Math 1201 **Date:**__________________

4.6 Applying the Laws of Exponents

Exponents laws that were developed for integral bases can also be applied to variable bases. The only difference is that the expression cannot be evaluated until a value for the variable is known. When simplifying expressions involving multiple exponent laws, you should consider the following guidelines:

1. Simplify the expression inside the brackets.
2. Remove the brackets by applying the exponent laws for products of powers, quotients of powers, or powers of powers.
3. Write the simplest expression using positive exponents.

Exponent Laws:

Product of Powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers:	$a^m \div a^n = a^{m-n}, a \neq 0$
Power of Power:	$(a^m)^n = a^{m \cdot n}$
Power of a Product:	$(ab)^m = a^m b^m$
Power of a Quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Example 1:
Simplify:

(A) $(x^3y^2)(x^2y^{-4})$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$

$= \dfrac{x^5}{y^2}$

(B) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

$= 5a^{5-2}b^{3-(-2)}$

$= 5a^3b^5$

(C) $m^4 n^{-2} \cdot m^2 n^3$

$= m^{4+2} n^{-2+3}$

$= m^6 n$

(D) $\frac{6x^4y^{-3}}{14xy^2}$

$= \frac{3x^{4-1}}{7} y^{-3-2}$

$= \frac{3x^3 y^{-5}}{7}$

$= \frac{3x^3}{7y^5}$

(E) $\frac{(t^2)^3}{t^{-3}}$

$= \frac{t^{2\cdot 3}}{t^{-3}}$

$= \frac{t^6}{t^{-3}}$

$= t^{6-(-3)}$

$= t^9$

(F) $\frac{2x(3xy)^2}{6xy^3}$

$= \frac{2x(3^2x^2y^2)}{6xy^3}$

$= \frac{\cancel{6}x^{1+2}y^2}{\cancel{6}xy^3}$

$= \frac{x^3y^2}{xy^3}$

$= x^{3-1}y^{2-3}$

$= x^2y^{-1}$

$= \frac{x^2}{y}$

(G) $\left(x^{\frac{1}{5}}y^{-\frac{2}{4}}\right)^{\frac{1}{2}}\left(x^{-\frac{1}{4}}y^{\frac{1}{2}}\right)^{-1}$

$= x^{\frac{1}{5}\cdot\frac{1}{2}}\, y^{-\frac{1}{2}\cdot\frac{1}{2}}\, x^{-\frac{1}{4}\cdot(-1)}\, y^{\frac{1}{2}\cdot(-1)}$

$= x^{\frac{1}{10}\cdot\frac{2}{2}}\, y^{-\frac{1}{4}}\, x^{\frac{1}{4}\cdot\frac{5}{5}}\, y^{-\frac{1}{2}\cdot\frac{2}{2}}$

$= x^{\frac{2}{20}}\cdot x^{\frac{5}{20}}\, y^{-\frac{1}{4}}\cdot y^{-\frac{2}{4}}$

$= x^{\frac{7}{20}}\, y^{-\frac{3}{4}}$

$= \dfrac{x^{\frac{7}{20}}}{y^{\frac{3}{4}}}$

(H) $\left(-\frac{3}{4}\right)^{\frac{2}{3}}\left(-\frac{3}{4}\right)^{2}$

$= \left(-\frac{3}{4}\right)^{\frac{2}{3}+\frac{2}{1}\cdot\frac{3}{3}}$

$= \left(-\frac{3}{4}\right)^{\frac{2}{3}+\frac{6}{3}}$

$= \left(-\frac{3}{4}\right)^{\frac{8}{3}}$

$= \left(-\frac{3}{4}\right)^{8\cdot\frac{1}{3}}$

$= \left[\left(-\frac{3}{4}\right)^{8}\right]^{\frac{1}{3}}$

$= \left(\frac{6561}{65536}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{6561}{65536}}$

Example 2:
A sphere has a volume of 1250 cm^3. What is the radius of the sphere to the nearest centimetre?

$V = \frac{4}{3}\pi r^3$

$1250 = \frac{4}{3}\pi r^3$

$1250 = 4.19 r^3$

$\frac{1250}{4.19} = \frac{\cancel{4.19} r^3}{\cancel{4.19}}$

$r^3 = 298.33$

$\sqrt[3]{r^3} = \sqrt[3]{298.33}$

$r = 6.68 \sim 7\text{cm}$

Example 3:
Simplify:

(A) $\left(\frac{2a^3b^{-2}}{8a^2b^7}\right)^{-2}$

$= \left(\frac{8a^2b^7}{2a^3b^{-2}}\right)^2$

$= \left(4a^{2-3}b^{7-(-2)}\right)^2$

$= \left(4a^{-1}b^9\right)^2$

$= 4^2a^{-1\cdot 2}b^{9\cdot 2}$

$= 16a^{-2}b^{18}$

$= \dfrac{16b^{18}}{a^2}$

(B) $\dfrac{(x^{-2}y^3)^{-4}}{(x^3y^{-2})^3}$

$= \dfrac{x^{-2(-4)}y^{3(-4)}}{x^{3\cdot 3}y^{-2\cdot 3}}$

$= \dfrac{x^8y^{-12}}{x^9y^{-6}}$

$= x^{8-9}\cdot y^{-12-(-6)}$

$= x^{-1}y^{-6}$

$= \dfrac{1}{xy^6}$

Example 4:
Determine the side length of a square with area $144x^4y^6$.

$A_{\square} = S^2$

$144x^4y^6 = S^2$

$(144x^4y^6)^{\frac{1}{2}} = (S^{\cancel{2}})^{\frac{1}{\cancel{2}}}$

$144^{\frac{1}{2}}x^{4\cdot\frac{1}{2}}y^{6\cdot\frac{1}{2}} = S$

$\sqrt{144}\,x^2y^3 = S$

$\rightarrow S = 12x^2y^3$

Example 5:
Identify the errors in each solution:

(A)
$$\left(a^3b^{-1}\right)\left(a^{\frac{1}{4}}b^{-3}\right)$$
$$a^3 \bullet a^{\frac{1}{4}} \bullet b^{-1} \bullet b^{-3}$$
$$a^{\frac{3}{4}}b^3 \quad \times$$

Multiplied exponents instead of adding.

(B)
$$\frac{(a^{-4}b^5)^{-1}}{ab^5}$$
$$= \frac{a^{-4}b^4}{ab^5}$$
$$= a^{-4}b$$

Various mistakes

(C)
$$(k^{\frac{1}{2}}h^{\frac{3}{2}})^{\frac{3}{2}}(k^2h)^{-1}$$
$$= (k^2h^3)(kh^{-1})$$
$$= k^3h^2$$

Added exponents instead of multiplying.

Example 6:
On a test, three students evaluated $2^{-\frac{1}{3}} \times 2^0$ as follows:

Bobby	Charlie	Michael
$2^{-\frac{1}{3}} \times 2^0$	$2^{-\frac{1}{3}} \times 2^0$	$2^{-\frac{1}{3}} \times 2^0$
$\times$ $4^{-\frac{1}{3}}$ Multiplied bases.	$2^{-\frac{1}{3}}$ $\frac{1}{2^3}$	4^0
$\frac{1}{4^{\frac{1}{3}}}$	$\frac{1}{8}$ $\times$	1

$2^{-\frac{1}{3}} \times 2^0$

$\frac{1}{2^{\frac{1}{3}}} \times 1$

$= \frac{1}{\sqrt[3]{2}}$

(i) What errors did each student make? What did each do correctly?

(ii) What is the correct answer?

Textbook Questions: page 241 – 243, #3 – 19, 21