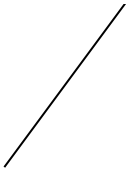
Math 1201

5.6B Rate of Change & Intercepts

Rate of Change

The **rate of change** of a relation is defined as the slant, or the change in the dependent variable over the change in the independent variable. The greater the slant of a line on a graph, the greater the rate of change:

steep slant – larger rate of change



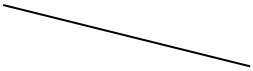
less steep slant – smaller rate of change

no slant – no rate of change or 0

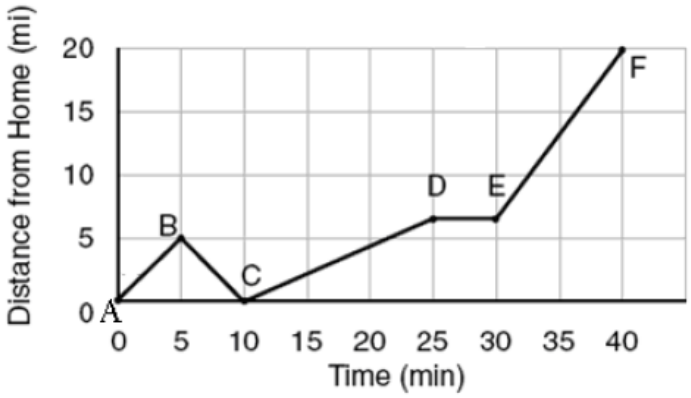
Rates of change can be positive or negative:

positive rate of change – up the hill from left to right

negative rate of change – down the hill from left to right



Example 1:
The graph shows Mitchell leaving home at point A and going to a party at point F.



(A) Over what interval was the slowest speed?

Moving: C-D 10 min – 25 min

(B) Over what interval was the fastest speed?

E - F 30 m.n – 40 m.h

(C) Which point represents when he turned around to go back home?

B 5 m.h. mark

(D) Explain what was he doing from 25-30 minutes.

He's stopped.

The rate of change of a linear function is defined as:

$$\text{roc} = \frac{\text{change in dependent varaible}}{\text{change in independent variable}}$$

rise

run

If we go back to the example of the original example from 5.6A, where the cost of a car rental is \$60, plus \$20 for every 100 km driven, we would get:

$$\text{roc} = \frac{\$20}{100 \text{ km}}$$

$$\text{roc} = \$0.20/\text{km}$$

We can now make an equation that represents the cost of renting the car for any trip as:

$$C = 0.20d + 60$$

- 60 — initial amount
- d — independent variable
- 0.20 — rate of change
- C — dependent variable

(A) How much will cost to rent the car and drive 800 km? BEDMAS

$d = 800$ $C(d) = 0.2d + 60$

$C = ?$

$C(800) = 0.2(800) + 60$

$C(800) = 160 + 60$

$C(800) = \$220$

(B) How many kilometers will you have driven if the rental costs $600?

$C = \$600$ $C = 0.2d + 60$

$d = ?$ $600 = 0.2d + 60$

$600 - 60 = 0.2d$

$\frac{540}{0.2} = \frac{0.2d}{0.2}$ $540 = 0.2d$ $d = 2700 km$

Calculating Rate of Change

Steps:

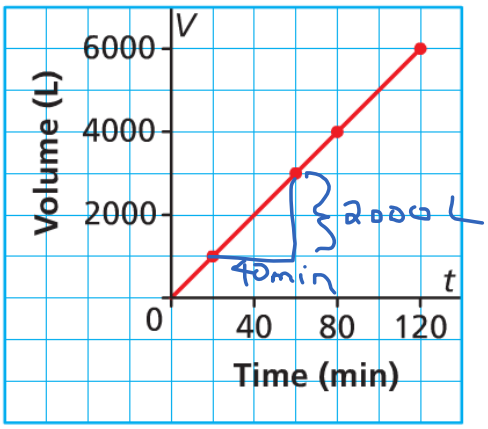
- Pick any two points that lie on a line.
- Between the two points, determine the change in value of the dependent variable.
- Between the two points, determine the change in value of the dependent variable.
- Calculate the rate of change by dividing the change in value of the dependent variable by the change in value of the independent variable.

Example 2:

A water tank on a farm near Cormack holds 6000 L. Graph A represents the tank being filled at a constant rate and Graph B represents the tank being emptied at a constant rate.

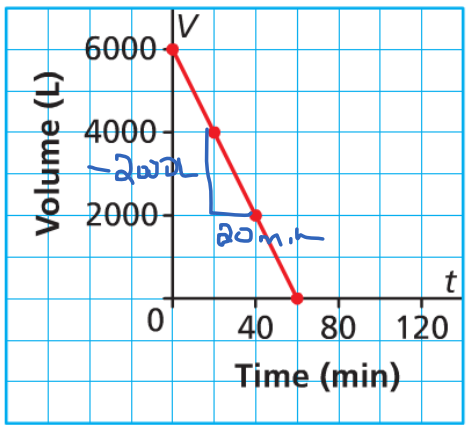
Graph A

Filling a Water Tank



Graph B

Emptying a Water Tank



$ROC = \frac{rise}{run} = \frac{2000L}{40 min}$

$= 50 L/min$

$ROC = \frac{rise}{run} = \frac{-2000L}{20 min}$

$= -100 L/min$

(A) Identify the independent and dependent variables.

