Math 1201 **Date:**___________________

4.5 Negative Exponents and Radicals

Recall from Grade 9, the division rule for exponents:

$$a^m \div a^n = a^{m-n}$$

$3^2 \div 3^4$

$= \frac{3^2}{3^4}$

$= \frac{3 \times 3}{3 \times 3 \times 3 \times 3}$

$= \frac{3}{3} \times \frac{3}{3} \times \frac{1}{3} \times \frac{1}{3}$

$= 1 \times 1 \times \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{3^2}$

$3^2 \div 3^4$

$= 3^{2-4}$

$= 3^{-2}$

$= \frac{1}{3^2}$

Negative Exponent Law

$$a^{-n} = \frac{1}{a^n}, a \neq 0$$ also $\frac{1}{a^{-n}} = a^n$

Example 1:
Write each of the following as a power with a positive exponent. Evaluate where possible.

(A) 4^{-2} (B) 2^{-5} (C) $10^{-\frac{1}{2}}$ (D) -3^{-6}

$= \frac{1}{4^2}$ $= \frac{1}{2^5}$ $= \frac{1}{10^{\frac{1}{2}}}$ $= \frac{1}{-3^6}$

$= \frac{1}{16}$ $= \frac{1}{32}$ $= \frac{1}{\sqrt{10}}$ $= \frac{1}{-729}$

Fractional Bases
Fractional bases follow the same pattern. However, an obvious short cut becomes apparent.

For example

$\left(\frac{2}{3}\right)^{-2}$ $\left(\frac{2}{3}\right)^{-2}$

$= \frac{1}{\left(\frac{2}{3}\right)^2}$ or $= \left(\frac{3}{2}\right)^2$

$= \frac{1}{\frac{4}{9}}$ $= \frac{9}{4}$

$= \frac{\frac{1}{1}}{\frac{4}{9}}$

$= \frac{1}{1} \cdot \frac{9}{4}$

$= \frac{9}{4}$

Example 2:
Evaluate:

(A) $\left(\frac{9}{4}\right)^{-2}$ (B) $\left(\frac{3}{2}\right)^{-3}$ (C) $\left(\frac{5}{2}\right)^{-2}$ (D) $\left(-\frac{3}{4}\right)^{-4}$

$= \left(\frac{4}{9}\right)^2$

$= \frac{16}{81}$

$= \left(\frac{2}{3}\right)^3$

$= \frac{8}{27}$

$= \left(\frac{2}{5}\right)^2$

$= \frac{4}{25}$

$= \left(-\frac{4}{3}\right)^4$

$= \frac{256}{81}$

Fractional Exponents
The exponent law will now be extended to include negative rational exponents. Here we will apply the meaning of rational exponents and negative exponents to rewrite, simplify and evaluate expressions.

$\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

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$= \left(\frac{27}{8}\right)^{\frac{2}{3}}$

$= \left(\frac{27}{8}\right)^{\frac{1}{3}\times 2}$

$= \left[\left(\frac{27}{8}\right)^{\frac{1}{3}}\right]^2$

$= \left(\frac{\sqrt[3]{27}}{\sqrt[3]{8}}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

Example 3:
Evaluate:

(A) $\left(\frac{9}{16}\right)^{-\cancel{\frac{2}{3}}\ -\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{1}{2} \times 3}$

$= \left(\frac{\sqrt{16}}{\sqrt{9}}\right)^{3}$

$= \left(\frac{4}{3}\right)^{3}$

$= \frac{64}{27}$

(B) $\left(\frac{16}{1}\right)^{-\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{1}{4} \times 5}$

$= \left(\frac{\sqrt[4]{1}}{\sqrt[4]{16}}\right)^{5}$

$= \left(\frac{1}{2}\right)^{5}$

$= \frac{1}{32}$

(C) $\left(\frac{25}{36}\right)^{-\frac{3}{2}}$

$= \left(\frac{36}{25}\right)^{\frac{3}{2}}$

$= \left(\frac{36}{25}\right)^{\frac{1}{2} \times 3}$

$= \left(\frac{\sqrt{36}}{\sqrt{25}}\right)^{3}$

$= \left(\frac{6}{5}\right)^{3}$

$= \frac{216}{125}$

(D) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$

$= \left(\frac{27}{8}\right)^{\frac{1}{3}}$

$= \frac{\sqrt[3]{27}}{\sqrt[3]{8}}$

$= \frac{3}{2}$

Textbook Questions: page 233, 234, #3, 5, 6, 7, 8, 9, 13, 16