Math 1201 **Date:**__________________

3.1 Factors and Multiples of Whole Numbers

Prime Number: a whole number greater than 1, whose only two whole-number factors are 1 and itself.

The first few **prime numbers** are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

2: 1, 2 5: 1, 5 17: 1, 17 29: 1, 29

The number **0** is not prime. While 0 is divisible by all prime numbers, for example $0 \div 2 = 0$, $0 \div 3 = 0$, at first glance, it seems that zero has an infinite number of prime factors.

However, if 2 is a factor of 0 because $2 \times 0 = 0$, then so is the number zero. Since division by zero is undefined, zero has no prime factors. $\frac{0}{0}$

The number **1** is not prime because it's only factor is 1. As well, when 1 is divided by a prime number, the quotient is never a whole number.

Composite number: a positive integer which is not prime. In other words, a number which has factors other than 1 and itself. 4, 6, 8, 9, 10 are all examples of composite numbers. A composite number will always have at least three factors.

4: 1, 4; 2, 2 10: 1, 10; 2, 5

Prime Factorization

A **prime** number can only be divided by 1 or itself, so it cannot be factored any further. Every other whole number can be broken down into **prime** number **factors**. It is like the **prime** numbers are the basic building blocks of all numbers. We can find all prime factors of any number by completing a **factor tree.**

Example 1:

(A) List all the factors of 12.

12: 1, 2, 3, 4, 6, 12

(B) What are the prime factors of 12?

12

$2 \cdot 6$

$2 \cdot 2 \cdot 3$

$12 = 2^2 \cdot 3$

(C) What is 12 as a product of prime factors?

$12 = 2 \cdot 2 \cdot 3$ or $2^2 \cdot 3$

Example 2:
Prime factorize the following:

(A) 54

$6 \cdot 9$

$2 \cdot 3 \; 3 \cdot 3$

or

$2 \cdot 3^3$

(B) 75

$5 \cdot 15$

$5 \; 3 \cdot 5$

or

$3 \cdot 5^2$

(C) 120

$10 \cdot 12$

$2 \cdot 5 \; 2 \cdot 6$

$2 \; 5 \; 2 \; 2 \cdot 3$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$2^3 \cdot 3 \cdot 5$

(C) 180

$18 \cdot 10$

$2 \cdot 9 \; 2 \cdot 5$

$2 \; 3 \cdot 3 \cdot 2 \; 5$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$2^2 \cdot 3^2 \cdot 5$

(D) 512

$4 \cdot 128$

$2 \cdot 2 \; 2 \cdot 64$

$2 \; 2 \; 2 \; 8 \cdot 8$

$2 \; 2 \; 2 \; 2 \cdot 4 \; 2 \cdot 4$

$2 \; 2 \; 2 \; 2 \; 2 \cdot 2 \; 2 \; 2 \cdot 2$

2^9

Greatest Common Factor – GCF
The greatest common factor, or GCF, is the greatest factor that divides into two numbers. To find the GCF of two numbers using prime factors:

1. List the prime factors of each number.
2. Multiply the factors both numbers have in common.
3. If there are no common prime factors, the GCF is 1.

Example 3:
Determine the GCF of 12 and 18.

Method 1: List all factors

12	18
1, 12	1, 18
2, 6	2, 9
3, 4	3, 6

GCF: 6

Method 2: Prime Factorize

12	18
$2 \cdot 6$	$2 \cdot 9$
$2 \cdot 2 \cdot 3$	$2 \cdot 3 \cdot 3$

$2 \cdot 3 = 6$

GCF: 6

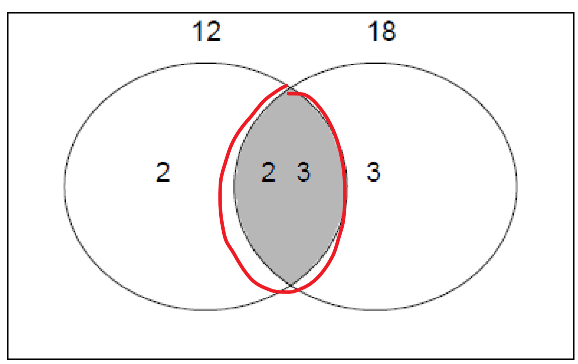
Venn Diagrams
A Venn diagram represents mathematical or logical sets pictorially as circles or closed curves within an enclosing rectangle, common elements of the sets being represented by the areas of overlap among the circles.

