Math 1201

6.1 Slope and Slope Properties

Slope refers to the steepness of a line. It means the same thing as the **rate of change** from Unit 5. Some examples of slope are:

- Roof of a house.
- A treadmill.
- A wheelchair ramp

In order to calculate the slope of a line, we need to know two points on the line. We could label these points as being (x_1, y_1) and (x_2, y_2), and substitute them into the slope formula.

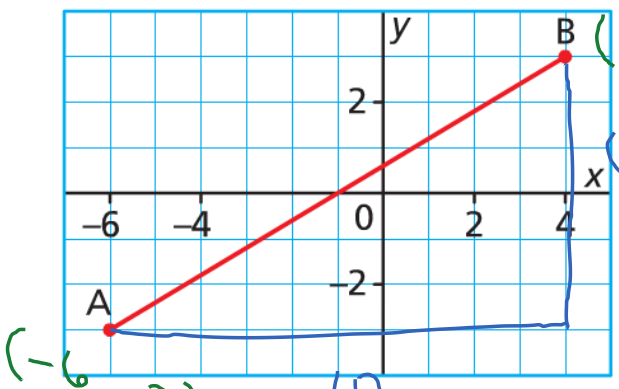
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Example 1:
Determine the slope of each line segment.

(A)



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{6}{10}$

$= \frac{3}{5}$

B $(4, 3)$, x_2 y_2

6

10

A $(-6, -3)$, x_1 y_1

$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$

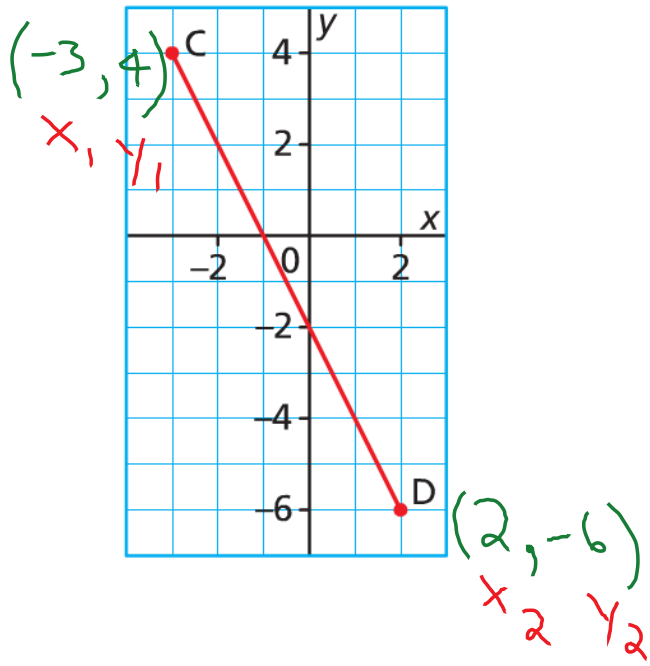
$= \frac{3-(-3)}{4-(-6)}$

$= \frac{6}{10}$

$= \frac{3}{5}$

(B)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-4}{2-(-3)} = \frac{-10}{5} = -2$$



Example 2:
Determine the slope of the line that passes through $(-5, -3)$ and $(2, 1)$.

$x_1, y_1 \quad x_2 y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{2-(-5)} = \frac{4}{7}$$

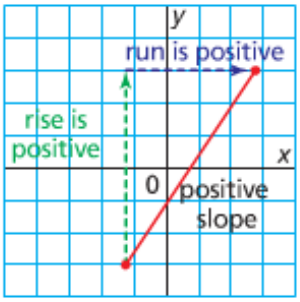
Example 3:
Determine the slop of the linear data in the following table:

Time (s)	0 (x_1)	5 (x_2)	10	15
Distance (s)	0 (y_1)	3 (y_2)	6	9

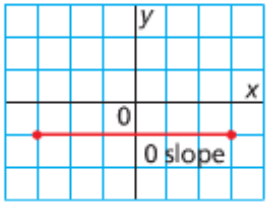
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{5-0} = \frac{3}{5}$$

Slopes of Different Types of Lines

When a line segment goes up to the right, both y and x increase; both the rise and run are positive, so the slope of the segment is positive.



