Math 1201 Date:__________________

4.4A Fractional Exponents and Radicals

In Grade 9 students developed and worked with powers having integral bases, excluding base 0, and whole number exponents. This will now be extended to include powers using fractional exponents and negative exponents, as well as powers with rational and variable bases.

Recall from Grade 9, the multiplicative property of exponents

$$a^m \times a^n = a^{m+n}, \text{ where } m \text{ and } n \text{ are whole numbers.}$$

Example 1:
Simplify:

(A) $2^2 \times 2^3$
$= 2^{2+3}$
$= 2^5$

(B) $3^3 \times 3^5$
$= 3^{5+5}$
$= 3^8$

(C) $5^{-2} \times 5^4$
$= 5^{-2+4}$
$= 5^2$

(D) $x^5 \times x^8$
$= x^{5+8}$
$= x^{13}$

Consider the following:

$\sqrt{3} \times \sqrt{3}$
$= \sqrt{3 \cdot 3}$
$= \sqrt{9}$
$= 3$

$3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$
$= 3^{\frac{1}{2}+\frac{1}{2}}$
$= 3^1$
$= 3$

Now consider:

$$\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} \qquad 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}}$$

$= \sqrt[3]{2 \cdot 2 \cdot 2}$

$= \sqrt[3]{8}$

$= 2$

$= 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 2^{1}$

$= 2$

This in fact holds true for all radicals. In general:

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \text{ where } n > 0$$

Example 2:
Re-write the following radicals in exponential form:

(A) $\sqrt{5}$ $= 5^{\frac{1}{2}}$

(B) $\sqrt[3]{11}$ $= 11^{\frac{1}{3}}$

(C) $\sqrt[4]{3}$ $= 3^{\frac{1}{4}}$

(D) $\sqrt[12]{5}$ $= 5^{\frac{1}{12}}$

Example 3:
Re-write the following radicals in radical form:

(A) $3^{\frac{1}{2}}$ $= \sqrt{3}$

(B) $4^{\frac{1}{3}}$ $= \sqrt[3]{4}$

(C) $6^{\frac{1}{7}}$ $= \sqrt[7]{6}$

(D) $6^{\frac{1}{21}}$ $= \sqrt[21]{6}$

Textbook Questions: page 227, #3, 4, 5, 6, 7, 13, 14