Math 1201 **Date:__________________**

4.4B Fractional Exponents and Radicals

Continuing on from 4.4 A, we will now investigate expressions where the numerator, m, in the fractional exponent is greater than 1.

Consider the following example:

$\sqrt[4]{16}$

$4 \cdot 4$

$(2 \cdot 2 \cdot 2 \cdot 2)$

$= 2$

$16^{\frac{3}{4}}$

$= 16^{\frac{1}{4} \times 3}$

$= \left(16^{\frac{1}{4}}\right)^3$

$= \left(\sqrt[4]{16}\right)^3$

$= 2^3$

$= 8$

$16^{\frac{3}{4}}$

$= 16^{3 \times \frac{1}{4}}$

$= \left(16^3\right)^{\frac{1}{4}}$

$= (4096)^{\frac{1}{4}}$

$= \sqrt[4]{4096}$

$= 8$

In general:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{(a^m)}, \text{ where } n > 0, m > 0$$

Example 1:

Express each power as a radical: (and evaluate)

(A) $8^{\frac{2}{3}}$

$= 8^{\frac{1}{3} \times 2}$

$= (\sqrt[3]{8})^2$

$= 2^2$

$= 4$

(B) $32^{\frac{1}{4}}$

$= \sqrt[4]{32}$

$4 \cdot 8$

$2 \cdot 2 \quad 2 \cdot 2 \cdot 2$

$= \sqrt[4]{\cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2}$

$= 2\sqrt[4]{2}$

(C) $5^{\frac{3}{4}}$

$= 5^{3 \times \frac{1}{4}}$

$= (5^3)^{\frac{1}{4}}$

$= (125)^{\frac{1}{4}}$

$= \sqrt[4]{125}$

(D) $(-4)^{\frac{2}{3}}$

$= (-4)^{2 \times \frac{1}{3}}$

$= \left[(-4)^2\right]^{\frac{1}{3}}$

$= 16^{\frac{1}{3}}$

$= \sqrt[3]{16}$

$\rightarrow \sqrt[3]{\cancel{2 \cdot 2 \cdot 2} \cdot 2}$

$= 2\sqrt[3]{2}$

(E) $-16^{\frac{2}{3}}$

$\rightarrow -(16)^{2 \times \frac{1}{3}}$

$= -(256)^{\frac{1}{3}}$

$= -\sqrt[3]{256}$

$= -\sqrt[3]{\cancel{2 \cdot 2 \cdot 2} \cdot \cancel{2 \cdot 2 \cdot 2} \cdot 2 \cdot 2}$

$= -2 \cdot 2\sqrt[3]{4}$

$= -4\sqrt[3]{4}$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

Example 2:

Express each radical as a power:

(A) $\sqrt[4]{4^3}$

$= (4^{\frac{3}{1}})^{\frac{1}{4}}$

$= 4^{\frac{3}{4}}$

(B) $\sqrt[5]{6^3}$

$= (6^3)^{\frac{1}{5}}$

$= 6^{\frac{3}{5}}$

Rational vs Irrational

We identified the value of a radical as either rational or irrational previously in this chapter. If $a^{\frac{1}{n}} = \sqrt[n]{a}$ is rational then $a^{\frac{m}{n}}$ is also rational.

For example, $32^{\frac{4}{5}}$ is a rational number because 32 has a perfect 5th root. $\sqrt[5]{32} = 2$

Example 3:

Identify each as rational or irrational:

(A) $4^{\frac{3}{2}}$ rational

(B) $9^{\frac{5}{3}}$ irrational

(C) $27^{\frac{2}{3}}$ rational

(D) $16^{\frac{4}{5}}$ irrational

You are expected to use mental math when dealing with rational numbers. When evaluating $8^{\frac{2}{3}}$, for example, students should initially be thinking about taking the cube root of 8 and then squaring it before actually writing it as a radical, $\left(\sqrt[3]{8}\right)^2$. While not always the case, it's often easier to reduce the radical first and then raise to the power after. This is because it's easier to work with smaller numbers when dealing with mental math.

Example 4:

Evaluate:

(A) $8^{\frac{2}{3}}$

$= 8^{\frac{1}{3} \times 2}$

$= \left(8^{\frac{1}{3}}\right)^2$

$= \left(\sqrt[3]{8}\right)^2$

$= 2^2$

$= 4$

(B) $27^{\frac{4}{3}}$

$= 27^{\frac{1}{3} \times 4}$

$= \left(\sqrt[3]{27}\right)^4$

$= 3^4$

$= 81$

(C) $81^{\frac{3}{4}}$

$= 81^{\frac{1}{4} \times 3}$

$= \left(81^{\frac{1}{4}}\right)^3$

$= \left(\sqrt[4]{81}\right)^3$

$= 3^3$

$= 27$

(D) $64^{\frac{2}{6}}$

$= 64^{\frac{1}{6} \times 2}$

$= \left(\sqrt[6]{64}\right)^2$

$= 2^2$

$= 4$

Example 5:
A cube-shaped storage container has a volume of 24.453 m^3. What are the dimensions of the container to the nearest tenth?

$V = 24.453 m^3$

$\sqrt[3]{24.453} = 2.9 m$

Example 6:
The area, A, of a face of a cube is given by, $A = V^{\frac{2}{3}}$ where V represents the volume of the cube. If $V = 64$ m^3, determine the value of A.

$A = 64^{\frac{2}{3}}$

$A = 64^{\frac{1}{3} \times 2}$

$A = (64^{\frac{1}{3}})^2$

$A = (\sqrt[3]{64})^2$

$A = 4^2$

$A = 16 m^2$

Example 7:
The value of a car, V, is given by the equation: $V = 32000(0.85)^{\frac{t}{2}}$, where t represents how old the vehicle is. Estimate the value of the car after 5 years.

$V = 32000(0.85)^{\frac{5}{2}}$

$V = 32000(0.85)^{2.5}$

BEDMAS

$V = 32000(0.66612087)$

$V = 21315.59$

Example 8:
Is $-32^{\frac{2}{5}}$ equal to $(-32)^{\frac{2}{5}}$?

$= -1 \cdot 32^{\frac{2}{5}}$

$= -1 \cdot 32^{\frac{1}{5} \times 2}$

$= -1(\sqrt[5]{32})^2$

$= -1 \cdot 2^2$

$= -1 \cdot 4$

$= -4$

$(-32)^{\frac{2}{5}}$

$= (-32)^{\frac{1}{5} \times 2}$

$= (\sqrt[5]{-32})^2$

$= (-2)^2$

$= 4$

NO!

Textbook Questions: page 227, #7, 12, 15, 17