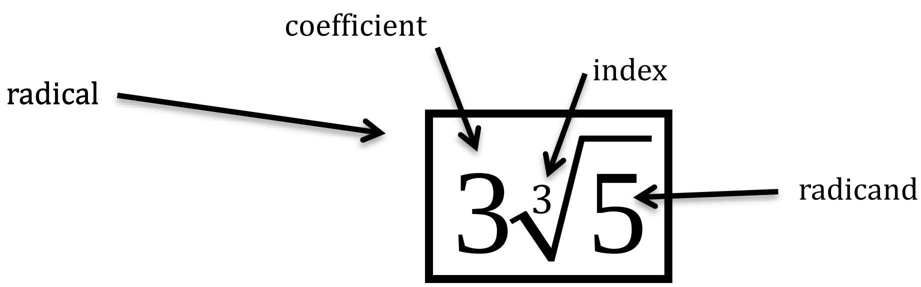
Math 1201 **Date:__________________**

3.2 Perfect Square, Perfect Cubes and Their Roots

Radicals

In **mathematics**, a **radical** expression is defined as any expression containing a **radical** √ symbol. Many people mistakenly call this a 'square root' symbol, and many times it is used to determine the square root of a number. However, it can also be used to describe a cube root, a fourth root, or higher.



In square root, an index of 2 is understood and usually not written.

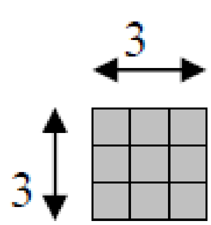
$$\sqrt{9} = \sqrt[2]{9}$$

In Grade 8, perfect square numbers were connected to the area of squares. When determining the **square root** of a whole number, you should view the area of the square as the perfect square number, and either dimension of the square as the square root.

Perfect Square – a number that can be expressed as the product of two equal integers. In other words, a whole number multiplied by itself. It can be written as a power with exponent 2.

Square Root - a factor of a number that when squared gives the number.

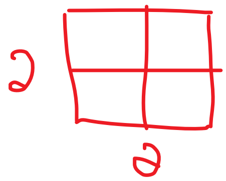
You can model perfect squares concretely using square tiles. For example, a square with side length 3 has an area of 9 tiles.



$$\sqrt{9} = 3 \quad or \quad 3^2 = 9$$

Example 1:

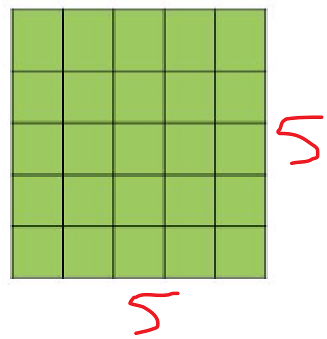
(A) Represent 2^2 by drawing algebra tiles.



$2^2 = 4 \therefore \sqrt{4} = 2$

(B) i. What perfect square is represented with the following algebra tiles?

25 or $5 \times 5 = 25$



ii. What is the square root of this number?

5

Find Square Roots Using Prime Factorization

The first method we will discuss is prime factorization. If a number is the product of equal groups of factors or if the factors can be grouped in sets of two, the number is a perfect square. For example,

16 is a perfect square since it can be written as $2 \times 2 \times 2 \times 2$. To find the square root, cross out each group of two and replace with a single 2. The product of all the single 2s is the square root.

16

$2 \cdot 8$

$2 \quad 2 \cdot 4$

$2 \cdot 2 \cdot 2 \cdot 2$

$\cancel{(2 \cdot 2)} \cdot \cancel{(2 \cdot 2)}$

$2 \cdot 2 = 4$

Example 2:
Determine if each number is a perfect square using prime factorization. If so, find the square root.

(A) 64

$8 \cdot 8$

$2 \cdot 4 \quad 2 \cdot 4$

$(2 \cdot 2)(2 \cdot 2)(2 \cdot 2)$

$2 \cdot 2 \cdot 2$

$= 8$

(B) 24

$6 \cdot 4$

$2 \cdot 3 \quad 2 \cdot 2$

$(2 \cdot 2) 2 \cdot 3$

Not a perfect square.

(C) 81

$3 \cdot 27$

$3 \quad 3 \cdot 9$

$(3 \cdot 3)(3 \cdot 3)$

$3 \cdot 3 = 9$

(D) 32

$2 \cdot 16$

$2 \quad 2 \cdot 8$

$2 \quad 2 \quad 2 \cdot 4$

$(2 \cdot 2)(2 \cdot 2) 2$

Not a perfect square.

Number of Factors
Another method involves whether a number has an odd or even number of factors. If a number has an **odd** number of factors, the number is a **perfect square**.

When the factors are listed in order, the middle factor is defined to be the square root.

For example, the factors of 36 are

1, 2, 3, 4, **6,** 9, 12, 18, 36.

There are 9 factors, which is an odd number. Therefore, 36 is a perfect square and 6, the middle factor, is the square root. Let's verify what we discovered in example 2 using the number of factors method.