independent: time

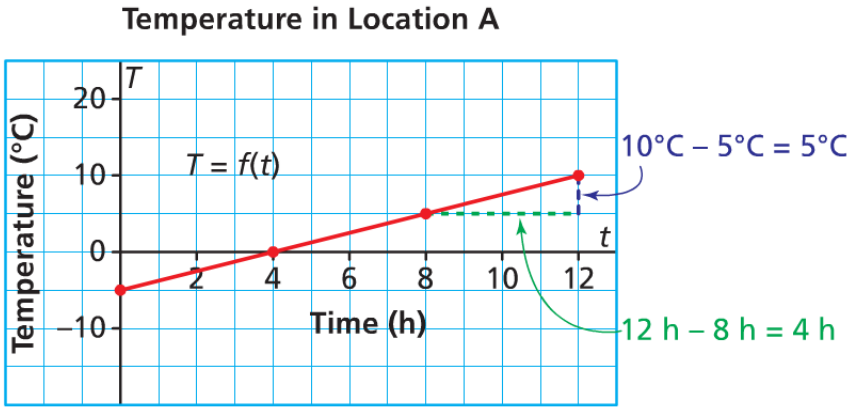
dependent: Volume

(B) Determine the rate of change of each relation, then describe what it represents.

$ROC = \frac{2000L}{40 min} = 50L/min$

$ROC = \frac{-2000L}{20 min} = -100L/min$

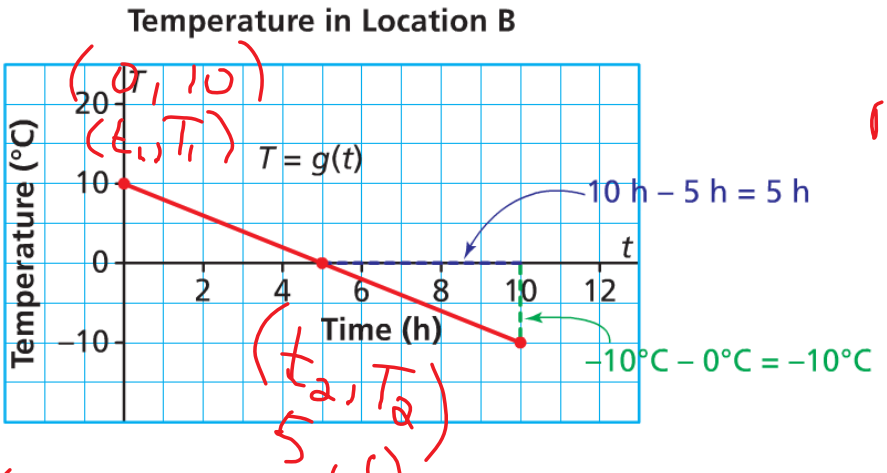
Example 3:
Each graph below shows the temperature, T, degrees Celsius, as a function of time, t hours, for two locations. What is the rate of change for each graph?



$ROC = \frac{rise}{run}$

$= \frac{5°C}{4h}$

$= 1.25°C/h$



(t_1, T_1) $(0, 10)$ (t_2, T_2) $(5, 0)$

$ROC = \frac{T_2 - T_1}{t_2 - t_1}$ $ROC = \frac{(0-10)^\circ C}{(5-0)\,h}$ $ROC = \frac{-10^\circ C}{5\,h}$ $ROC = -2^\circ C/h$

$* \; ROC = \frac{rise}{run} = \frac{-10^\circ C}{5\,h} = -2^\circ C/h$

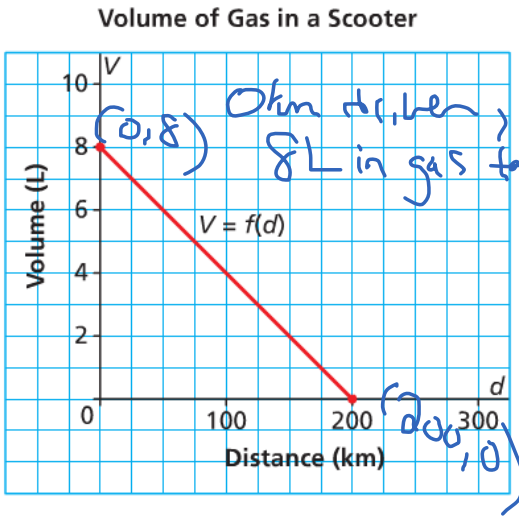
Intercepts
Intercepts are points where a graph crosses the horizontal or vertical axis.

Horizontal Intercept: where a graph crosses the horizontal axis or x-axis.

Vertical Intercept: where a graph crosses the vertical axis or y-axis.

Example 4:
The graph below shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

(A) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. What do these points represent?



(B) What are the domain and range of the function?

Domain: $\{d \mid 0 \le d \le 200, d \in R\}$

Range: $\{v \mid 0 \le v \le 8, v \in R\}$

D: $[0, 200]$

R: $[0, 8]$

Textbook Questions: page 319 - 323 #4, 5, 7, 8, 9, 10, 12, 16