Math 1201

5.2 Properties of Functions

Domain: the first set of elements of a relation.

Range: the second set of elements of a relation

Function: a special type of relation where each element in the domain is associated with exactly one element in the range.

Example 1:

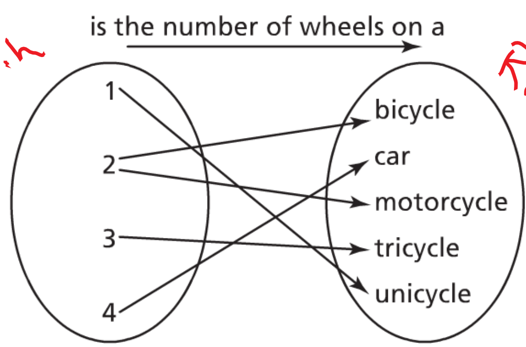
We want to examine the relation between vehicle types and the number of wheels on the vehicle. We will do this in two different ways.

(A)

i.

This relation associates a number with a vehicle with that number of wheels.

Domain



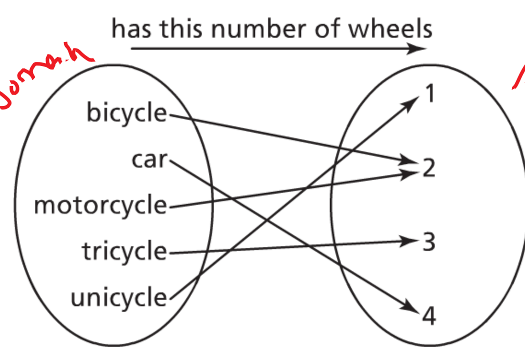
Range

This is not a function since there is an element in the first set that is associated with two elements in the second.

ii.

This relation associates a vehicle with the number of wheels.

Domain



Range

This is a function since all elements in the first set that are associated with only one element in the second.

(B) Represent both as ordered pairs:

i.

{(1, unicycle), (2, bicycle), (2, motorcycle), (3, tricycle), (4, car)}

If you see a domain value repeated, the relation is not a function.

ii.

{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)}

Function (no repeats in domain)

(C) Write the Domain for both:

i.
Domain: {1, 2, 3, 4}

ii.
Domain: {unicycle, bicycle, motorcycle, tricycle, car}

(D) Write the Range for both:

i.
Range: {unicycle, bicycle, motorcycle, tricycle, car}

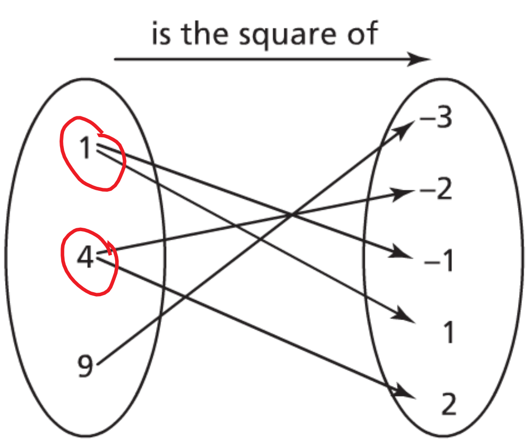
ii.
Range: {1, 2, 3, 4}

Example 2:
Determine whether the following relations are functions.

(A) A relation that associates given shapes with the number of right triangles in the shape: {(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}

Function.

(B)



Not a function.

Example 3:
Determine whether the following sets of ordered pairs represents functions. Explain your answer.

(A) $\{(2, 4), (3, 6), (4, 6), (5, 6), (6, 12), (7, 14)\}$

Yes. There are no repetitions of elements in the domain.

(B) $\{(-3, 7), (0, 10), (3, 13), (3, -5), (6, 16), (9, 19)\}$

No. There is repetition of elements in the domain.

Independent and Dependent Variables

Independent Variable:

- Does **not** depend on the other variable.
- Represents the domain.
- Listed in the first column of a table.
- Plotted on the horizontal axis of a graph.

Dependent Variable:

- Depends on the other variable.
- Represents the range.
- Listed in the second column of a table.
- Plotted on the vertical axis of a graph.

For example

In the workplace, a person's gross pay, P dollars, often depends on the number of hours worked, h.
So, we say P is the *dependent variable*. Since the number of hours worked, h, does not depend on the gross pay, P, we say that h is the *independent variable*.

independent variable → dependent variable

Hours Worked, h	Gross Pay, P ($)
1	12
2	24
3	36
4	48
5	60

domain

range

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

Example 4:
The table shows the masses, m grams, of different identical marbles, n.

Number of Marbles, n	Mass of Marbles, m (g)
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

(A) Why is this relation also a function?

Each element in the domain is associated with only one element in the range.

(B) Identify the independent and dependent variables.

Independent: number of marbles
Dependent: mass of marbles

(C) Write the domain and range.

Domain: $\{1, 2, 3, 4, 5, 6\}$

Range: $\{1.27, 2.54, 3.81, 5.08, 6.35, 7.12\}$

Example 5:
For each situation described below, identify the independent and dependent variables.

