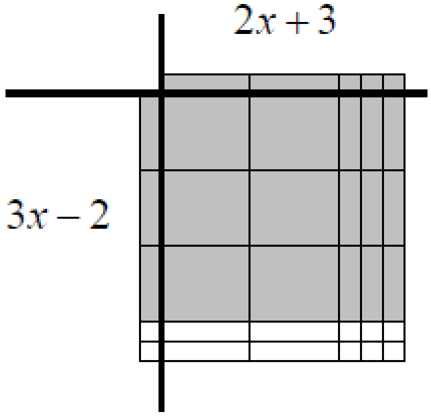
Math 1201 **Date:**________________

3.6 Polynomials of the Form $ax^2 + bx + c$

In this section we will adapt the strategies for multiplying binomials of the form $(x+e)(x+g)$ to multiply binomials of the form $(dx+e)(fx+g)$. Be sure to continue the use of a rectangle model and algebra tiles if it helps you to understand the concept. Continue to check your work symbolically through the use of the distributive property.

For example, the product $(3x-2)(2x+3) = 6x^2 + 5x - 6$ can be modeled as follows:



Distributive Property:

$(3x-2)(2x+3)$

$= 3x(2x+3) - 2(2x+3)$

$= 6x^2 + 9x - 4x - 6$

$= 6x^2 + 5x - 6$

Example 1:
Expand, using algebra:

(A) $(2x+3)(3x+1)$

$= 2x(3x+1) + 3(3x+1)$

$= 6x^2 + 2x + 9x + 3$

$= 6x^2 + 11x + 3$

(B) $(3x+4)(x-2)$

$= 3x(x-2) + 4(x-2)$

$= 3x^2 - 6x + 4x - 8$

$= 3x^2 - 2x - 8$

(C) $(2x+1)(2x+1)$

$= 2x(2x+1) + 1(2x+1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

Example 2:
Expand, using algebra:

(A) $(-5x+3)(-2x+4)$ (B) $(-3x+4)(-8-3x)$ (C) $(2x+11)(-7x-8)$

$= -5x(-2x+4)+3(-2x+4)$
$= 10x^2-20x-6x+12$
$= 10x^2-26x+12$

$= -3x(-8-3x)+4(-8-3x)$
$= 24x+9x^2-32-12x$
$= 9x^2+12x-32$

$= 2x(-7x-8)+11(-7x-8)$
$= -14x^2-16x-77x-88$
$= -14x^2-93x-88$

Factoring Trinomials of the Form ax^2+bx+c
When factoring trinomials of the form, ax^2+bx+c, we will revisit the rectangle model. Be sure to record the process symbolically when using concrete materials or sketches. Use logical reasoning when factoring trinomials. This is an efficient method if the values of the coefficients are relatively small. The ability to multiply binomials mentally is beneficial when using this method. You can produce a possible binomial products relating the constant term and the leading coefficient of the trinomial.

For example, when factoring $2x^2+7x+6$ the coefficient of the 1st terms in the binomial are factors of 2 and the coefficient of the 2nd terms in the binomial are factors of 6. Check the various representations of the binomial products by using the distributive property. This is called the "Guess and Check" method. Let's give it a try:

2
1, 2

6
1, 6
2, 3

$(2x+2)(x+3)$
Check:
$= 2x(x+3)+2(x+3)$
$= 2x^2+6x+2x+6$
$= 2x^2\cancel{+8x}+6$

$(2x+1)(x+6)$
Check:
$= 2x(x+6)+1(x+6)$
$= 2x^2+12x+x+6$
$= 2x^2\cancel{+13x}+6$

$(2x+3)(x+2)$
Check:
$= 2x(x+2)+3(x+2)$
$= 2x^2+4x+3x+6$
$= 2x^2+7x+6$ ✓

As you can see, this is not an efficient method for factoring. Especially when there's a lot of factors for a and c.

