Math 1201 Date:__________________

4.3B Changing Mixed Radicals to Entire Radicals

Here we will learn how to express a mixed radical as an entire radical, limited to numerical radicands (no variables). Any number can be written as the square root of its square, the cube root of its cube and so on. You can use this strategy to write a mixed radical as an entire radical by writing the leading number as a radical, then multiplying the radicands.

Example 1:

Express $3\sqrt{2}$ as an entire radical.

$3\sqrt{2}$ index: 2

$= \sqrt{3^2} \cdot \sqrt{2}$

$= \sqrt{9}\sqrt{2}$

$= \sqrt{18}$

or

$3\sqrt{2}$

$= \sqrt{3 \cdot 3}\sqrt{2}$

$= \sqrt{9}\sqrt{2}$

$= \sqrt{18}$

Example 2:

Express $3\sqrt[3]{2}$ as an entire radical.

index: 3

$\sqrt[3]{3^3}\sqrt[3]{2}$

$\sqrt[3]{27}\sqrt[3]{2}$

$= \sqrt[3]{54}$

Example 3:

Express $3\sqrt[4]{2}$ as an entire radical.

$\sqrt[4]{3^4}\sqrt[4]{2}$

$= \sqrt[4]{81}\sqrt[4]{2}$

$= \sqrt[4]{162}$

Example 4:

Express $3\sqrt[5]{2}$ as an entire radical.

$\sqrt[5]{3^5}\sqrt[5]{2}$

$= \sqrt[5]{243}\sqrt[5]{2}$

$= \sqrt[5]{486}$

Example 5:
Change the following mixed radicals to entire radicals.

(A) $2\sqrt{5}$

$= \sqrt{2^2}\sqrt{5}$
$= \sqrt{4}\sqrt{5}$
$= \sqrt{20}$
3

(B) $3\sqrt{3}$

$= \sqrt{3^2}\sqrt{3}$
$= \sqrt{9}\sqrt{3}$
$= \sqrt{27}$
4

(C) $2\sqrt[3]{3}$

$\sqrt[3]{2^3}\sqrt[3]{3}$
$= \sqrt[3]{8}\sqrt[3]{3}$
$= \sqrt[3]{24}$
2

(D) $3\sqrt[3]{-2}$

$\sqrt[3]{3^3}\sqrt[3]{-2}$
$= \sqrt[3]{27}\sqrt[3]{-2}$
$= \sqrt[3]{-54}$
1

Example 6:
Which is bigger, $2\sqrt{3}$ or $3\sqrt{2}$?

$\sqrt{2^2}\sqrt{3}$
$= \sqrt{4}\sqrt{3}$
$= \sqrt{12}$

$\sqrt{3^2}\sqrt{2}$
$= \sqrt{9}\sqrt{2}$
$= \sqrt{18}$

Example 7:
Order the mixed radicals in example 5 from smallest to largest.

$\sqrt[3]{-54}, \sqrt[3]{24}, \sqrt{20}, \sqrt{27}$

Mixed Radicals with Negative Coefficients
Converting from mixed radicals with negative leading coefficients to entire radicals have to be approached differently depending on if the index of the radical is odd or even. For example,

$-2\sqrt[3]{5}$ can be written as an entire radical in two different ways, both being correct:

$\sqrt[3]{(-2)^3}\sqrt[3]{5}$ or $-\sqrt[3]{2^3}\cdot\sqrt[3]{5}$

$=\sqrt[3]{-8}\sqrt[3]{5}$ $\quad = -\sqrt[3]{8}\sqrt[3]{5}$

$=\sqrt[3]{-40}$ $\quad = -\sqrt[3]{40}$

However, $-2\sqrt{5}$ cannot be written as $\sqrt{\cancel{-20}}$ for the following reason:

$\sqrt{(-2)^2}\sqrt{5} = \sqrt{4}\sqrt{5} = \sqrt{20}$

Again, we can't take the square root of a negative radicand in the **Real Number System.** The correct solutions is:

$-\sqrt{2^2}\sqrt{5} = -\sqrt{4}\sqrt{5} = -\sqrt{20}$

In summary, for **odd** indices, a negative leading coefficient can be left outside the radical or brought inside. For **even** indices, the negative, **must remain outside** the radical.

Example 8:
Write each mixed radical as an entire radical:

(A) $-4\sqrt{3}$
$-\sqrt{4^2}\sqrt{3}$
$=-\sqrt{16}\sqrt{3}$
$=-\sqrt{48}$

(B) $-2\sqrt[3]{5}$
$\sqrt[3]{(-2)^3}\sqrt[3]{5}$
$=\sqrt[3]{-8}\sqrt[3]{5}$
$=\sqrt[3]{-40}$

(C) $-3\sqrt[4]{3}$
$=-\sqrt[4]{3^4}\sqrt[4]{3}$
$=-\sqrt[4]{81}\sqrt[4]{3}$
$=-\sqrt[4]{243}$

(D) $-2\sqrt[5]{4}$
$=-\sqrt[5]{2^5}\sqrt[5]{4}$
$=-\sqrt[5]{32}\sqrt[5]{4}$
$=-\sqrt[5]{128}$

Textbook Questions: page 218, 219 #5, 12, 18, 20, 22