For a horizontal line segment, the change in y is 0 and x increases. The rise is 0 and the run is positive.



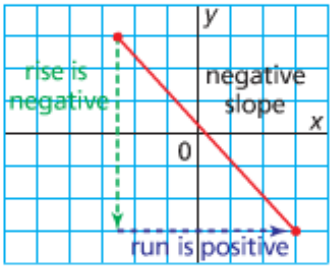
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{0}{\text{run}}$$

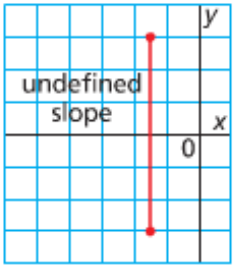
$$\text{Slope} = 0$$

So, any horizontal line segment has slope 0.

When a line segment goes down to the right, y decreases and x increases; the rise is negative and the run is positive, so the slope of the segment is negative.



For a vertical line segment, y increases and the change in x is 0. The rise is positive and the run is 0.



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

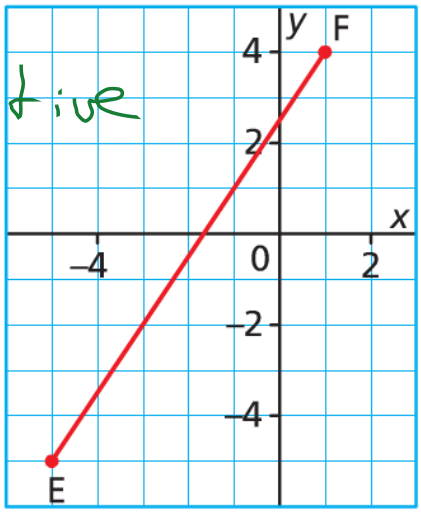
$$\text{Slope} = \frac{\text{rise}}{0}$$

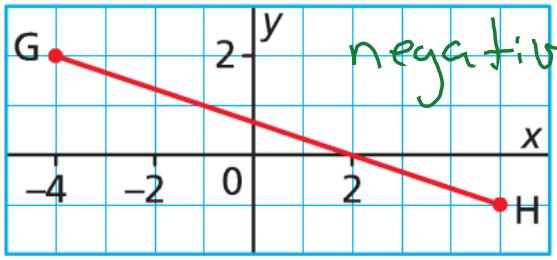
A fraction with denominator 0 is not defined.

So, any vertical line segment has a slope that is undefined.

Example 4:
Tell whether the following graphs have positive or negative slopes.

positive





negative

Drawing a Line, Given its Slope and a Point on the Line

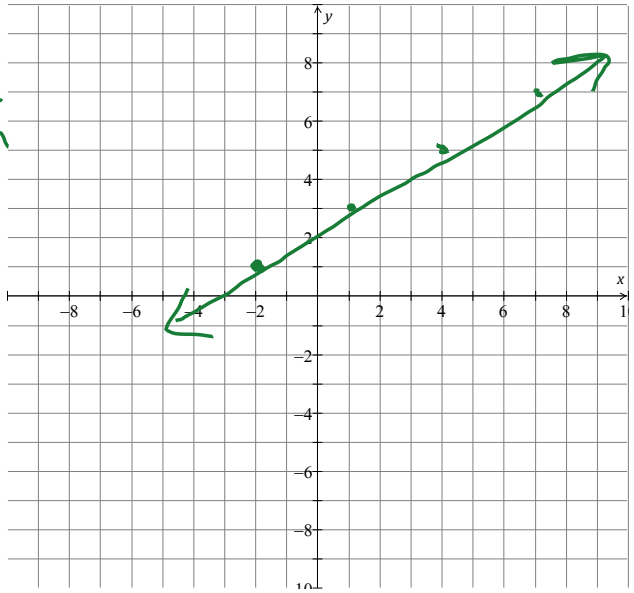
Step 1: Plot the given point on the line.

