

## Math 1201

### 5.5 Graphs and Relations of Functions

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So far we've seen what a function looks like when it's represented as a table of values, a set of ordered pairs and an arrow diagram. The fundamental idea is that every domain value is associated with one and only one range value.

We will now investigate what a function looks like as a graph.

Our example will be a function that associates every whole number with its double. The table of values for this function would look like this:

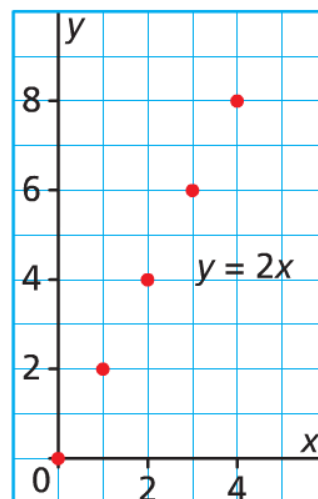
Whole Number, $x$	Double the Number, $y$
0	0
1	2
2	4
3	6
4	8

This table continues for a set for all whole numbers. The domain is the set of whole numbers and the range is the set of even whole numbers. If we plot these points on an axis we get:

We know the relation  $y = 2x$  is a function because each value of  $x$  associates with exactly one value of  $y$  and each ordered pair has a different first element.

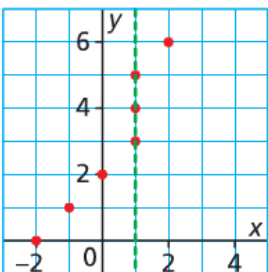
The **domain** of a function is the set of values of the Independent variable. For this graph the domain is the set of  $x$ -values.

The **range** function is the set of values of the dependent variable. For this graph the range is the  $y$ -values.

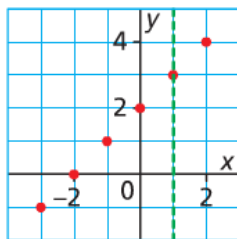


In summary:

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.



A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.



### Vertical Line Test for a Function

A graph represents a function when no two points on the graph lie on the same vertical line.

- Place a ruler or pencil vertically on a graph.
- Slide the ruler or pencil across the graph.
- If one edge of the ruler always intersects the graph at no more than one point, the graph does represent a function.

### Example 1:

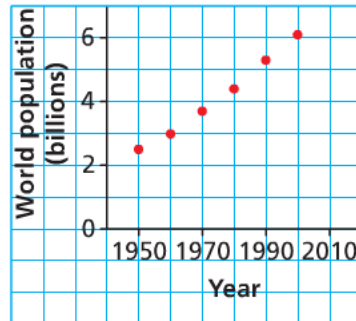
Which of these graphs represents a function? How do you know?

a) Height against Shoe Size



Not a function

b) World Population



Function

### Domain and Range for Discrete Data

**Domain:** the values of the independent variable,  $x$ .

**Range:** the values of the dependent variable,  $y$ .

Earlier in this unit we did examples in which we wrote the domain and range for sets of discrete data. The domain and range can be written in words, or as a list.

### Example 2:

Write the domain and range for the following:  $\{(1,5), (2, 10), (3, 15), (4, 20), (5, 25)\}$

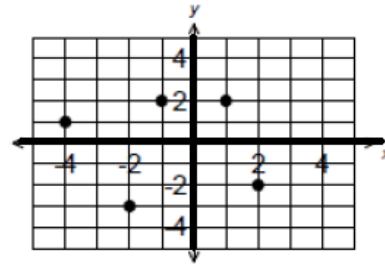
Domain:  $\{1, 2, 3, 4, 5\}$

Range:  $\{5, 10, 15, 20, 25\}$

**Example 3:**

Write the domain and range for the following.