Factoring By Decomposition

Students can also use the visual representation of decomposition through a rectangle model. When factoring the trinomial $ax^2 + bx + c$, you will split the middle term into two seperate terms and then find factors of the product ac which have a sum of b. Consider the following example:

Factor: $2x^2 - 11x + 5$ 10

10
1, 10
2, 5

Draw a two-by-two grid. Place the first term in the upper left-hand corner and the last term in the lower right-hand corner as shown. Then place the factors – 1 and -10, complete with their signs and variables, in the diagonal corners. The placement of the diagonal entries will not affect the factors. Be sure to remove a negative common factor when the leading coefficient is negative.

$2x^2$	$-10x$
$-x$	5

$2x^2$	$-10x$
$-1x$	5

	x	-5
$2x$	$2x^2$	$-10x$
-1	$-1x$	5

$(2x-1)(x-5)$

Check:

$2x(x-5) - 1(x-5)$

$= 2x^2 - 10x - x + 5$

$= 2x^2 - 11x + 5$

Example 3:

Factor using the rectangle model:

(A) $4x^2 + 11x + 6$

24
1, 24
2, 12
3, 8
4, 6

	x	$+2$
$4x$	$4x^2$	$8x$
$+3$	$3x$	6

$(4x+3)(x+2)$

Check:

$= 4x(x+2) + 3(x+2)$

$= 4x^2 + 8x + 3x + 6$

$= 4x^2 + 11x + 6$

(B) $6x^2 - 7x - 10$

60
1, 60
2, 30
3, 20
4, 15
5, 12
6, 10

	$6x$	5
x	$6x^2$	$5x$
-2	$-12x$	-10

$(6x+5)(x-2)$

Check:

$6x(x-2) + 5(x-2)$

$= 6x^2 - 12x + 5x - 10$

$= 6x^2 - 7x - 10$

The method of decomposition is an inverse procedure from multiplying binomials. The trinomial $ax^2 + bx + c$ is decomposed by replacing the middle term bx with two terms whose integer coefficients have a sum of b and a product of ac.

For example, when factoring $2x^2 + 7x + 6$ you will rewrite the expression as $2x^2 + 4x + 3x + 6$. You will then factor by removing a common factor from the first two terms and the last two terms, such as $2x(x + 2) + 3(x + 2)$. Notice the relationship to the double distributive property. Therefore the factors are $(x + 2)(2x + 3)$.

The solution would be as follows:

$$2x^2 + 7x + 6$$

12
1, 12
2, 6
3, 4

$(2x^2 + 4x)(+3x + 6)$

$2x(x + 2) + 3(x + 2)$

$(2x + 3)(x + 2)$

Example 4:
Factor by decomposition:

(A) $3x^2 + 11x + 6$

18
1, 18
2, 9
3, 6

$(3x^2 + 9x)(+2x + 6)$

$3x(x + 3) + 2(x + 3)$

$(3x + 2)(x + 3)$

(B) $4x^2 + 4x + 1$

4
1, 4
2, 2

$(4x^2 + 2x)(+2x + 1)$

$2x(2x + 1) + 1(2x + 1)$

$(2x + 1)(2x + 1)$

(C) $2x^2 - x - 10$

20
1, 20
2, 10
4, 5

$(2x^2 - 5x)(+4x - 10)$

$x(2x - 5) + 2(2x - 5)$

$(x + 2)(2x - 5)$

(D) $8x^2 - 18x - 5$

40
1, 40
2, 20
4, 10
5, 8

$(8x^2 - 20x)(+2x - 5)$

$4x(2x - 5) + 1(2x - 5)$

$(2x - 5)(4x + 1)$

(E) $4x^2 + 13x + 3$

12
1, 12
2, 6
3, 4

$(4x^2 + 12x)(+x + 3)$

$4x(x + 3) + 1(x + 3)$

$(4x + 1)(x + 3)$

(F) $2x^2 + 5x + 2$

4
1, 4
2, 2

$(2x^2 + 4x)(+x + 2)$

$2x(x + 2) + 1(x + 2)$

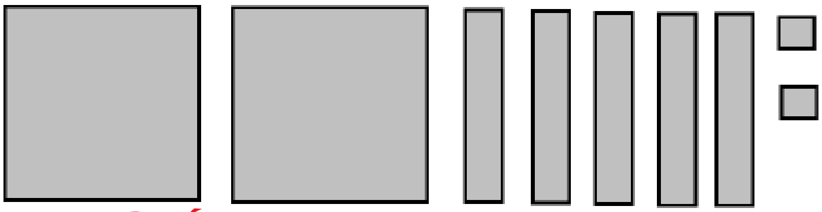
$(2x + 1)(x + 2)$

Factoring With Algebra Tiles

There's the option of factoring with algebra tiles as well. Just like the previous sections, you have to arrange the tiles in a rectangle and then count the top and left edges to obtain the factors.

