Math 1201 **Date:**___________________

3.3 Common Factors of a Polynomial

A **term** can be a number, a variable, the product of variables, or the product of numbers and variables.

Some examples of terms are: 10, x, $2a^2$, $-3xy$

Polynomial - an expression of more than two algebraic terms, especially the sum of several terms that contain different powers of the same variable.

Types of Polynomials

Monomial – a single termed polynomial. For example: $5x$, $3x^2$

Binomial – two termed polynomial. For example: $3x^2 - 4x$, $-2x - 7$

Trinomial – three termed polynomial: For example: $3xy^4 - 2xy - 7y$, $3x^2 - 2x - 7$

In Grade 9, students multiplied a monomial by a polynomial using the distributive property. For all numbers a, b and c,

$$a(b + c) = ab + ac$$

Example 1:
Expand: $3(x + 5)$

$3(x+5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$

Example 2:
Expand:

(A) $5(a - 3)$ (B) $c(c + 7)$ (C) $2j(5 + j)$ (D) $-3f(2f - 7)$

(A) $= 5 \cdot a - 5 \cdot 3$
$= 5a - 15$

(B) $= c \cdot c + c \cdot 7$
$= c^2 + 7c$

(C) $= 2j \cdot 5 + 2j \cdot j$
$= 10j + 2j^2$

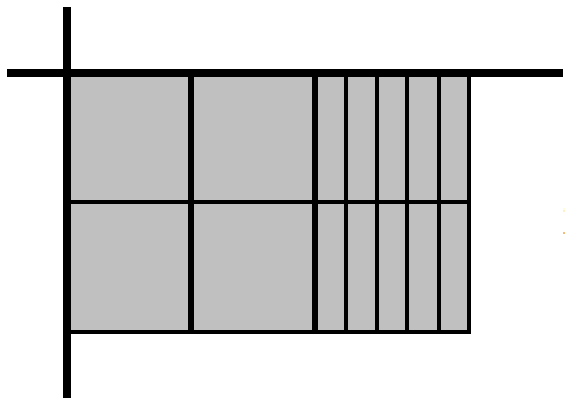
(D) $= -6f^2 + 21f$

You will now be exposed to a polynomial written in expanded form and will be expected to write the polynomial as a product of its factors

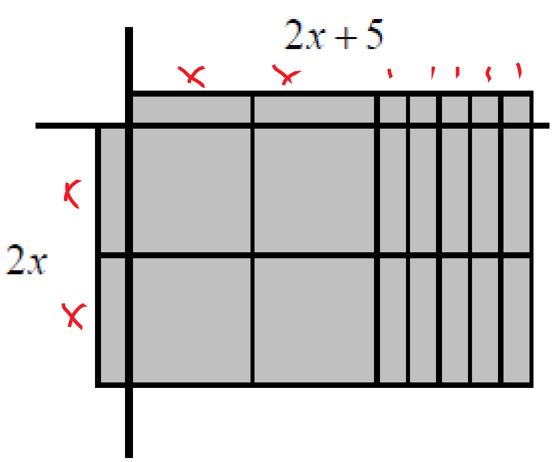
$$ab + ac \;=\; a(b + c)$$

Factoring Using Algebra Tiles

Factoring polynomials is the inverse operation of multiplying polynomials. The initial step when factoring a polynomial is identifying a common factor in every term. Because there are a variety of methods available, you will be given the opportunity to apply your own personal strategies. Factoring out a common term can be visualized, for example, using a rectangle model with algebra tiles. For example, when factoring $4x^2 + 10x$, arrange the tiles as follows:



The factors of the polynomial are then found by counting the right and top side of the rectangle.