As an alternative, you could use both prime factorization and a Venn diagram. The Venn diagram is a different way to organize the data to show common and unique prime factors. For example, the Venn diagram shown below can be used to demonstrate the GCF of 12 and 18.

The prime factors that 12 and 18 have in common are 2 and 3, as seen in the intersection portion of the Venn diagram.

$2 \cdot 3 = 6$

GCF: 6



Example 4:
Find the GCF of the following:

(A) 10 and 15

10: 1, 10; 2, 5
15: 1, 15; 3, 5
or
10: 2 · 5
15: 3 · 5
5

GCF: 5

GCF: 5

(B) 12 and 24

12: 2 · 6; 2 · 2 · 3
24: 4 · 6; 2 · 2 · 2 · 3

$2 \cdot 2 \cdot 3 = 12$

GCF: 12

(C) 24 and 32

24: 3 · 8; 3 · 2 · 4; 3 · 2 · 2 · 2
32: 4 · 8; 2 · 2 · 2 · 4; 2 · 2 · 2 · 2 · 2

$2 \cdot 2 \cdot 2 = 8$

GCF: 8

(D) 45 and 80

45: 9 · 5; 3 · 3 · 5
80: 2 · 40; 2 · 8 · 5; 2 · 2 · 4 · 5; 2 · 2 · 2 · 2 · 5

GCF: 5

(E) 20, 54 and 72

20: 4 · 5; 2 · 2 · 5
54: 6 · 9; 2 · 3 · 3 · 3
72: 8 · 9; 2 · 4 · 3 · 3; 2 · 2 · 2 · 3 · 3

GCF: 2

(F) 75, 200 and 250

75: 25 · 3; 5 · 5 · 3
200: 20 · 10; 4 · 5 · 2 · 5; 2 · 2 · 5 · 2 · 5
250: 25 · 10; 5 · 5 · 2 · 5

$5 \cdot 5 = 25$

GCF: 25

GCF and Reducing Fractions
The GCF can be used to reduce fractions. Simply prime factorize the numerator and the denominator. Find the GCF of the two numbers and then divide both numbers by that GCF.

For example, we can reduce $\frac{32}{48}$:

32: $4 \cdot 8$ → $2 \cdot 2 \cdot 2 \cdot 4$ → $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

48: $3 \cdot 16$ → $3 \cdot 2 \cdot 8$ → $3 \cdot 2 \cdot 2 \cdot 4$ → $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

GCF: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

$32 \div 16 = 2$, $48 \div 16 = 3$ → $\boxed{\frac{2}{3}}$

Example 5:
Use the GCF to simplify the following fractions:

(A) $\frac{165}{385}$

$165 \div 55 = 3$, $385 \div 55 = 7$ → $\boxed{\frac{3}{7}}$

165: $5 \cdot 33$ → $5 \cdot 3 \cdot 11$

385: $35 \cdot 11$ → $5 \cdot 7 \cdot 11$

GCF: $5 \cdot 11 = 55$

(B) $\frac{1260}{2310} =$

$1260 \div 210 = 6$, $2310 \div 210 = 11$ → $\boxed{\frac{6}{11}}$

1260: $21 \cdot 60$ → $3 \cdot 7 \cdot 6 \cdot 10$ → $3 \cdot 7 \cdot 2 \cdot 3 \cdot 2 \cdot 5$

2310: $231 \cdot 10$ → $21 \cdot 11 \cdot 2 \cdot 5$ → $3 \cdot 7 \cdot 11 \cdot 2 \cdot 5$

GCF: $2 \cdot 3 \cdot 5 \cdot 7 = 210$

Example 6:
What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume the squares cannot be cut. Sketch the rectangles and squares.

16: $2 \cdot 8$ → $2 \cdot 2 \cdot 4$ → $2 \cdot 2 \cdot 2 \cdot 2$

40: $8 \cdot 5$ → $2 \cdot 4 \cdot 5$ → $2 \cdot 2 \cdot 2 \cdot 5$

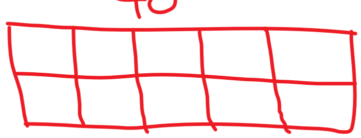
GCF: $2 \cdot 2 \cdot 2 = 8$

tiles

$\frac{16}{8} = 2$ $\frac{40}{8} = 5$

40

16



*Bigger → Smaller

Least Common Multiple - LCM

The **least common multiple** of two numbers is the **smallest** number, not including 0, that is a **multiple** of both.