Example 3:
Determine if each number is a perfect square using the number of factors. If so, find the square root.

(A) 64 odd #

1, 2, 4, 8, 16, 32, 64

Square root: 8

(B) 24 even #

1, 2, 3, 4, 6, 8, 12, 24

Not a perfect square.

(C) 81 odd #

1, 3, 9, 27, 81

Square root: 9

(D) 32 even #

1, 2, 4, 8, 16, 32

Not a perfect square

Example 4:
List the factors of 256. Use the factors to determine the square root of 256.

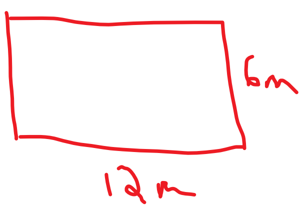
256
1, 256
2, 128
4, 64
8, 32
16, 16

256: 1, 2, 4, 8, 16, 32, 64, 128, 256

odd # ∴ perfect square

Square root: 16

Example 5:
David's rectangular living room is 12m by 6m. He has a square rug that covers half the area of the floor. What is the side length of the square rug?



$A = l \cdot w$
$A = (12m)(6m)$
$A = 72m^2$

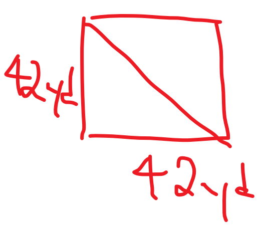
$\frac{72m^2}{2} = 36m^2$

36
3 · 12
3 3 · 4
$(\cancel{3 \cdot 3})(\cancel{2 \cdot 2})$

$3 \cdot 2 = 6$

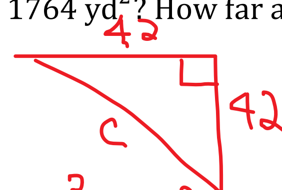
Rug has side length of 6m.

Example 6:
What is the length of the side of a square farm which contains 1764 yd^2? How far apart are its opposite corners?



1764
7 · 252
7 7 · 36
7 7 6 · 6
7 7 2 · 3 2 · 3

$(\cancel{2 \cdot 2})(\cancel{3 \cdot 3})(\cancel{7 \cdot 7})$
$2 \cdot 3 \cdot 7$
$= 42$

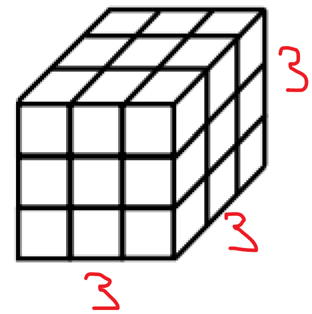


$c^2 = 42^2 + 42^2$
$c^2 = 1764 + 1764$
$\sqrt{c^2} = \sqrt{3528}$
$c = 59.4\,yd \sim 59\,yd$

It's a good idea to know as many perfect squares and their roots as possible.

Perfect Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Square Root	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Perfect Cubes and Cube Roots
When determining the cube root of a whole number, you should view the perfect cube number as the volume and the cube root as any one of the three equivalent dimensions. Students can model perfect cubes concretely using linking cubes.



For example, a cube with side length 3 has a volume of 27 cubes.

Therefore $\sqrt[3]{27} = 3$

Find Cube Roots Using Prime Factorization
Similar to square root, students can use prime factorization as a method to find the cube root of whole numbers as well. If a number is the product of equal factors or when the factors can be grouped in sets of three, the number is a perfect cube.

For example, 64 is a perfect cube because it can be written as $2 \times 2 \times 2 \times 2 \times 2 \times 2$. To find the cube root, cross out each group of three and replace with a single 2. The product of all the single 2s is the cube root.

64
8 · 8
2 · 4 2 · 4
(2 2 2)(2 2 2)

2 · 2
= 4

Example 7:
Determine if each number is a perfect cube using prime factorization. If so, find the cube root.

(A) 27
3 · 9
(3 3 · 3)
cube root: 3

(B) 81
3 · 27
3 3 · 9
(3 3 3) 3
Not a perfect cube.

(C) 216
8 · 27
2 · 4 3 · 9
(2 2 2)(3 3 3)
2 · 3
= 6

(D) 32
2 · 16
2 2 · 8
2 2 2 · 4
(2 · 2 · 2) 2 · 2
Not a perfect cube.

Example 8:

If 1000 linking cubes were combined to make a giant cube, what is the area of each face?

