3 - 33512\,cm^3 = 3821\,cm^3$

Example 8:
A spherical Christmas ornament measures 12 cm in circumference. What is the approximate volume of the cubed box that will hold this ornament?

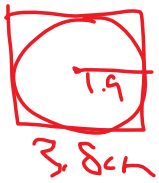
$C = 12\text{cm}$

$C = 2\pi r$

$\frac{12}{2\pi} = \frac{2\pi r}{2\pi}$

$r = 1.9\text{cm}$

$2(1.9) = 3.8\text{cm}$



$V = s^3$

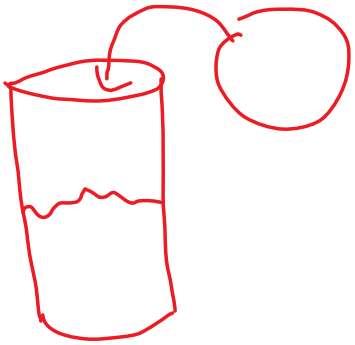
$V = (3.8\text{cm})^3$

$V = 54.9\text{cm}^3 \sim 55\text{cm}^3$

Example 9:
A heavy sphere with diameter 20 cm is dropped into a right circular cylinder with a base radius of 10 cm and a height of 34 cm.

$r = 10\text{cm}$

(A) If the cylinder is half full of water, what is the total volume of the water and the sphere?



$V_{sphere} = \frac{4}{3}\pi(10\text{cm})^3$

$= 4189\text{cm}^3$

$\frac{1}{2}V_{cyl} = \pi r^2 h$

$= \pi(10\text{cm})^2(34\text{cm})$

$= \frac{1}{2}1068\text{1cm}^3$

$\approx 5341\text{cm}^3$

$V_T = 4189\text{cm}^3 + 5341\text{cm}^3$

$= 9530\text{cm}^3$

(B) How high will the water rise once the sphere is completely under the water? Note: Once the sphere is dropped into the water, the water level will rise to a height that represents the volume of the water plus the volume of the sphere.

$V = \pi r^2 h$

$9530 = \pi(10\text{cm})^2 h$

$9530 = 314h$

$\frac{9530}{314} = \frac{314h}{314}$

$h = 30.3\text{cm} \sim 30\text{cm}$

Textbook Questions: page 50 - 52 #3, 4, 5, 7, 8, 9, 10, 12, 15, 19