Math 1201 **Date:__________________**

4.2 Irrational Numbers

The value of a radical is either a rational or irrational number. Radicals that are square roots of perfect squares, cube roots of perfect cubes, and so on, are **rational numbers**.

An **irrational** number is a number that cannot be expressed as a fraction, $\frac{m}{n}$ for any integers and m and n. **Irrational numbers** have decimal expansions that neither terminate nor repeat.

The set of irrational numbers is now added to the family of natural numbers, whole numbers, integers, and rational numbers. The irrational numbers together with the rational numbers form the real number system.

Real Number System

Natural Numbers – numbers greater than zero with no decimals or fractions. These are also called the counting numbers. {1, 2, 3, 4, 5, …}

Whole Numbers – the natural numbers combined with zero. {0, 1, 2, 3, 4, 5, …}

Integers - the set of integers includes, positive numbers, zero and negative numbers without decimals or fractions. {…-3, -2, -1, 0, 1, 2, 3…}

Rational Numbers - any number that can be expressed as the quotient or fraction $\frac{m}{n}$ of two integers, a numerator m and a non-zero denominator n. Since n may be equal to 1, every integer is a rational number.

Example 1:
Sort the following numbers into rational and irrational:

$$\frac{1}{2}, 4, -6, \sqrt{9}, \sqrt{17}, \pi, -\frac{2}{3}, \sqrt[3]{\frac{8}{27}}$$

Rational

$\frac{1}{2}, 4, -6, \sqrt{9}, -\frac{2}{3}, \sqrt[3]{\frac{8}{27}}$

Irrational

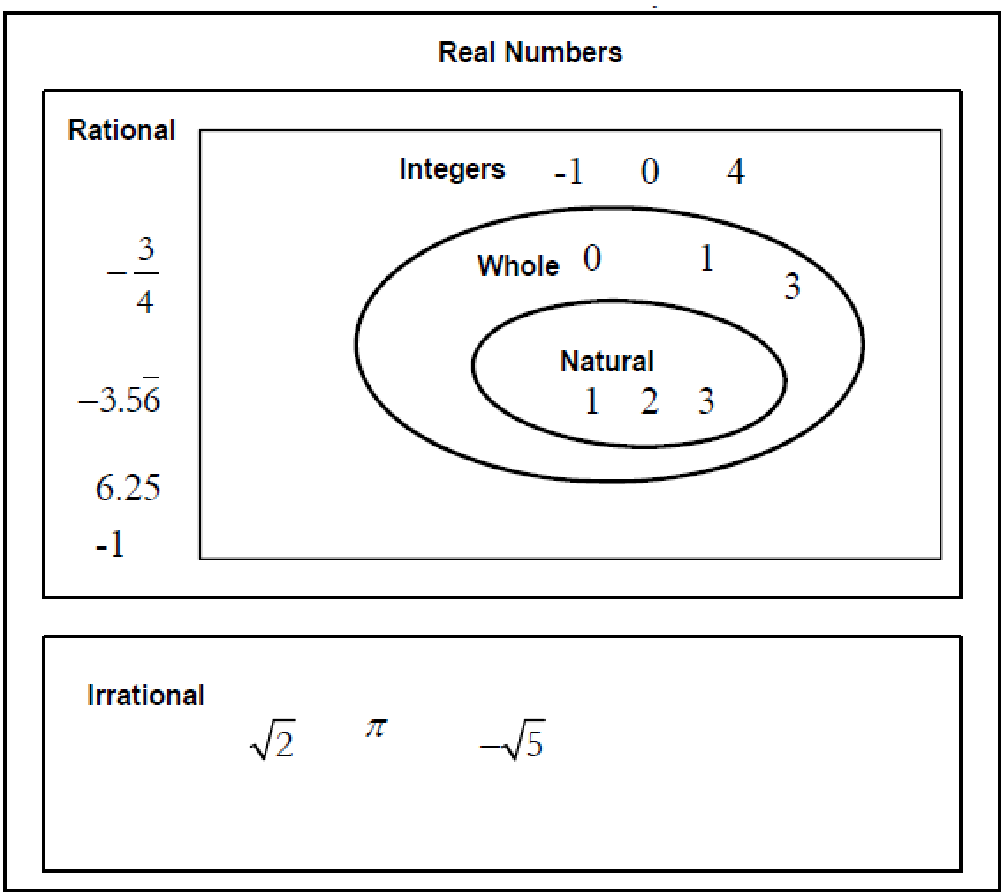
$\sqrt{17}, \pi$

Example 2:
Complete the following:

DEFINITION	CHARACTERISTICS/FACTS
• a number that can't be written as a fraction • non-terminating decimal	• non-repeating / non-terminating numbers • roots of numbers that are not perfect squares, cubes, etc.
EXAMPLES	**NON-EXAMPLES**
π $\sqrt{17}$ $\sqrt[3]{\frac{2}{60}}$ $2.4132\ldots$	$\frac{2}{3}$ $\sqrt{16}$ $3.454545\ldots$ $3.\overline{45}$

IRRATIONAL NUMBERS

The irrational numbers together with the rational numbers form the real number system. A graphic organizer, such as a Venn diagram, is a good way to visualize the various subsets of the real number system.



Example 3:

Determine if each of the following statements are sometimes true, always true or never true, and justify their choices.

(A) All whole numbers are integers.

True

(B) All integers are whole numbers.

Never true

(C) If a number is a rational number, then it is also an integer.

Sometimes true

(D) If a number is an integer, then it is also a rational number.

True

(E) There is a number which is both rational and irrational.

Never true

Example 4:
Use a number line to order the following numbers from least to greatest:

$$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \sqrt[4]{27}, \sqrt[3]{-5}$$

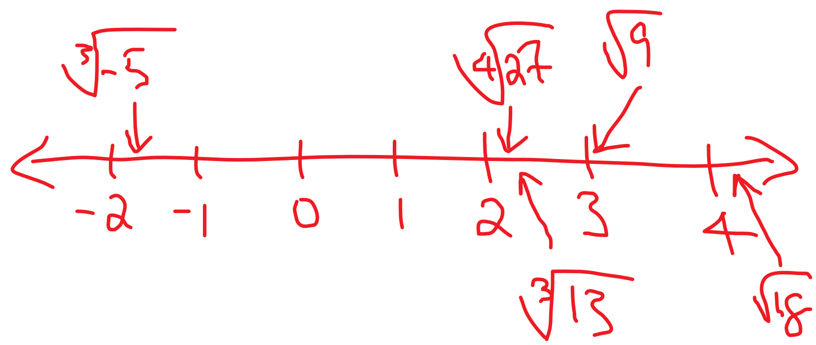
$\sqrt[3]{13} \sim 2.351$

$\sqrt{18} \sim 4.243$

$\sqrt{9} = 3$

$\sqrt[4]{27} \sim 2.280$

$\sqrt[3]{-5} \sim -1.710$



Textbook Questions: page 211, #1, 4, 5, 8, 9, 10, 11, 17