Math 1201 **Date:**__________________

3.7 Multiplying Polynomials

In this section we will extend the strategies for multiplying binomials to multiplying polynomials. We will be multiplying a binomial by a trinomial and multiplying a trinomial by a trinomial, where the polynomials may contain more than one variable. Although it is important to model the product using a rectangle diagram, the focus here is on symbolic manipulation.

The process for multiplying polynomials with more than two terms is similar to multiplying binomials. You will distribute each term of the first polynomial by each term of the other polynomial. In both cases, the distributive property is applied.

Example 1:

(A) Expand using the rectangle model: $(2x - 5)(2x^2 + 3x - 4)$

	$2x^2$	$3x$	-4
$2x$	$4x^3$	$6x^2$	$-8x$
-5	$-10x^2$	$-15x$	20

$4x^3 + 6x^2 - 8x - 10x^2 - 15x + 20$

$= 4x^3 - 4x^2 - 23x + 20$

(B) Expand using the distributive property: $(2x - 5)(2x^2 + 3x - 4)$

$= 2x(2x^2 + 3x - 4) - 5(2x^2 + 3x - 4)$

$= 4x^3 + 6x^2 - 8x - 10x^2 - 15x + 20$

$= 4x^3 - 4x^2 - 23x + 20$

Example 2:
Expand using the distributive property:

(A) $(4x - 3y)(2x - 6y + 2)$

$= 4x(2x - 6y + 2) - 3y(2x - 6y + 2)$

$= \cancel{8x}^2 - \cancel{24}xy + \cancel{8x} - \cancel{6x}y + 18y^2 - 6y$

$= 8x^2 + 8x - 30xy - 6y + 18y^2$

(B) $(3a + 4)(a^2 - 2a - 7)$

$= 3a(a^2 - 2a - 7) + 4(a^2 - 2a - 7)$

$= 3a^3 - 6a^2 - 21a + 4a^2 - 8a - 28$

$= 3a^3 - 2a^2 - 29a - 28$

(C) $(b - 6)(2b^2 - 3b - 7)$

$= b(2b^2 - 3b - 7) - 6(2b^2 - 3b - 7)$

$= 2b^3 - 3b^2 - 7b - 12b^2 + 18b + 42$

$= 2b^3 - 15b^2 + 11b + 42$

(D) $(2x - 3y)^2$

$= (2x-3y)(2x-3y)$

$= 2x(2x-3y) - 3y(2x-3y)$

$= 4x^2 - 6xy - 6xy + 9y^2$

$= 4x^2 - 12xy + 9y^2$

(E) $(3b^2 + 2b + 6)(2b^2 - 3b - 7)$

$= 3b^2(2b^2-3b-7) + 2b(2b^2-3b-7) + 6(2b^2-3b-7)$

$= \cancel{6b^4} - \cancel{9b^3} - \cancel{21b^2} + \cancel{4b^3} - \cancel{6b^2} - \cancel{14b} + \cancel{12b^2} - \cancel{18b} - 42$

$= 6b^4 - 5b^3 - 15b^2 - 32b - 42$

Since multiplication is commutative, there are different ways $a(bx + c)(dx + e)$ can be multiplied. You should carefully organize your work when performing operations on products of polynomials and use brackets where appropriate.

Example 3:
Expand:

(A) $2(x - 2)(3x + 1)$

$= 2[x(3x+1) - 2(3x+1)]$

$= 2(3x^2 + x - 6x - 2)$

$= 2(3x^2 - 5x - 2)$

$= 6x^2 - 10x - 4$

(B) $-3(-x-4)(5-x)$

$= -3[-x(5-x)-4(5-x)]$

$= -3(-5x+x^2-20+4x)$

$= -3(x^2-x-20)$

$= -3x^2+3x+60$

(C) $(x+2)(x+3)(x+4)$

$=(x+2)[x(x+4)+3(x+4)]$

$=(x+2)(x^2+4x+3x+12)$

$=(x+2)(x^2+7x+12)$

$= x(x^2+7x+12)+2(x^2+7x+12)$

$= x^3+7x^2+12x+2x^2+14x+24$

$= x^3+9x^2+26x+24$

(D) $5(2x+3)(x^2-2x-1)$

$= 5[2x(x^2-2x-1)+3(x^2-2x-1)]$

$= 5(2x^3-4x^2-2x+3x^2-6x-3)$

$= 5(2x^3-x^2-8x-3)$

$= 10x^3-5x^2-40x-15$

(E) $(2a-4)(a+7)+3(a+2)(2a-1)$

$= 2a(a+7)-4(a+7)+3[a(2a-1)+2(2a-1)]$

$= 2a^2+14a-4a-28+3(2a^2-a+4a-2)$

$= 2a^2+10a-28+3(2a^2+3a-2)$

$= 2a^2+10a-28+6a^2+9a-6$

$= 8a^2+19a-34$

Example 4:

(A) How many terms are created when $(x + 1)(x + 2)$ is multiplied? How many sets of like terms can be combined?