Domain:  $\{-4, -2, -1, 1, 2\}$   
Range:  $\{-3, -2, 1, 2\}$

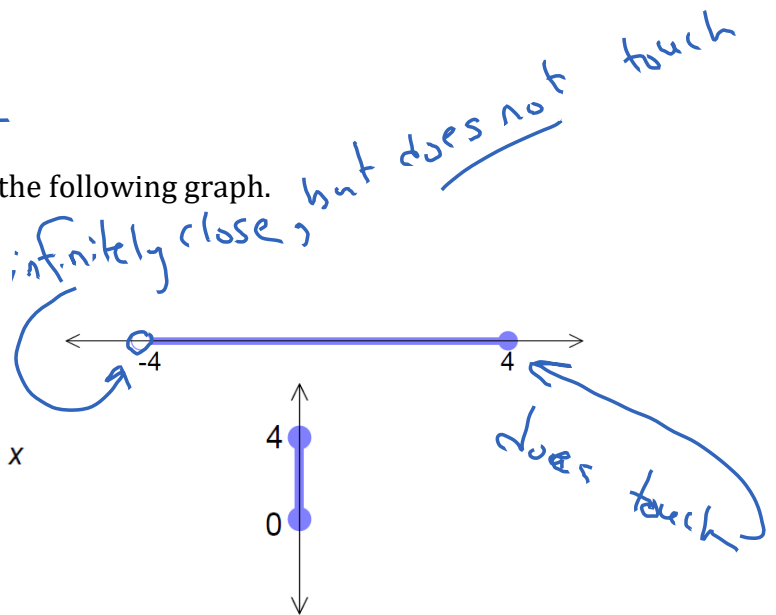
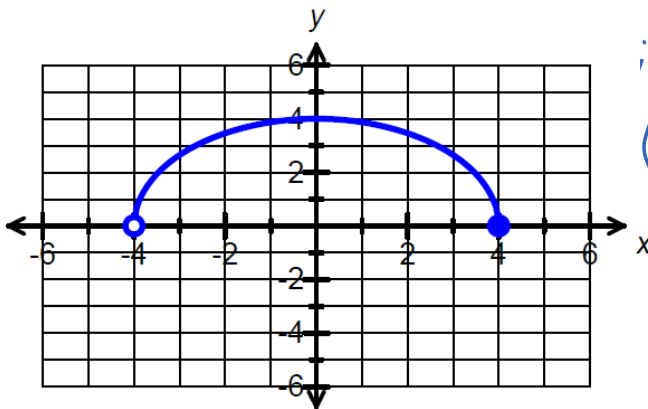


**Domain and Range of Continuous Data**

This can be written in three ways:

1. using words
2. interval notation ✓
3. set notation *advanced math*

**Example:** Write the domain and range for the following graph.



(A) Using words.

Domain: All numbers from, but not including -4 up to and including 4.

Range: All numbers between, and including 0 and 4.

(B) Using Set Notation.

Recall inequality signs:

- > - greater than
- < - less than
- ≥ - greater than or equal to
- ≤ - less than or equal to

/: such that

∈: belongs to

$\mathbb{R}$ : the set of real numbers

$$\text{Domain: } \{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$$
$$\text{Range: } \{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$$

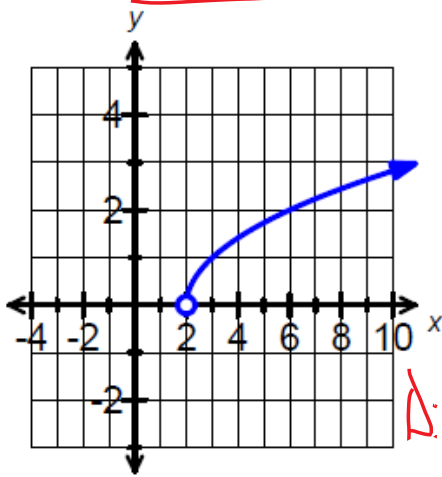
(C) In interval notation, brackets are put around the smallest and largest numbers in the domain & range. If the number belongs to the domain or range, put a square bracket next to it. If it is not included or in other words shows up as an open circle on a graph, put a round bracket next to it.

$$\text{Domain: } (-4, 4]$$
$$\text{Range: } [0, 4]$$

**Example 4:**

Determine the domain and range of each graph, writing your answer ~~in words~~, in set notation and in interval notation.

(A)



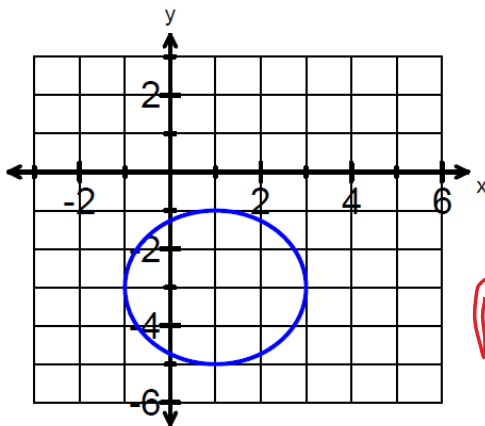
$$D: \{x \mid x > 2, x \in \mathbb{R}\}$$

$$R: \{y \mid y > 0, y \in \mathbb{R}\}$$

$$D: (2, \infty)$$

$$R: (0, \infty)$$

(B)