(A) The amount of vitamin C one consumers can influence life expectancy.

independent: vitamin C
dependent: life expectancy

(B) A farmer wants to determine the influence of different quantities of fertilizer on plant growth.

independent: fertilizer
dependent: plant growth

(C) The time spent studying will influence test scores.

independent: time studying
dependent: test score

(D) The weight of a letter will determine the amount of postage paid.

independent: mass of letter
dependent: amount of postage paid

(E) Shots on net will influence the number of goals scored.

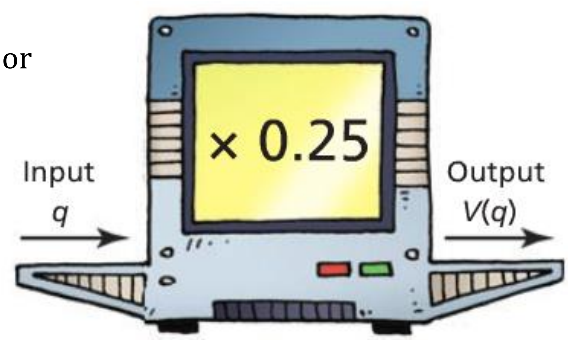
indpendent: shots on net
dependent: goals scored

Equations and Functions
We can think of a function as an input/output machine. The input can be any number in the domain and the output number depends on the input number. So, the input is the independent variable and the output is the dependent variable.

For example, consider the following machine which calculates the value of any given number of quarters:

When the input is q quarters, the output or Value, V, in dollars is $0.25q$. Therefor the equation that describes this function is:

$$V = 0.25q$$



Function V

Example 6:
What is the value of 3 quarters?

$q = 3$

$V = 0.25(3) = \$0.75$

Function Notation
Function notation is a different way of writing an equation. If we were to write the equation $y = 2x + 1$ using function notation, it would be $f(x) = 2x + 1$. Notice y gets replaced with $f(x)$. We read this as "f of x", and it means that the value of the function $f(x)$ depends on the value of x.

The variable x refers to the domain or the input of the function, while $f(x)$ is the range or the output of the function.
y

A benefit of functional notation is the domain value is presented as a function of the range. For example, if $f(x) = 2x + 1$, then $f(3)$ is:

$x = 3$

$f(3) = 2(3) + 1$

$f(3) = 6 + 1$

$f(3) = 7$ $\rightarrow$ $y = 7$

We can then write this as an **ordered pair** of $(3, 7)$. (independent, dependent)
x y

We can also calculate the value of x when given the value of $f(x)$. Using the previous example, what value of x makes $f(x) = 9$?

Think: $y = 9$

$f(x) = 2x + 1$

$9 = 2x + 1$

$9 - 1 = 2x$

$8 = 2x$

$\frac{8}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$x = 4$

$(4, 9)$

Example 7:

(A) Write $V = 0.25q$ in function notation and find the value of 8 quarters.

$V(q) = 0.25q$ $\quad V(8) = 0.25(8)$

$q = 8$ $\quad V(8) = \$2.00$

(B) How many quarters are there in \$3.25?

$V(q) = 3.25$ $\quad 3.25 = 0.25q$

$\frac{3.25}{0.25} = \frac{\cancel{0.25}q}{\cancel{0.25}}$ $\Rightarrow q = 13$

Example 8:

The equation $V = -0.08d + 50$ represents the volume, V litres, of gasoline in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it's empty.

(A) Describe the function and write it in function notation.

$V(d) = -0.08d + 50$

(B) Determine the value of $V(600)$. What does this number represent? BEDMAS

$d = 600$

$V(600) = -0.08(600) + 50$

$= -48 + 50$

$= 2L$

(C) Determine the value of d when $V(d) = 26$. What does this number represent?

$26 = -0.08d + 50$

$26 - 50 = -0.08d$

$-24 = -0.08d$

$\frac{-24}{-0.08} = \frac{\cancel{-0.08}d}{\cancel{-0.08}}$ $\Rightarrow d = 300\text{ km}$

Example 9:
Evaluate the following and write your answer as an ordered pair:

(A) $d(t) = 3t + 4$, determine $d(3)$ $t = 3$

$d(3) = 3(3) + 4$

$d(3) = 9 + 4$

$d(3) = 13$

$(3, 13)$

(B) $f(x) = x^2 - 2x - 24$, determine $f(-2)$ $x = -2$

$f(-2) = (-2)^2 - 2(-2) - 24$

$f(-2) = 4 + 4 - 24$

$f(-2) = -16$

$(-2, -16)$

(C) $f(x) = 5x - 11$, find the value of x that makes $f(x) = 9$

$9 = 5x - 11$

$9 + 11 = 5x$

$20 = 5x$

$\frac{20}{5} = \frac{\cancel{5}x}{\cancel{5}}$

$x = 4$

$(4, 9)$

(D) $g(x) = -2x + 5$, find the value of x that makes $g(x) = -7$

$-7 = -2x + 5$

$-7 - 5 = -2x$

$-12 = -2x$

$\frac{-12}{-2} = \frac{\cancel{-2}x}{\cancel{-2}}$

$x = 6$

$(6, -7)$

Example 10:
A boat travelling at 8 m/s begins to accelerate. Its new speed, S, in metres per second,is modelled by the function $S(t) = 8 + 1.5t$, where t is the length of time, in seconds, that it accelerates.