Example 5:

Factor this trinomial using algebra tiles and record the result.



$2x^2+5x+2$

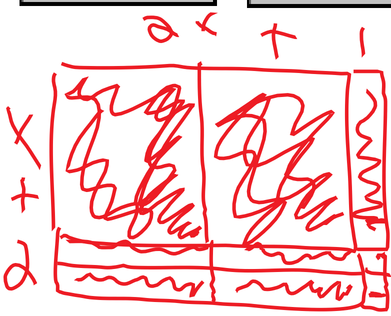
$=(2x+1)(x+2)$

Check:

$2x(x+2)+1(x+2)$

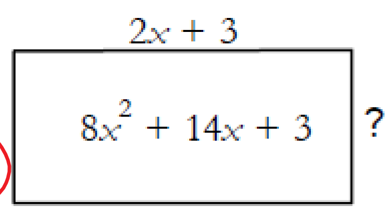
$=2x^2+4x+x+2$

$=2x^2+5x+2$



Example 6:

The area of a rectangle is represented by the product $8x^2 + 14x + 3$. If the length of one side is $(2x + 3)$, determine the width of the other side. How did knowing one of the factors help you determine the other factor?



$8x^2+14x+3=(2x+3)(4x+1)$

Check: $(2x+3)(4x+1)$

$=2x(4x+1)+3(4x+1)$

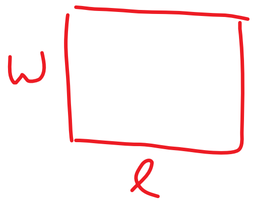
$=8x^2+2x+12x+3$

2

We knew what was needed to multiply to get a and c.

Example 7:
A rectangle has a product given by the expression $15x^2 - 7x - 30$. If the product represents the area of the rectangle, complete the following:

(A) Find an expression for the length and width of the rectangle.



$\ell \cdot w = 15x^2 - 7x - 30$

$\frac{450}{18, 25}$

$(15x^2 - 25x)(+18x - 30)$

$5x(3x-5) + 6(3x-5)$

$(5x+6)(3x-5)$

$\ell = 5x+6$

$w = 3x-5$

(B) Find the length of the rectangle given that its width is 4cm.

$3x - 5 = 4 \rightarrow 3x = 9$

$3x = 4 + 5$

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

$\ell = 5(3) + 6 = 15 + 6 = 21\text{cm}$

Example 8:
How many integer values are there for k for which $6x^2 + kx - 7$ is factorable? Explain.

$6x^2 + kx - 7$

$\frac{42}{1,42}$

$2,21$

$3,14$

$6,7$

Homework!

When factoring trinomials of the form $ax^2 + bx + c$, first factor out the greatest common factor, if possible. Common student errors occur when they do not factor out the GCF from all terms. When the common factor removed is negative, students sometimes neglect to divide all the terms in the trinomial by the negative.

Example 5:

(A) Factor: $-5x^2 - 25x + 30$

$$= -5(x^2 + 5x - 6)$$

$$\frac{6}{\begin{array}{c}1,6\\2,3\end{array}}$$

$$= -5(x - 1)(x + 6)$$

Check: $x(x+6) - 1(x+6)$

$$= x^2 + 6x - x - 6$$

$$= x^2 + 5x - 6 \checkmark$$

(B) $4x^2 + 10x - 6$

$$= 2(2x^2 + 5x - 3)$$

$$\frac{6}{\begin{array}{c}1,6\\2,3\end{array}}$$

$$= 2(2x^2 + 6x - x - 3)$$

$$= 2[2x(x+3) - 1(x+3)]$$

$$= 2(2x - 1)(x + 3)$$

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