Math 1201 **Date:**__________________

3.5A Polynomials of the Form $x^2 + bx + c$

In this section we will use different strategies to multiply binomials of the form:

$$(x + d)(x + e)$$

and to factor polynomials of the form:

$$x^2 + bx + c$$

In algebra, terms are separated by addition or subtraction signs. A term or group of terms is a polynomial.

Monomial: any polynomial with one term. Examples: $3x$, $-2x^2$, $3xy^3$

Binomial: any polynomial with two terms. Examples: $3x - 2$, $4x^2 - 9$, $4x^5y^6 + 12x^7y^8$

Trinomial: any polynomial with three terms. Examples: $3x^2 + 7x - 9$, $x^2 - 9x + 11$

Multiplying polynomials is also called **expansion.**

Multiplying Binomials Using the Rectangle Model
Before we multiply two binomials, let's look at the rectangle model to illustrate multiplication of two digit numbers.

When calculating 15×12, for example, the number 15 can be expressed as the sum of $10 + 5$ and 12 as the sum of $10 + 2$. The sum of the products of the four smaller rectangles equals the product of the large rectangle.

	10	2
10	10x10=100	10x2=20
5	5x10=50	5x2=10

15×12=100+20+50+10
=180

Check: 15×12=180

Example 1:
Use the rectangle model to illustrate the multiplication of the following two digit numbers:

(A) 17×14

	10	4
10	$10 \times 10 = 100$	$10 \times 4 = 40$
7	$7 \times 10 = 70$	$7 \times 4 = 28$

$100 + 40 + 70 + 28$
$= 238$

Check: $17 \times 14 = 238$

(B) 11×16

	10	6
10	100	10
1	60	6

$100 + 10 + 60 + 6$
$= 176$

(C) 21×12

	10	2
20	200	40
1	10	2

$200 + 10 + 40 + 2$
$= 252$

We can adopt this strategy when multiplying binomials of the form $(x + d)(x + e)$

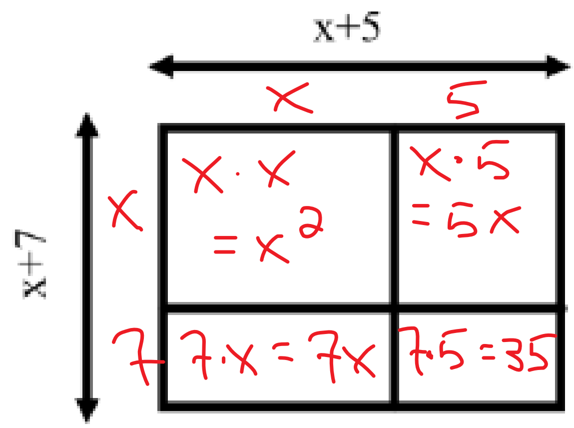
Let's multiply (expand) using the rectangle model the following binomials: $(x + 2)(x - 3)$:

	x	-3
x	$(x)(x)=x^2$	$(x)(-3)=-3x$
2	$(2)(x)=2x$	$(2)(-3)=-6$

$(x+2)(x-3) = x^2-3x+2x-6$
$(x+2)(x-3) = x^2-x-6$

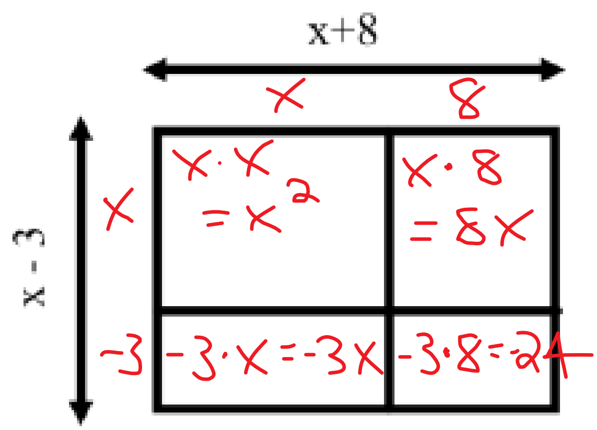
Example 2:
Use the rectangle model to aid in the multiplication of the following binomials:

(A)



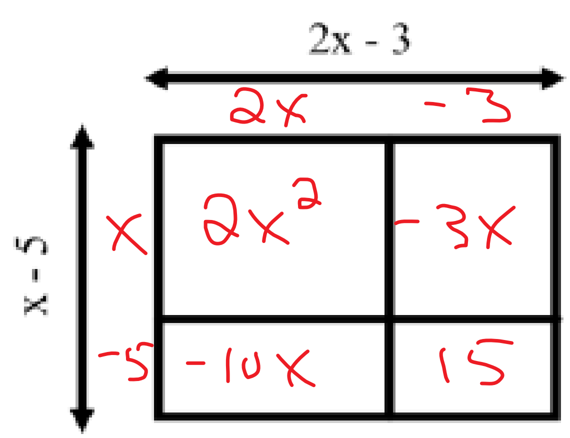
$x^2 + 5x + 7x + 35$
$= x^2 + 12x + 35$

(B)



$x^2 + 8x - 3x - 24$
$= x^2 + 5x - 24$

(C)



$2x^2 - 3x - 10x + 15$
$= 2x^2 - 13x + 15$

Example 3:
Use the rectangle model to multiply the following:

(A) $(4x + 6)(3x - 5)$

	$4x$	6
$3x$	$12x^2$	$18x$
-5	$-20x$	-30

$12x^2 + 18x - 20x - 30$

$= 12x^2 - 2x - 30$

(B) $(2m + 2)(5m - 4)$

	$5m$	-4
$2m$	$10m^2$	$-8m$
2	$10m$	-8

$10m^2 - 8m + 10m - 8$

$= 10m^2 + 2m - 8$

(C) $(-3x + 2)(2x - 7)$

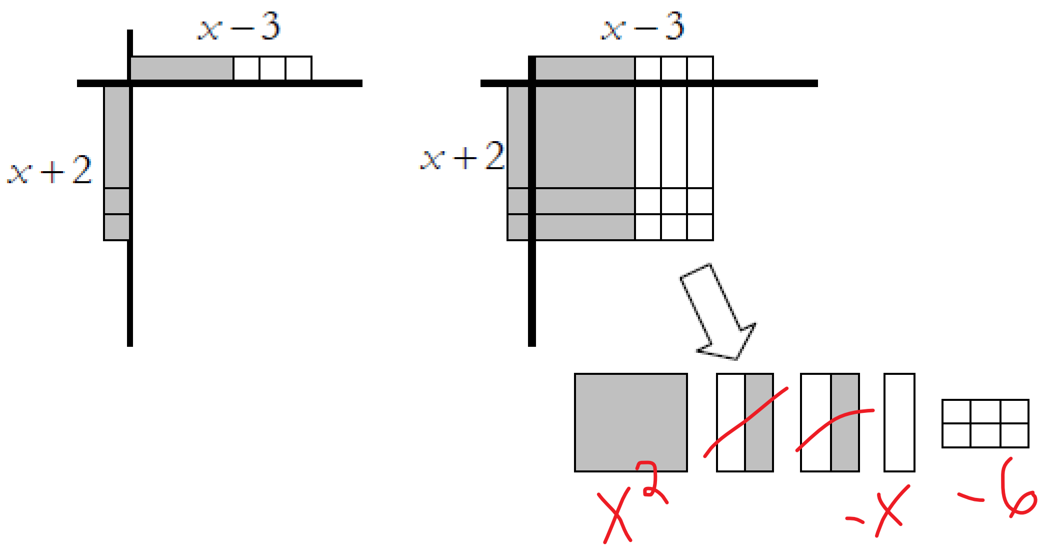
	$2x$	-7
$-3x$	$-6x^2$	$21x$
2	$4x$	-14

$-6x^2 + 21x + 4x - 14$

$= -6x^2 + 25x - 14$

Multiplying Binomials with Algebra Tiles

Concrete models are critical in mathematics because most mathematical ideas are abstract. The algebra tile model will create a rectangle diagram resulting in the product of two binomial factors. You can multiply, for example, $(x + 2)(x - 3)$:



The algebra tile model allows you to visualize the process. The product of two binomials is the sum of the tiles in the grid. In this model, the pairs of opposite x-tiles cancel, resulting in the product: $x^2 - 6x - 6$.

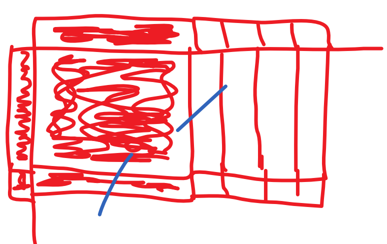
Example 4:

Use algebra tiles to find the product of the following binomials:

(A) $(x + 3)(x + 5)$

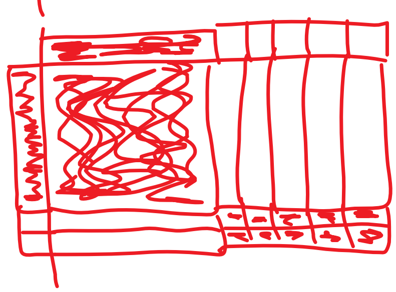
$x^2 + 8x + 15$

(B) $(x + 1)(x - 4)$



$x^2 - 3x - 4$

(C) $(x - 2)(x - 5)$



$x^2 - 7x + 10$

Multiplying Binomials Using the Distributive Property

The distributive property tells us how to solve expressions in the form of $a(b+c)$. The distributive property is sometimes called the distributive law of multiplication and division. When multiplying two binomials, you are applying the distributive property twice. Let's compare to the distributive property to the rectangle model:

	c	d
a	ac	ad
b	bc	bd

$$(a+b)(c+d)=a(c+d)+b(c+d)$$
$$=ac+ad+bc+bd$$

The distributive property can make more complex calculations easier. For example:

$$(x+2)(x+3)=x(x+3)+2(x+3)$$
$$=x^2+3x+2x+6$$
$$=x^2+5x+6$$

Example 5:

Use the distributive property to find the product of the following binomials:

(A) $(x+5)(x+4)$

$$=x(x+4)+5(x+4)$$
$$=x^2+4x+5x+20$$
$$=x^2+9x+20$$

(B) $(d-1)(d+14)$

$$=d(d+14)-1(d+14)$$
$$=d^2+14d-d-14$$
$$=d^2+13d-14$$

(C) $(2+f)(f-7)$

$= 2(f-7) + f(f-7)$

$= 2f - 14 + f^2 - 7f$

$= f^2 - 5f - 14$

(D) $(9-w)(5-w)$

$= 9(5-w) - w(5-w)$

$= 45 - 9w - 5w + w^2$

$= w^2 - 14w + 45$

(E) $(k+10)(k-40)$

$= k(k-40) + 10(k-40)$

$= k^2 - 40k + 10k - 400$

$= k^2 - 30k - 400$

Textbook Questions: page 166 - 167 #4, 5,, 9, 10, 12, 13, 18