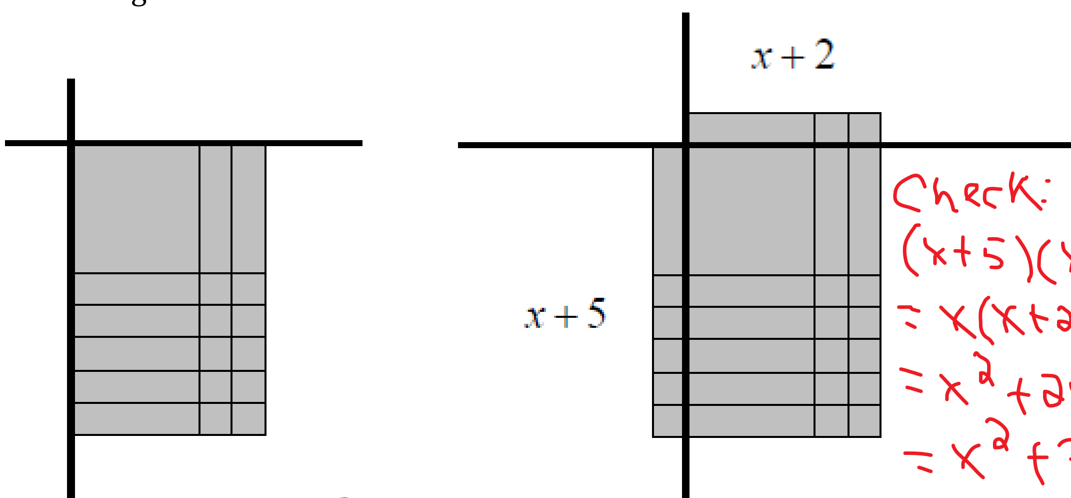
Math 1201 Date:__________________

3.5B Polynomials of the Form $x^2 + bx + c$

When we multiplied two binomials, the factors of the rectangle were given and the product was determined. The process for determining the common factors of a polynomial is very similar to the process of trinomial factoring. In both situations, the product of the rectangle will be given but the factors will need to be determined.

Consider the example: Factor $x^2 + 7x + 10$

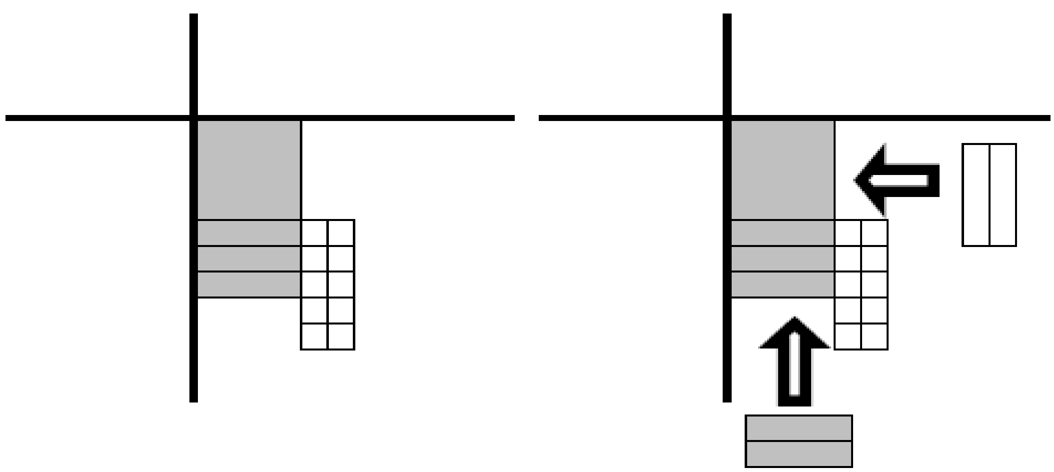
Using one positive x^2 - tile, seven positive x-tiles, and ten positive unit tiles, we will arrange the tiles in a rectangle.