(A) Determine the speed of the boat at 7 seconds. $t = 7$

$S(7) = 8 + 1.5(7)$

$S(7) = 8 + 10.5$

$S(7) = 18.5\ m/s$

The speed of the boat at 7s is 18.5 m/s.

(B) Determine the time it takes the boat to reach 26 m/s.

$26 = 8 + 1.5t$

$26 - 8 = 1.5t$

$18 = 1.5t$

$\frac{18}{1.5} = \frac{1.5t}{\cancel{1.5}}$

$\rightarrow t = 12s$

The boat is travelling 26m/s at 12s.

(C) What is the domain of the function?

$t \geq 0$

Your turn:

1. Evaluate the following:

(A) $d(t) = -6t + 7$, determine $d(-2)$ $t=-2$

$d(-2) = -6(-2) + 7$

$d(-2) = 12 + 7$

$d(-2) = 19$

$(2, 19)$

(B) $f(x) = x^2 + 5x - 13$, determine $f(3)$ $x = 3$

$f(3) = (3)^2 + 5(3) - 13$

$f(3) = 9 + 15 - 13$

$f(3) = 11$

$(3, 11)$

(C) $h(t) = 4t^2 - 3t$, determine $h(1) + h(-2)$

$h(1) = 4(1)^2 - 3(1)$

$h(1) = 4 - 3$

$h(1) = 1$

$h(-2) = 4(-2)^2 - 3(-2)$

$h(-2) = 16 + 6$

$h(-2) = 22$

$1 + 22 = \underline{23}$

(D) $f(x) = -6x - 15$, find the value of x that makes $f(x) = 9$

$9 = -6x - 15$

$9 + 15 = -6x$

$24 = -6x$

$\frac{24}{-6} = \frac{-6x}{-6}$

$x = -4$

$(-4, 9)$

(E) $g(x) = 3x + 3$, find the value of x that makes $g(x) = -12$

$-12 = 3x + 3$

$-12 - 3 = 3x$

$-15 = 3x$

$\frac{-15}{3} = \frac{3x}{3}$

$x = -5$

$(-5, -12)$

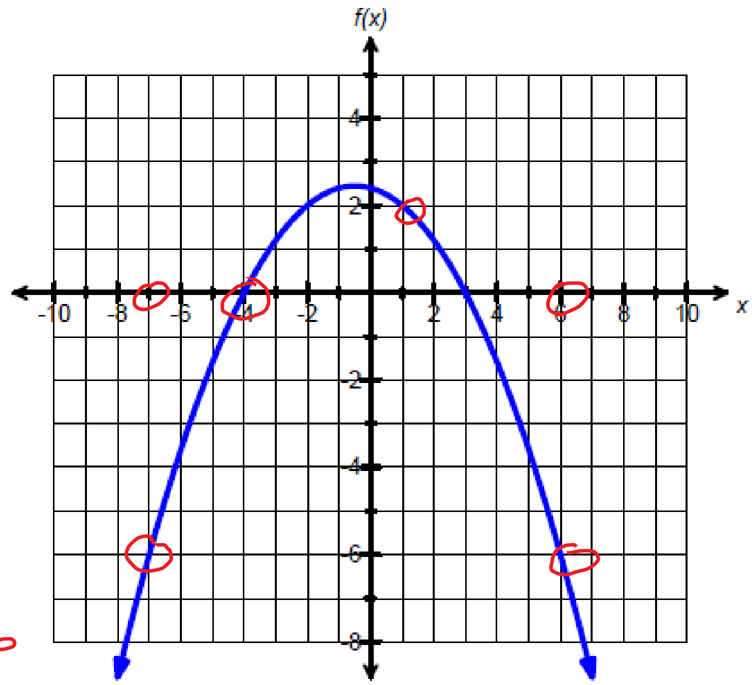
(F) $f(x+1)$ if $f(x) = 3x - 5$

$f(x+1) = 3(x+1) - 5$

$f(x+1) = 3x + 3 - 5$

$f(x+1) = 3x - 2$

2. Given the function $f(x)$, find:



(A) $f(-4) = 0$

$x = -4$

(B) $f(1) = 2$

$x = 1$

(C) x such that $f(x) = -6$

$x = -7$ or $x = 6$

3. The perimeter of a rectangle is $P = 2l + 2w$. If it is known that the length must be 6 ft, then the perimeter is a function of the width. Write this function using function notation.

$l = 6$

$P = 2(6) + 2w$

$P = 12 + 2w$

$P(w) = 12 + 2w$

Textbook Questions: page 270 - 271 #4, 5, 6(a, b), 7(a, b), 8, 9(a), 14, 15