Math 1201

5.6A Properties of Linear Functions

Determining Whether a Relation is Linear

The term linear is defined as being arranged in or extending along a line. Therefore linear functions are functions that can be represented by lines. We can use different techniques to determine if a function is linear depending on how the data is presented.

We will use the following example to demonstrate this. The cost of a car rental is $60, plus $20 for every 100 km driven. The independent variable is the distance driven and the dependent variable is the cost.

Table of Values

As long as the independent variable values, x, increases by the same amount, then the dependent variable values must also increase by the same amount each time in order for the relation to be linear.

Independent variable →

Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

← Dependent variable

+100, +100, +100, +100

+20, +20, +20, +20

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

Set of Ordered Pairs

This form works exactly the same way as the table of values.

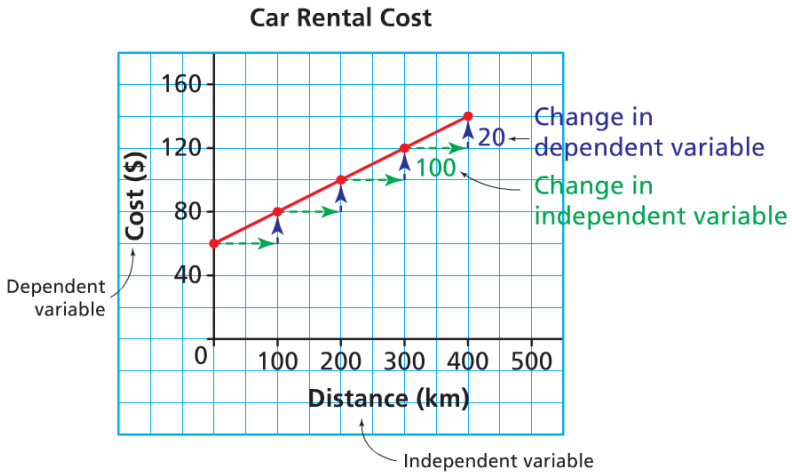
+100 +100 +100 +100

{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

Graph

As long as the rate of change is constant regardless of which points you use on the graph, then the relation is linear. If the graph is a single line, then the rate of change is constant.



From an Equation

You can also determine if a function is linear by the highest power in the corresponding equation. If the highest power is 1, it is linear. If 0 or more than one it is not linear. The highest power of a function is also called the degree of a function.

For example, $y = 2x - 1$ is linear because the largest power is 1. The degree is 1.

$y = 3x^2 + 5x - 4$ is not linear because the largest power is 2. The degree is 2.

$y = 9x^3 - 13$ is not linear because the largest power is 3. The degree is 3.

Example 1:

Which of these three functions is linear?

(i) $f(x) = -4x + 7$

(ii) $d(t) = t^2 + t - 2$

(iii) $g(t) = \frac{1}{2}t + 8$

Textbook Questions: page 308 - 310 #3, 4, 5, 8, 9, 12, 16, 17