1000

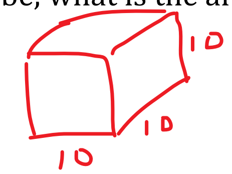
$10 \cdot 100$

$2 \cdot 5 \quad 10 \cdot 10$

$2 \quad 5 \quad 2 \cdot 5 \quad 2 \cdot 5$

$(\cancel{2 \cdot 2 \cdot 2})(\cancel{5 \cdot 5 \cdot 5})$

cube root: $2 \cdot 5 = 10$



$A = s^2 = 10^2 = 100 \text{ units}^2$

Example 9:

Simplify the following:

(A) $\sqrt[3]{343} = 7$

343

$7 \cdot 49$

$(\cancel{7 \cdot 7 \cdot 7})$

(B) $\sqrt{121} - \sqrt[3]{216} = 11 - 6 = 5$

$(\cancel{11 \cdot 11})$

11

$6 \cdot 36$

$2 \cdot 3 \quad 6 \cdot 6$

$2 \quad 3 \quad 2 \cdot 3 \quad 2 \cdot 3$

$(\cancel{2 \cdot 2 \cdot 2})(\cancel{3 \cdot 3 \cdot 3})$

$2 \cdot 3 = 6$

BEDMAS!!

(C) $\sqrt[3]{64} + \sqrt[3]{1000} \div \sqrt{25} = 4 + 10 \div 5$

$8 \cdot 8$

$2 \cdot 4 \quad 2 \cdot 4$

$(\cancel{2 \cdot 2 \cdot 2})(\cancel{2 \cdot 2 \cdot 2})$

$2 \cdot 2 = 4$

$10 \cdot 100$

$2 \cdot 5 \quad 10 \cdot 10$

$2 \quad 5 \quad 2 \cdot 5 \quad 2 \cdot 5$

$(\cancel{2 \cdot 2 \cdot 2})(\cancel{5 \cdot 5 \cdot 5})$

$2 \cdot 5 = 10$

$(\cancel{5 \cdot 5})$

5

$= 4 + 2$

$= 6$

Example 10:
If the volume of a cube is 125 m^3, what is the expression for the length of each side?

125

$25 \cdot 5$

$\cancel{(5 \cdot 5 \cdot 5)}$

$5m$

Example 11:
A large aquarium has a volume of 125 m^3. It has glass on the bottom and four sides but no top. The edges are reinforced with angle iron.

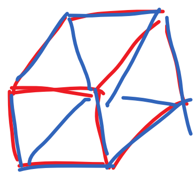
(A) What is the area of glass required?

125

$5 \cdot 25$

$\cancel{(5 \cdot 5 \cdot 5)}$

5



$A = 5(s^2)$

$A = 5(5m)^2$

$A = 125m^2$

(B) What is the length of angle iron required?

$12 \times 5m = 60m$

Example 12:
A right rectangular prism measures 9 in × 8 in × 24 in. What are the dimensions of a cube with the same volume?

$9in \times 8in \times 24in = 1728 in^3$

$3 \cdot 3 \quad 2 \cdot 4 \quad 4 \cdot 6$

$2 \quad 2 \cdot 2 \quad 2 \cdot 2 \quad 2 \cdot 3$

$\cancel{(2 \cdot 2 \cdot 2)}\cancel{(2 \cdot 2 \cdot 2)}\cancel{(3 \cdot 3 \cdot 3)}$

$2 \cdot 2 \cdot 3 = 12$

Cube: $12in \times 12in \times 12in$

x^3

Example 13:
Determine the cube root of 3375 in a variety of ways. This could include the use of prime factorization, the use of benchmarks and/or the use of a calculator. $\boxed{y^x}$.

3375
75 45
25·3 9·5
5·5 3 3·3 5

$(\cancel{3\cdot3\cdot3})(\cancel{5\cdot5\cdot5})$

$3\cdot5=15$

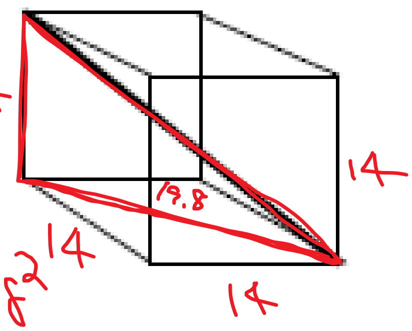
$(3375)\boxed{\wedge}(1\div3)$
$=15$

Example 14:
A cube has a volume of 2744 cm^3. What is the diagonal distance through the cube from one corner to the opposite corner?

2744
8·343
2·4 7·49
$(\cancel{2\cdot2\cdot2})(\cancel{7\cdot7\cdot7})$
$2\cdot7=14$

$c^2=14^2+14^2$
$\sqrt{c^2}=\sqrt{392}$
$c=19.8$

$z^2=14^2+19.8^2$
$\sqrt{z^2}=\sqrt{588.04}$
$\boxed{z=24.2}$



Example 15:
The lowest whole number which is both a perfect square and a perfect cube is the number 1. Determine the next lowest number which is both a perfect square and a perfect cube, and explain the strategy you used.

PS: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

PC: 1, 8, 27, 64

It's a good idea to know as many perfect cubes and their roots as possible.

Perfect Cube	1	8	27	64	125	216	343	512	729	1000
~~Square~~ Cube Root	1	2	3	4	5	6	7	8	9	10

Textbook Questions: page 146, 147 #4, 5, 6, 7, 8, 10, 11, 13,