3

1

$(x+1)(x+2)$

$= x(x+2)+1(x+2)$

$= x^2+2x+x+2$

$= x^2+3x+2$

(B) How many terms are created when $(x + 1)(x^2 + 4x + 2)$ is multiplied? How many sets of like terms can be combined?

4

2

$(x+1)(x^2+4x+2)$

$= x(x^2+4x+2)+1(x^2+4x+2)$

$= x^3+4x^2+2x+x^2+4x+2$

$= x^2+5x^2+6x+2$

(C) How many terms are created when $(x + 1)(x^3 + x^2 + 4x + 2)$ is multiplied? How many sets of like terms can be combined?

5

3

$(x+1)(x^3+x^2+4x+2)$

$= x(x^3+x^2+4x+2)+1(x^3+x^2+4x+2)$

$= x^4+x^3+4x^2+2x+x^3+x^2+4x+2$

$= x^4+2x^3+5x^2+6x+2$

(D) What pattern can you find in the above answers?

The number of terms increase by 1.

Number of sets of like terms increase by 1.

Example 5:
Why is $(x+4)(x^2-3x+2) = (x^2-3x+2)(x+4)$?
Commutative property, $a \cdot b = b \cdot a$, applies to polynomial multiplication.

You can verify the polynomial product by substituting a number for each variable in both the polynomial factors and their product simplification. If the value of the left side equals the value of the right side, the product is likely correct. Zero would not be a good value to substitute because any term that contains a variable would have a value of zero. Thus errors would go unnoticed.

Example 6:
How can you check that $(b-1)(b-2)(b-3) = b^3 - 6b^2 + 11b - 6$?
LHS RHS Let $b = 4$
$(4-1)(4-2)(4-3) = 4^3 - 6(4)^2 + 11(4) - 6$
$3 \cdot 2 \cdot 1 = 64 - 96 + 44 - 6$
$6 = 6$
LHS = RHS ✓

Example 7:
Dean solved the following multiplication problem:

$$(3x+4)(x+3) = 3x + 9x + 4x + 12 = 16x + 12$$

(A) Is Dean's answer correct? No. A binomial x binomial yields a highest power 2.

(B) Dean checked his work by substituting $x = 1$. Was this a good choice to verify the multiplication? No. $(1)^2 = 1$

(C) When verifying work, what other numbers should be avoided?

0

Example 8:
Find a shortcut for multiplying $(x+5)^2$. Why does this work? Will the same type of shortcut work for multiplying $(x+5)^3$?

$(x+5)(x+5)$
$= x(x+5)+5(x+5)$
$= x^2+5x+5x+25$
$= x^2+10x+25$

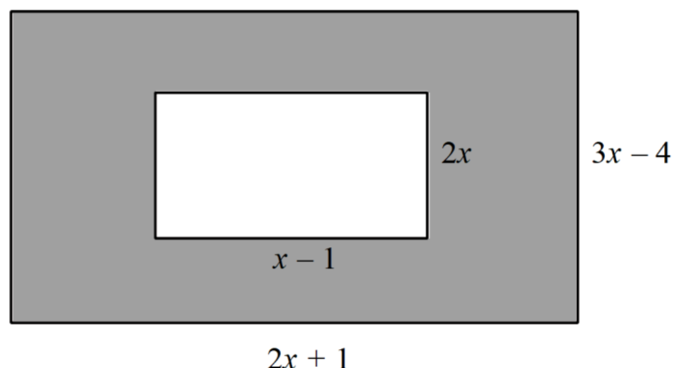
- Square first term
- multiply terms and double
- Square last term

for example: $(x+6)^2$
$= x^2+12x+36$

Won't work for $(x+5)^3$

Example 9:
(A) Determine the area of the shaded region:

$A = l \cdot w$



$A_B = (2x+1)(3x-4)$
$= 2x(3x-4)+1(3x-4)$
$= 6x^2-8x+3x-4$
$= 6x^2-5x-4$

$A_S = 2x(x-1)$
$= 2x^2-2x$

$A_{shaded} = A_B - A_S$
$= 6x^2-5x-4-(2x^2-2x)$
$= 6x^2-5x-4-2x^2+2x$
$= 4x^2-3x-4$

(B) What is the area of the shaded region if $x = 2$?

$A = 4(2)^2-3(2)-4 = 4(4)-6-4 = 6 \text{ units}^2$.

Textbook Questions: page 186 - 187 #4 – 15, 17 – 19, 21, 22