Check:
$(x+5)(x+2)$
$= x(x+2)+5(x+2)$
$= x^2+2x+5x+10$
$= x^2+7x+10$

$x^2+7x+10=(x+5)(x+2)$

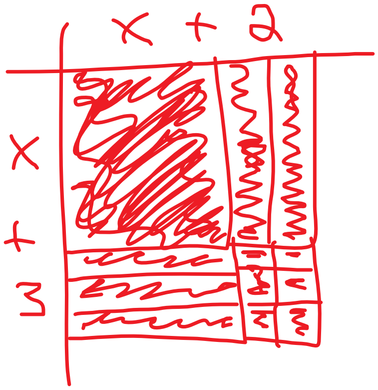
We can also factor trinomials with negative terms. These particular examples involve using the Zero Principle. Pairs of opposite tiles are added to form a rectangle. For example, to model the factoring of $x^2 + 3x - 10$, you should attempt to form a rectangle. In order to complete the rectangle, 2 pairs of opposite x-tiles must be added.



$x^2+3x-10=(x+5)(x-2)$

Example 1:
Factor using algebra tiles:

(A) $x^2 + 5x + 6$



$x^2 + 5x + 6 = (x+2)(x+3)$

Check:

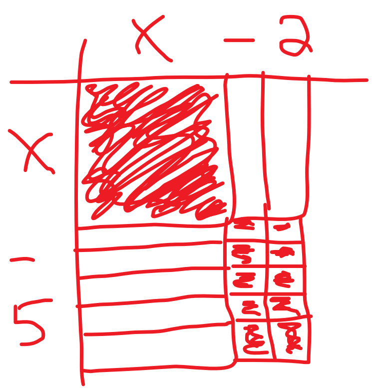
$(x+2)(x+3)$

$= x(x+3) + 2(x+3)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

(B) $x^2 - 7x + 10$



$x^2 - 7x + 10 = (x-5)(x-2)$

Check:

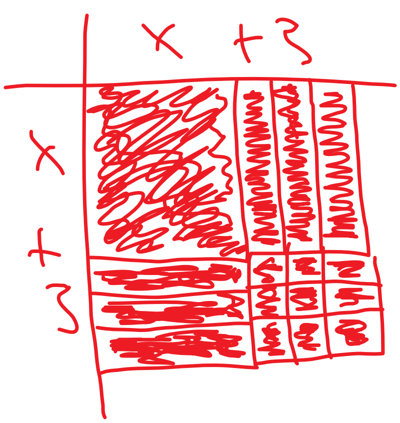
$(x-5)(x-2)$

$= x(x-2) - 5(x-2)$

$= x^2 - 2x - 5x + 10$

$= x^2 - 7x + 10$

(C) $x^2 + 6x + 9$



$x^2 + 6x + 9 = (x+3)(x+3)$

Check:

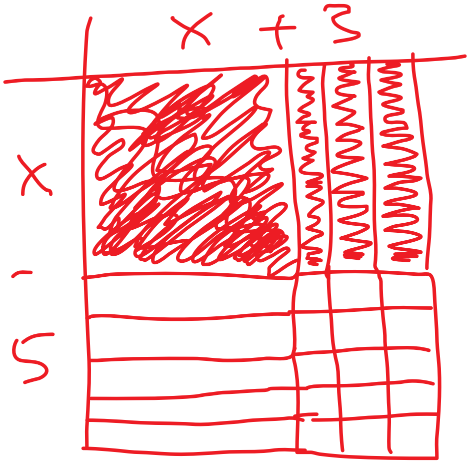
$(x+3)(x+3)$

$= x(x+3) + 3(x+3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

(D) $x^2 - 2x - 15$



$x^2 - 2x - 15 = (x+3)(x-5)$

Check:

$x(x-5) + 3(x-5)$

$= x^2 - 5x + 3x - 15$

$= x^2 - 2x - 15$

Factoring Algebraically

As mentioned previously, factoring is the reverse process of expanding. Essentially we are using the distributive property in reverse. To factor $x^2 + 7x + 6$, we use the product - sum method. We need to find two numbers that: multiply to give us 6, add to give us 7. Let's give it a try:

$$\overset{a}{1}x^2 + \overset{b}{7}x + \overset{c}{6}$$

6
1, 6
2, 3

Step 1: List factors of c

$(x+1)(x+6)$

Step 2: which factors can combine to add up to b.

