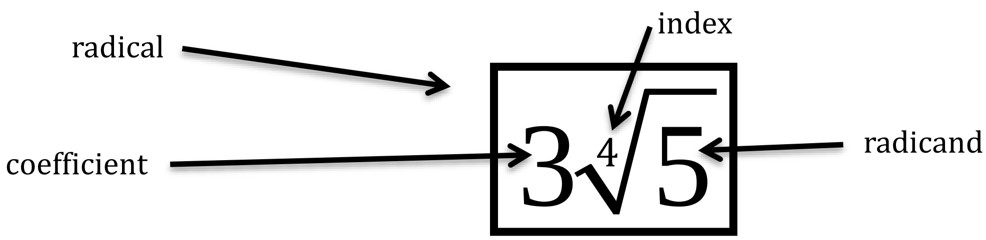
Math 1201 **Date:**__________________

4.3A Mixed vs Entire Radicals

Radicals

A radical is any expression containing the root symbol: $\sqrt{\ }$. A radical, itself, can be made up of different parts.



The concept of square root and cube root was developed earlier and will now be extended to roots including the n^{th} root using the notation $\sqrt[n]{x}$ where n is the index and x is the radicand.

Principal and Secondary Square Roots

Every positive number has two square roots. For example, the square root of 49 is 7 since $7^2 = 49$. Likewise $(-7)^2 = 49$ so -7 is also a square root of 49. The value $\sqrt{49} = 7$ is called the **principal square root** and $\sqrt{49} = -7$ is the **secondary square root**.

Even Index vs Odd Index

If a radical has an even index, the radicand must be non-negative. If a radical has an odd index, the radicand can be any real number, including negative numbers. The reason for this is as follows:

$(-2)(-2) = 4 \therefore \sqrt{4} = -2$

Because two negatives multiplied will always be positive, you can never have a negative radicand by multiplying a negative number by itself.

$(-2)(-2)(-2) = -8 \therefore \sqrt[3]{-8} = -2$

Entire Radicals – a radical with a coefficient of 1. For example:

$$\sqrt{5} \quad \sqrt{2} \quad \sqrt{9}$$

Mixed Radicals – a radical with a leading coefficient other than 1. For example:

$$2\sqrt{5} \quad \tfrac{2}{3}\sqrt{6} \quad -3\sqrt{11}$$

Converting Entire Radicals to Mixed Radicals
This process is also called **radical reduction** because it reduces an entire radical to a lowest terms mixed radical. There are two methods that we will learn in this unit.

Method 1 - Prime Factorization
This method is an extension of determining if a number is a perfect square or perfect cube through prime factorization. Here we will prime factorize the radicand. If a square root we will then looks for groups of two. As before, we will cross out these groups of two and place one on the outside. Any prime factors that aren't groups of two remain under the root sign and are multiplied back together. The process is the same for cube roots but instead we look for groups of three. For fourth roots we would look for groups of four, and so on.

Example 1:
Convert to a reduced, mixed radical by prime factorizing the radicand:

(A) $\sqrt{8} = \sqrt{\cancel{2\cdot 2}\cdot 2}$ $= 2\sqrt{2}$

$8 \to 2\cdot 4 \to 2\cdot 2\cdot 2$

(B) $\sqrt{12} = \sqrt{\cancel{2\cdot 2}\,3}$ $= 2\sqrt{3}$

$12 \to 4\cdot 3 \to 2\cdot 2\cdot 3$

(C) $\sqrt{32} = \sqrt{\cancel{2\cdot 2}\,\cancel{2\cdot 2}\,2}$ $= 2\cdot 2\sqrt{2}$ $= 4\sqrt{2}$

$32 \to 8\cdot 4 \to 2\cdot 4\; 2\; 2 \to 2\,2\,2\,2\,2$

Example 2:
Convert to a reduced, mixed radical by prime factorizing the radicand:

(A) $\sqrt[3]{16} = \sqrt[3]{\cancel{(2\cdot 2\cdot 2)}\cdot 2}$

$= 2\sqrt[3]{2}$

16
$4 \cdot 4$
$2\cdot 2\cdot 2\cdot 2$

(B) $\sqrt[3]{32} = \sqrt[3]{\cancel{(2\cdot 2\cdot 2)}\cdot 2\cdot 2}$

$= 2\sqrt[3]{2\cdot 2}$

$= 2\sqrt[3]{4}$

32
$16 \cdot 2$
$4 \cdot 4 \cdot 2$
$2\cdot 2\cdot 2\cdot 2\cdot 2$

(C) $\sqrt[3]{81} = \sqrt[3]{\cancel{(3\cdot 3\cdot 3)}\cdot 3}$

$= 3\sqrt[3]{3}$

81
$9 \cdot 9$
$3\cdot 3\cdot 3\cdot 3$

Example 3:
Convert to a reduced, mixed radical by prime factorizing the radicand:

(A) $\sqrt[4]{32} = \sqrt[4]{\cancel{(2\cdot 2\cdot 2\cdot 2)}\cdot 2}$

$= 2\sqrt[4]{2}$

32
$8 \cdot 4$
$2\cdot 4\cdot 2\cdot 2$
$2\cdot 2\cdot 2\cdot 2\cdot 2$

(B) $\sqrt[4]{243} = \sqrt[4]{\cancel{(3\cdot 3\cdot 3\cdot 3)}\cdot 3}$

$= 3\sqrt[4]{3}$

243
$9 \cdot 27$
$3\cdot 3\cdot 3\cdot 9$
$3\cdot 3\cdot 3\cdot 3\cdot 3$

(C) $\sqrt[4]{64} = \sqrt[4]{\cancel{(2\cdot 2\cdot 2\cdot 2)}\cdot 2\cdot 2}$

$= 2\sqrt[4]{2\cdot 2}$

$= 2\sqrt[4]{4}$

64
$8 \cdot 8$
$2\cdot 4\cdot 2\cdot 4$
$2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$

Method 2 – Biggest Perfect Square/Perfect Cube

Before we get into this method, we need to introduce the multiplicative property of radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$, where n is a natural number and a and b are real numbers.

ie $\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

When a mixed radical is expressed in simplest form, the radicand contains no perfect factors other than 1. Students will write an equivalent radical expression as the product of two factors, one of which is the greatest perfect square, cube, etc., depending on the index of the radical. You will only be expected to reduce square and cube roots with this method.

It's a good idea to know as many perfect squares and cubes and possible to help with this method.

Perfect Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Square Root	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Perfect Cube	1	8	27	64	125	216	343	512	729	1000
Cube Root	1	2	3	4	5	6	7	8	9	10

Example 4:

Change $\sqrt{32}$ to mixed radical form using the biggest perfect square method.

$\sqrt{32}$

$= \sqrt{16} \cdot \sqrt{2}$

$= 4\sqrt{2}$

Example 5:

Change $\sqrt[3]{81}$ to mixed radical form using the biggest perfect square method.

$\sqrt[3]{81}$

$= \sqrt[3]{27} \cdot \sqrt[3]{3}$

$= 3\sqrt[3]{3}$

Example 6:
Reduce using the biggest perfect square method:

(A) $\sqrt{72}$
$= \sqrt{36}\sqrt{2}$
$= 6\sqrt{2}$

(B) $\sqrt{24}$
$= \sqrt{4}\sqrt{6}$
$= 2\sqrt{6}$

(C) $\sqrt{80}$
$= \sqrt{16}\sqrt{5}$
$= 4\sqrt{5}$

(D) $\sqrt[3]{24}$
$= \sqrt[3]{8}\sqrt[3]{3}$
$= 2\sqrt[3]{3}$

(E) $\sqrt[3]{144}$
$= \sqrt[3]{8}\sqrt[3]{18}$
$= 2\sqrt[3]{18}$

(F) $\sqrt[3]{108}$
$= \sqrt[3]{27}\sqrt[3]{4}$
$= 3\sqrt[3]{4}$

(G) $\sqrt[3]{36}$

(H) $\sqrt{150}$
$= \sqrt{25}\sqrt{6}$
$= 5\sqrt{6}$

(I) $\sqrt[3]{375}$
$= \sqrt[3]{125}\sqrt[3]{3}$
$= 5\sqrt[3]{3}$

Textbook Questions: page 218, 219, #3, 4, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 21