You can check your solution by expanding the factors using the distributive property as follows:

$4x^2 + 10x = 2x(2x+5)$

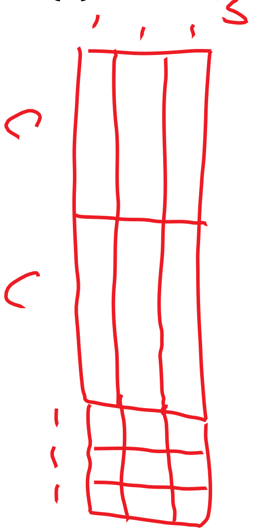
Check: $2x(2x+5)$

$= 2x \cdot 2x + 2x \cdot 5$

$= 4x^2 + 10x$ ✓

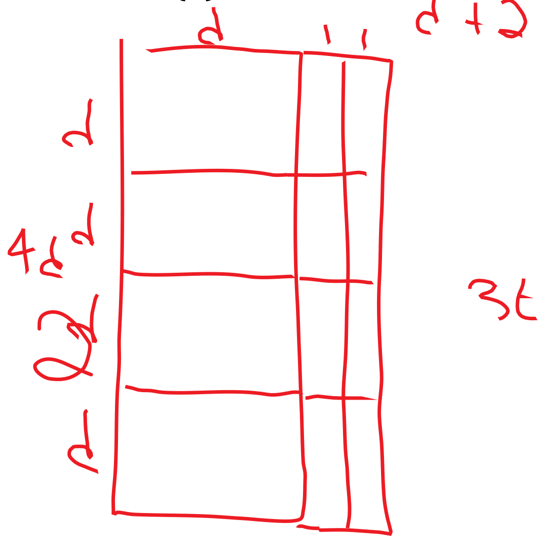
Example 3:
Factor the following polynomials using algebra tiles:

(A) $6c + 9$



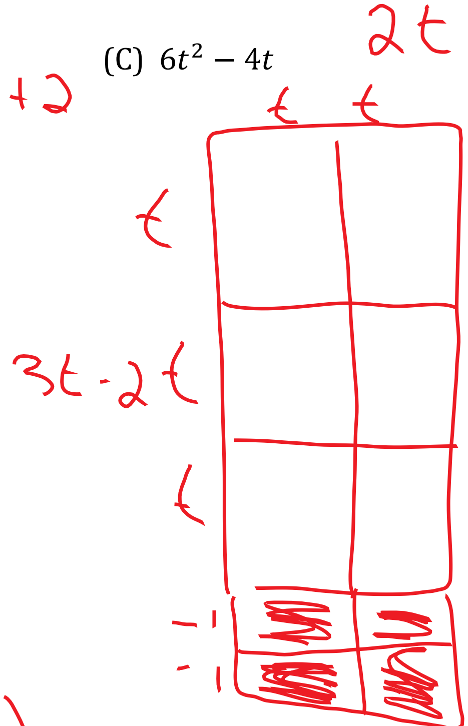
Check: $3(2c+3)$
$= 6c+9$

(B) $4d^2 + 8d$



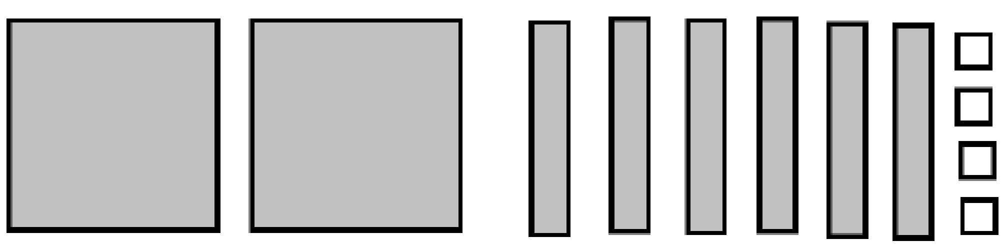
Check: $4d(d+2)$
$= 4d^2 + 8d$

(C) $6t^2 - 4t$



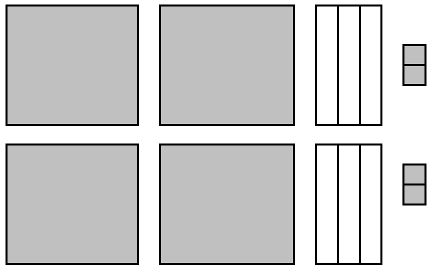
Check: $2t(3t-2)$
$6t^2 - 4t$

Example 4:
What trinomial is represented by the following tiles?



