Math 1201 **Date:__________________**

4.1 Estimating Roots

The concept of square root and cube root was developed earlier and will now be extended to roots including the n^{th} root using the notation:

$\sqrt[n]{x}$, where n is the index and x is the radicand

It should be emphasized that when there is no numerical index stated, the value of the index is 2. For example, $\sqrt{36}$ indicates a square root with an index of 2.

$\sqrt[2]{36} \longleftrightarrow \sqrt{36}$

$= 6 \qquad = 6$

Since $3^2 = 9$, 3 is the square root of 9. So we write $3 = \sqrt[2]{9}$.

Since $3^3 = 27$, 3 is the cube root of 27. So we write $3 = \sqrt[3]{27}$.

Since $3^4 = 81$, 3 is the fourth root of 81. So we write $3 = \sqrt[4]{81}$.

How could you write 5 as a square root? $5 = \sqrt{25}$

A cube root? $5 = \sqrt[3]{125}$

A fourth root? $5 = \sqrt[4]{625}$

There is a relationship between the power and the index or the root. What is it?

index = power

Example 1:

If $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$, what would be the index be for $\sqrt[x]{\frac{16}{81}}$ $= \frac{2}{3}$

$x = 4$

Example 2:
Identify the index and the radicand for each of the following:

(A) $\sqrt[3]{5}$

index: 3
radicand: 5

(B) $\sqrt[7]{17}$

index: 7
radicand: 17

(C) $\sqrt{28}$

index: 2
radicand: 28

Estimating Roots Using Calculators
In Grade 9, you used upper and lower benchmarks to approximate square roots of non-perfect square rational numbers. This strategy can be adapted to approximate the *n*th root of rational numbers.

For example, to approximate $\sqrt[3]{10}$, students should identify the closest perfect cubes $\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$.

Since 10 is closer to 8, the value of $\sqrt[3]{10}$ would be closer to 2.

Students can select numbers close to 2 and check with a calculator:

$2.1^3 = 9.261$

$2.2^3 = 10.648$

$(2.19)^3 = 10.5$

$(2.16)^3 = 10.077$

$(2.15)^3 = 9.93$

$(2.155)^3 = 10.007$

$\sqrt[3]{10} \sim 2.155$

Example 3:
Determine the approximate root of the following:

(A) $\sqrt{20}$ $\sqrt{16}=4$, $\sqrt{25}=5$ $(4.48)^2=20.07$

$(4.4)^2=19.36$ $(4.475)^2=20.02$

$(4.5)^2=20.25$

$\sqrt{20}\sim 4.475$

$(4.45)^2=19.8$

$(4.47)^2=19.98$

(B) $\sqrt[3]{16}$ $\sqrt[3]{8}=2$ $\sqrt[3]{27}=3$

$(2.5)^3=15.625$ $\sqrt[3]{16}\sim 2.52$

$(2.6)^3=17.576$

$(2.51)^3=15.81$

$(2.52)^3=16.003$

(C) $\sqrt[3]{50}$ $\sqrt[3]{27}=3$ $\sqrt[3]{64}=4$

$(3.685)^3=50.04$ $\therefore \sqrt[3]{50}=3.685$

Textbook Questions: page 206 #2 (identify the index and the radicand), 3, 5 Homework!