Step 3: Set up two sets of brackets

Step 4: Add your "x"s

Step 5: Add your factors

Step 6: Add your signs

Check:

$(x+1)(x+6)$

$= x(x+6) + 1(x+6)$

$= x^2 + 6x + x + 6$

$= x^2 + 7x + 6$

You should observe patterns that result from factoring polynomials of the form $x^2 + bx + c$ and record your work symbolically. When the polynomial is written in descending order, they should recognize relationships between the two binomial factors and the coefficients of the trinomial. When the coefficient of the x^2 term is one, students should recognize that:

- the coefficient of the squared term in the trinomial is always 1
- the sum of the constant terms in the binomial factors is the coefficient of the middle term, b, in the trinomial
- the linear product of the constant terms in the binomial factors is the constant term, c, in the trinomial

Example 3:
Factor:

(A) $x^2 + 15x + 14$

14: 1, 14; 2, 7

$(x+1)(x+14)$

Check:

$x(x+14)+1(x+14)$

$= x^2+14x+x+14$

$= x^2+15x+14$

(B) $x^2 + 7x + 12$

12: 1, 12; 2, 6; 3, 4

$(x+3)(x+4)$

$x(x+4)+3(x+4)$

$= x^2+4x+3x+12$

$= x^2+7x+12$

(C) $x^2 + 9x + 20$

20: 1, 20; 2, 10; 4, 5

$(x+4)(x+5)$

(D) $x^2 + 2x - 8$

8: 1, 8; 2, 4

$(x-2)(x+4)$

Check:

$x(x+4)-2(x+4)$

$= x^2+4x-2x-4$

$= x^2+2x-4$

(E) $x^2 + 8x - 48$

48: 1, 48; 2, 24; 3, 16; 4, 12; 6, 8

$(x-4)(x+12)$

(F) $x^2 - x - 72$

72: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12; 8, 9

$(x+8)(x-9)$

Be sure you understand that sometimes a trinomial has a common factor that should be factored out before the trinomial is factored into two binomials.

When factoring $2x^2 + 14x + 24$, for example, the greatest common factor is 2 resulting in $2(x^2 + 7x + 12)$. You can then continue to factor the trinomial using the sum and product method:

$2x^2+14x+24$
$= 2(x^2+7x+12)$
$= 2(x+3)(x+4)$

12
1, 12
2, 6
3, 4

Example 4:
Factor:

(B) $2x^2 - 8x - 24$

$= 2(x^2-4x-12)$
$= 2(x+2)(x-6)$

12
1, 12
2, 6
3, 4

(B) $4x^2 + 16x - 128$

$4(x^2+4x-32)$
$4(x-4)(x+8)$

(C) $-5x^2 - 20x + 60$

$= -5(x^2+4x-12)$
$= -5(x-2)(x+6)$

12
1, 12
2, 6
3, 4

Example 4:
Explain why $x^2 + 3x + 4$ cannot be factored.

$$\frac{4}{\begin{matrix}1,4\\2,2\end{matrix}}$$

$(x-1)(x+4)$

Check:

$x(x+4)-1(x+4)$

$= x^2+4x-x-4$

$\rightarrow = x^2+3x-4$

Not factorable b/c there aren't two numbers that multiply to get 4 and add to get 3.

Example 5:
How many integer values are there for k for which $x^2 + kx + 24$ is factorable?

$$\frac{24}{\begin{matrix}1,24\\2,12\\3,8\\4,6\end{matrix}}$$

$1+24=25$ $\quad -1+(-24)=-25$

$2+12=14$ $\quad -2+(-12)=-14$

$3+8=11$ $\quad -3+(-8)=-11$

$4+6=10$ $\quad -4+(-6)=-24$

8 possible values for k.

Example 6:

(A) Given $s^2 - 3s - 10$, find and correct the mistake in the factoring below.

Sum $= -3$, product $= -10$
numbers are -2 and 5
$(s-2)(s+5)$

Check:

$s(s+5)-2(s+5)$

$= s^2+5s-2s-10$

$= s^2+3s-10$

Signs are reversed.

$(s+2)(s-5)$

(B) Given $2x^2 + 6x - 40$, find and correct the mistakes in the factoring below.

$2x^2 + 6x - 40$
$2(x^2 + 3x - 40)$
$2(x - 8)(x + 5)$

Forgot to factor 2 out of 40.

$2(x^2+3x-20)$

Not factorable.

$$\frac{20}{\begin{matrix}1,20\\2,10\\4,5\end{matrix}}$$

Textbook Questions: page 166 - 167 #6, 7, 8, 11, 14, 15, 19, 20, 21