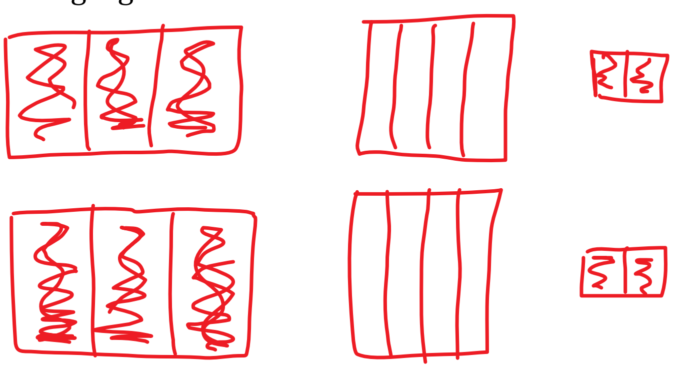
$2x^2 + 6x - 4$

Depending on the polynomial given, a common factor may not be able to be removed by arranging the tiles as a rectangle. Instead the tiles will be arranged into equal groups. For example, the trinomial $4x^2 - 6x + 4$ can be broken into two equal groups of:



Therefore, $4x^2 - 6x + 4 = 2(2x^2 - 3x + 2)$

Example 5:
Factor using algebra tiles $6x^2 - 8x + 4$



$2(3x^2 - 4x + 2)$

Not all polynomials have a common factor. For example, the polynomial $2x^2 + 3x + 7$ has no common factors, other than 1. Therefore, this is considered a polynomial that does not have a GCF.

Factoring Algebraically

Algebra tiles are time consuming and not always practical. Most mathematicians would choose to factor algebraically. The GCF can be determined using prime factorization. You are familiar with finding the GCF of two or more numbers from Chapter 3. This will now be extended to include variables. Consider the following example:

$4x^2 + 10x$:

GCF: $2x$

$4x^2 + 10x$

$2 \cdot 2 \cdot x \cdot x + 2 \cdot 5 \cdot x$

$2x \cdot 2x + 2x \cdot 5$

$2x(2x + 5)$

Check:

$2x(2x+5)$

$= 2x \cdot 2x + 2x \cdot 5$

$= 4x^2 + 10x$

The product of the common prime factors results in the GCF. You will write each term as a product of the GCF and its remaining factors and then use the distributive property:

Example 6:
Factor:

(A) $5m^2 - 35m$

$= 5m(m-7)$

$5m^2 - 35m$

$5 \cdot m \cdot m - 5 \cdot 7 \cdot m$

$5m \cdot m - 5m \cdot 7$

$5m(m - 7)$

Check: $5m^2 - 35m$ ✓

(B) $3x^4 + x^2$

$x^2(3x^2 + 1)$

$3x^4 + x^2$

$3 \cdot x \cdot x \cdot x \cdot x + x \cdot x$

$x x \cdot 3 x x + x \cdot x \cdot 1$

$x^2 \cdot 3x^2 + x^2 \cdot 1$

$x^2(3x^2 + 1)$

(C) $15e^2g^5 - 20e^7g^2$

$5e^2g^2(3g^3 - 4e^5)$

$15e^2g^5 - 20e^7g^2$

$= 3 \cdot 5eeggggg - 2 \cdot 2 \cdot 5eeeeeeegg$

$= 5eegg \cdot 3ggg - 5eegg\, 2 \cdot 2eeeee$

$= 5e^2g^2 \cdot 3g^3 - 5e^2g^2 \cdot 4e^5$

$= 5e^2g^2(3g^3 - 4e^5)$

(D) $5x^2 + 15x + 10$

$= 5(x^2 + 3x + 2)$

$5x^2 + 15x + 10$

$5xx + 3 \cdot 5x + 2 \cdot 5$

$5 \cdot x \cdot x + 5 \cdot 3x + 5 \cdot 2$

$5(x^2 + 3x + 2)$

Check: $5x^2 + 15x + 10$

(E) $3 - 6x - 9x^2$

$= 3(1 - 2x - 3x^2)$

Check: $3 - 6x - 9x^2$

(F) $10x^3 + 15x^2 + 20x$

$5x(2x^2 + 3x + 4)$

(G) $14c^2 + 49c - 21$

$7(2c^2 + 7c - 3)$

Check:

$14c^2 + 49c - 21$ ✓

(H) $24 + 8m^2 - 16m$

$8(3 + m^2 - 2m)$

Check: $24 + 8m^2 - 16m$ ✓

(I) $-15n^2 - 20n - 30$

$= -5(3n^2 + 4n + 6)$

Check: $-15n^2 - 20n - 30$ ✓

(J) $6x^2 + 10x + 8$

$2(3x^2 + 5x + 4)$

Check: $6x^2 + 10x + 8$ ✓

(K) $3m^2 + 5m - 7 + m^2 + m - 1$

$4m^2 + 6m - 8$

$= 2(2m^2 + 3m - 4)$

Check: $4m^2 + 6m - 8$

(L) $2c^2 - 7c + 3 + 3c^2 - 3c + 12$

$= 5c^2 - 10c + 15$

$= 5(c^2 - 2c + 3)$

Check:

$5c^2 - 10c + 15$

A polynomial with two different variables can also be factored algebraically by finding the GCF:

$16m^3n^2 + 12mn^2 + 20m^2n^2$

$4mn^2(4m^2 + 3 + 5m)$

Check: $16m^3n^2 + 12mn^2 + 20m^2n^2$ ✓

Example 7:
Factor:

(A) $-20a^4b + 30a^3b^2 - 25ab$

$= -5ab(4a^3 - 6a^2b + 5)$

(B) $72m^3n + 18mn - 24m^3$

$= 6m(12m^2n + 3n - 4m^2)$

(C) $-30a^2b - 12a^2b - 6ab$

$= -42a^2b - 6ab$

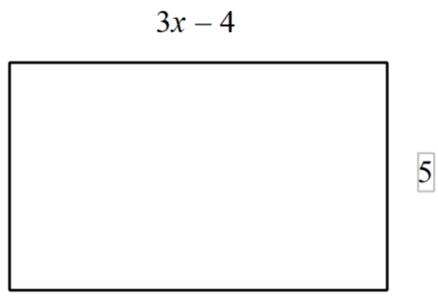
$= -6ab(7a - 1)$

(D) $18a^5b^3 + 9a^3b^2 - 27b$

$9b(2a^5b^2 + a^3b - 3)$

Example 8:
What is the area of the each rectangle?

(A)

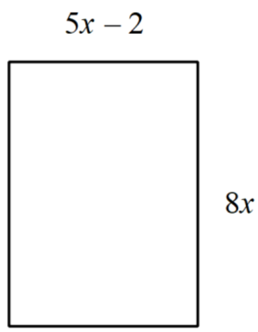


$A = l \cdot w$

$A = (3x - 4)5$

$A = 15x - 20$

(B)

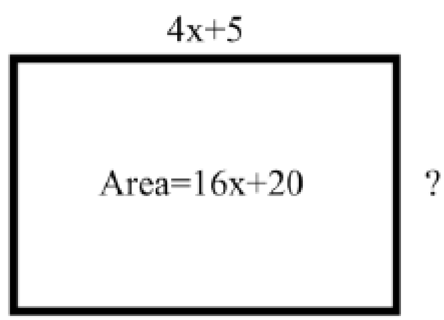


$A = l \cdot w$

$A = (5x - 2) \cdot 8x$

$A = 40x^2 - 16x$

Example 9:
Determine the value of the missing side of the rectangle:



$A = 16x + 20$

$A = 4(4x + 5)$

width is 4

Textbook Questions: page 155 - 156 #4 - 12, 14 - 18