Step 2: Use the slope to get another point on the line.

Step 3: Connect the points.

Example 5:

(A) Draw a line that passes through the point $(-2, 1)$ and has slope $\frac{2}{3}$. rise / run



(B) Determine the coordinates of two other points that lie on the line.

(4, 5), (7, 7)

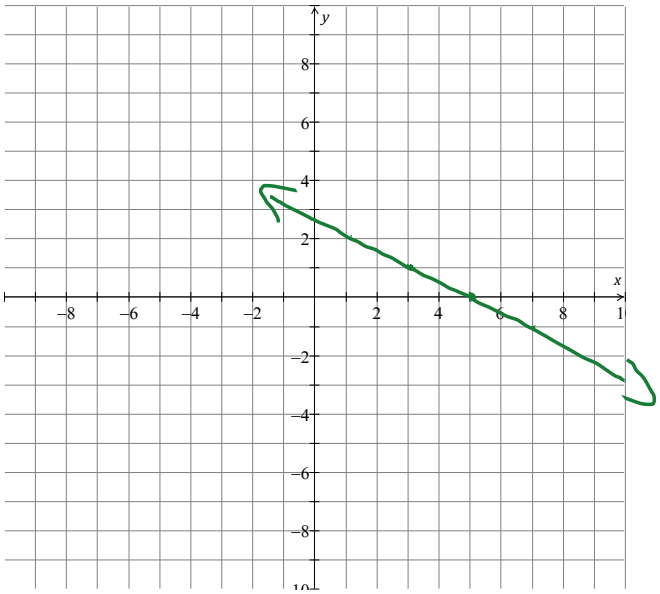
Example 6:

(A) Draw a line that passes through the point $(3, 1)$ and has slope $-\frac{1}{2}$.

$\frac{-1}{2}$ rise / run

(B) Determine the coordinates of two other points that lie on the line.

$(5, 0)$, $(7, -1)$



Word Problems Involving Slope

Example 7:
Jacob was charged $7.00 to travel 7 km. Later in the week, he was charged $12.50 for 18 km. What is the average cost per kilometre for the trip?

$(7, 7)$, $(18, 12.50)$
$x_1\ y_1$ $x_2\ y_2$

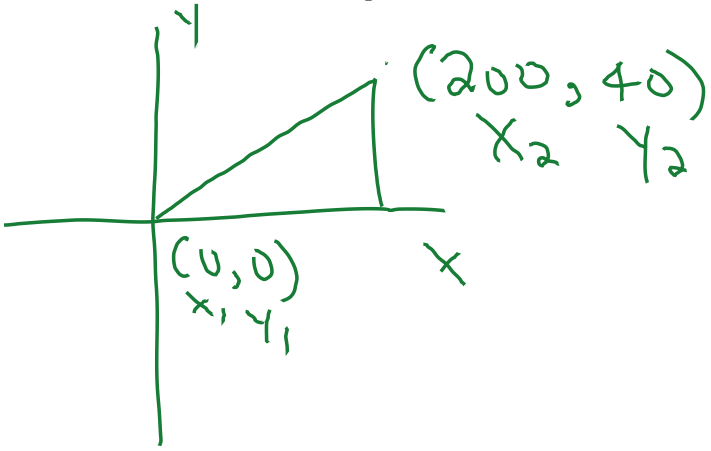
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12.5 - 7}{18 - 7}$$

$$= \frac{5.5\ \$}{11\ km}$$

$= \$0.5/km$

Example 8:
Marine Park is the home of two giant waterslides which measure 40 ft high by 200 ft long. Determine the slope of the slides.



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{40 - 0}{200 - 0}$$

$$= \frac{40}{200}$$

$$= \frac{1}{5} \text{ or } 0.2$$

Textbook Questions: page 339 - 340 #5, 6, 7, 8, 9, 11, 15