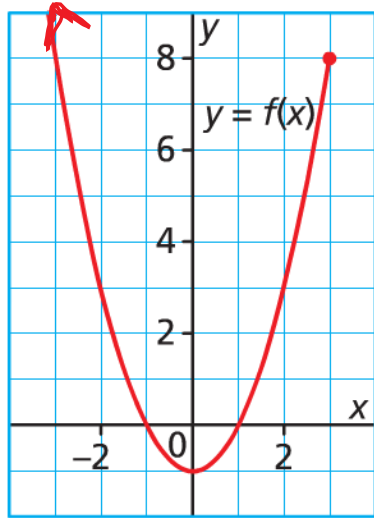
$$D: \{x \mid -1 \leq x \leq 3, x \in \mathbb{R}\}$$

$$R: \{y \mid -5 \leq y \leq -1, y \in \mathbb{R}\}$$

$$D: [-1, 3]$$

$$R: [-5, -1]$$

(C)



$$D: \{x \mid x \leq 3, x \in \mathbb{R}\}$$

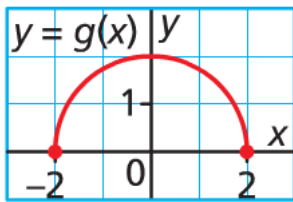
$$R: \{y \mid y \geq -1, y \in \mathbb{R}\}$$

$$D: (-\infty, 3]$$

$$R: [-1, \infty)$$

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(D)



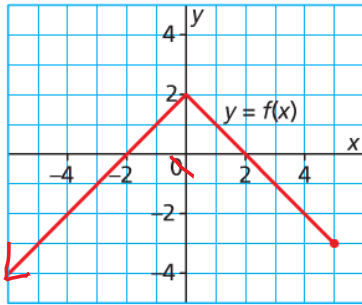
$$D: \{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$$

$$R: \{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}$$

$$D: [-2, 2]$$

$$R: [0, 2]$$

(E)



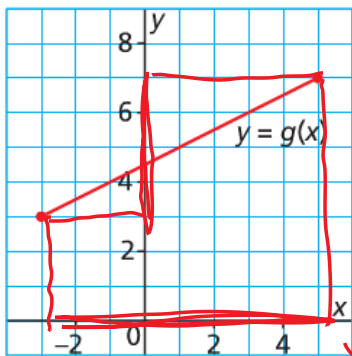
$$D: \{x \mid x \leq 5, x \in \mathbb{R}\}$$

$$R: \{y \mid y \leq 2, y \in \mathbb{R}\}$$

$$D: (-\infty, 5]$$

$$R: (-\infty, 2]$$

(F)



$$D: \{x \mid -3 \leq x \leq 5, x \in \mathbb{R}\}$$

$$R: \{y \mid 3 \leq y \leq 7, y \in \mathbb{R}\}$$

$$D: [-3, 5]$$

$$R: [3, 7]$$



**Example 5:**

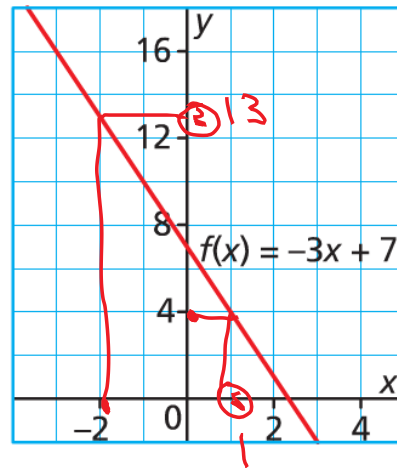
Determine the domain and range values from the graph of the function  $f(x) = -3x + 7$ .

- (A) Determine the range value when the domain value is  $-2$ . ~~X~~

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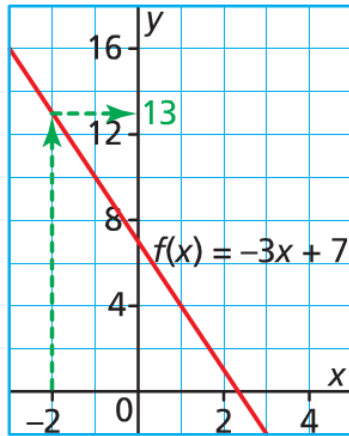
- (B) Determine the domain value when the range value is  $4$ . Y

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**Solution:**

- (A)



- (B)