One way to find the LCM is to list out multiples of each number until the first multiple common to both numbers is discovered.

Another way to find the LCM of two numbers is by using prime factors:

1. List the prime factors of each number.
2. Select the set of prime numbers from each set.
3. Multiply the prime factors from this set.

Example 7:

Determine the LCM of 12 and 18 using both methods.

Method 1

12: 12, 24, 36, 48, 60, . . .

18: 18, 36

Method 2

12 → 4 · 3 → 2 · 2 3

18 → 2 · 9 → 2 3 · 3

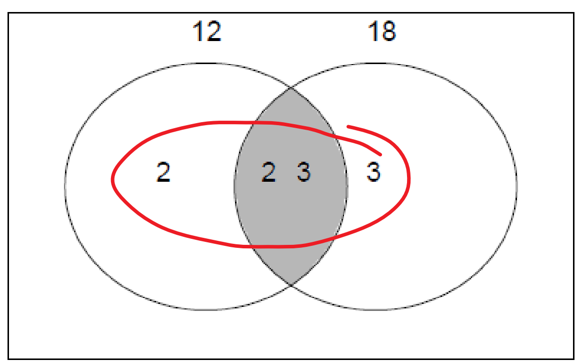
LCM: 2·2·3·3

= 36

As with the GCF, using prime factorization and a Venn diagram, students can determine the LCM. The diagram below can be used to demonstrate the LCM of 12 and 18.

This strategy should help students visualize that the remaining numbers, including the factors which make up the GCF, multiplied together equals the LCM.

LCM: 2·2·3·3=36



Example 8:
Find the LCM using two methods:

(A) 18 and 24

18: 18, 36, 54, 72
24: 24, 48, 72

18 → $2 \cdot 9$ → $2 \cdot 3 \cdot 3$

24 → $4 \cdot 6$ → $2 \cdot 2 \cdot 2 \cdot 3$

Lcm: $2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 = 72$

(B) 16 and 40

16: 16, 32, 48, 64, 80, 96
40: 40, 80

16 → $2 \cdot 8$ → $2 \cdot 2 \cdot 4$ → $2 \cdot 2 \cdot 2 \cdot 2$

40 → $4 \cdot 10$ → $2 \cdot 2 \cdot 2 \cdot 5$

Lcm: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 80$

(C) 27 and 45

27: 27, 54, 81, 108, 135, 162
45: 45, 90, 135

27 → $9 \cdot 3$ → $3 \cdot 3 \cdot 3$

45 → $9 \cdot 5$ → $3 \cdot 3 \cdot 5$

Lcm: $3 \cdot 3 \cdot 3 \cdot 5 = 135$

Example 9:
Pencils come in packages of 10. Erasers come in packages of 12. Jason wants to purchase the smallest number of pencils and erasers so that he will have exactly 1 eraser per pencil. How many packages of pencils and erasers should Jason buy?

10 → 2 · 5

12 → 2 · 6 → 2 · 2 · 3

LCM: 2·2·3·5 = 60

$\frac{60}{10}$ = 6 packs of pencils

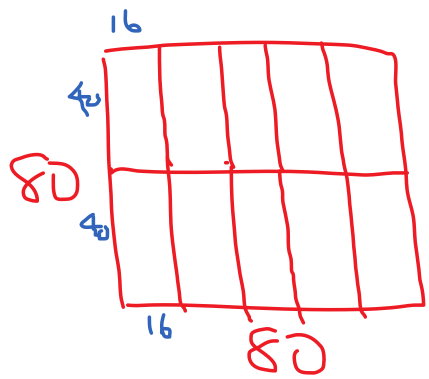
$\frac{60}{12}$ = 5 packs of erasers.

Example 10:
What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm. Assume the rectangles cannot be cut. Sketch the square and rectangles.

Smaller → bigger LCM: 2·2·2·2·5 = 80

16 → 4 · 4 → 2·2 2·2

40 → 4 · 10 → 2·2 2·5



Example 11:
One trip around a track is 440 yards. One runner can complete one lap in 8 minutes, the other runner can complete it in 6 minutes. How long will it take for both runners to arrive at their starting point together if they start at the same time and maintain their pace?

6: 6, 12, 18, 24, 30, . . .
8: 8, 16, 24

or

8 → 2 · 4 → 2 2 · 2

6 → 2 · 3

LCM: 2·2·2·3 = 24 minutes.

Textbook Questions: page 140, 141 #3 4, 5, 8, 9, 10, 11, 13, 14, 15, 17, 19, 20