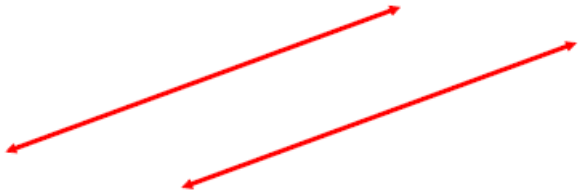
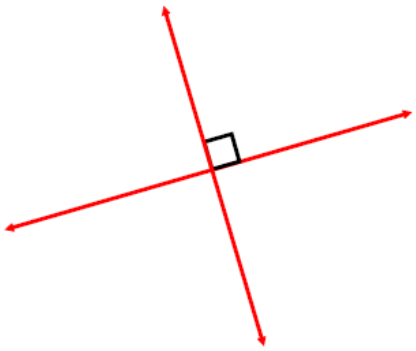
Math 1201

6.2 Slopes of Parallel and Perpendicular Lines

Parallel Lines: lines that never meet.



Perpendicular Lines: lines that meet at a 90° angle.

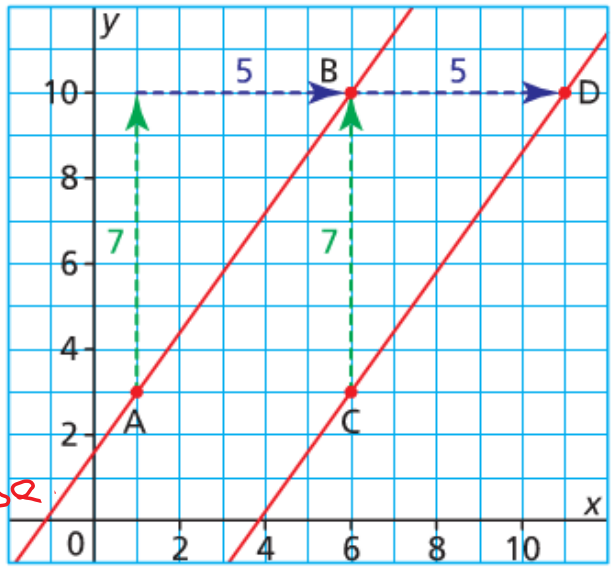


What is the relationship between the slopes of parallel lines? Consider the parallel lines shown in the graph:

$\text{Slope}_{\overline{AB}} = \frac{\text{rise}}{\text{run}} = \frac{7}{5}$

$\text{Slope}_{\overline{CD}} = \frac{\text{rise}}{\text{run}} = \frac{7}{5}$

// have the same slope

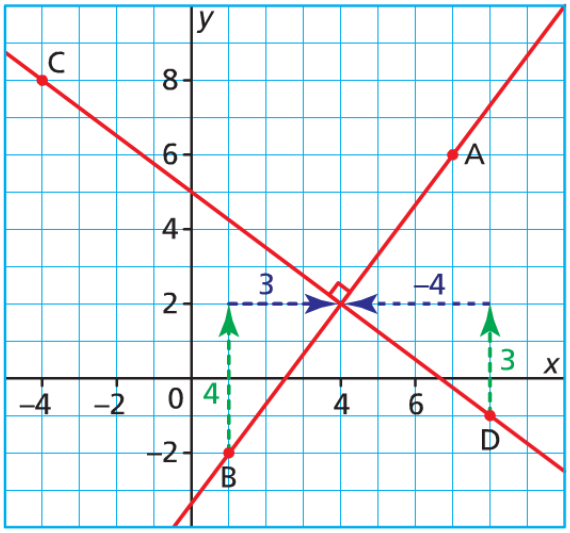


What is the relationship between the slopes of perpendicular lines? Consider the perpendicular lines shown in the graph:

$\text{Slope}_{\overline{AB}} = \frac{\text{rise}}{\text{run}} = \frac{4}{3}$

$\text{Slope}_{\overline{CD}} = \frac{\text{rise}}{\text{run}} = \frac{3}{-4} = -\frac{3}{4}$

$\frac{4}{3} \times \left(-\frac{3}{4}\right) = -\frac{12}{12} = -1$



Negative Reciprocal: two numbers, a and b, are negative reciprocals if $a \cdot b = -1$

Summary

- **Parallel Lines** have equal slope.
- **Perpendicular Lines** have slopes that are negative reciprocals.

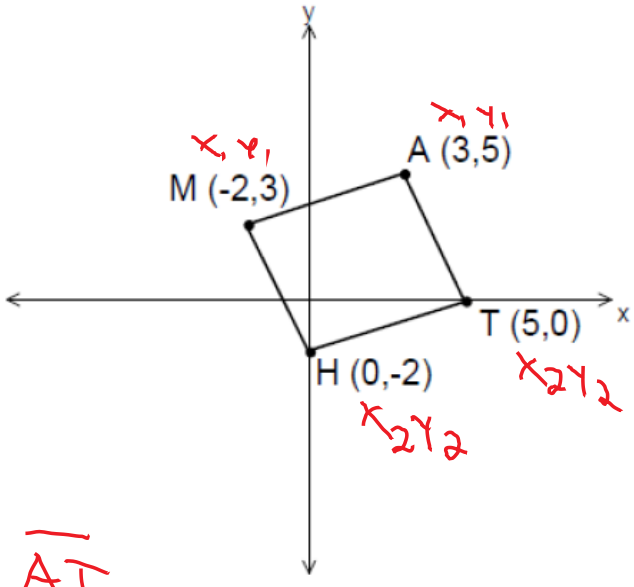
Example 1:

Prove $\overline{MH}$ and $\overline{AT}$ are parallel:

$m_{\overline{MH}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{0-(-2)} = -\frac{5}{2}$

$m_{\overline{AT}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{5-3} = -\frac{5}{2}$

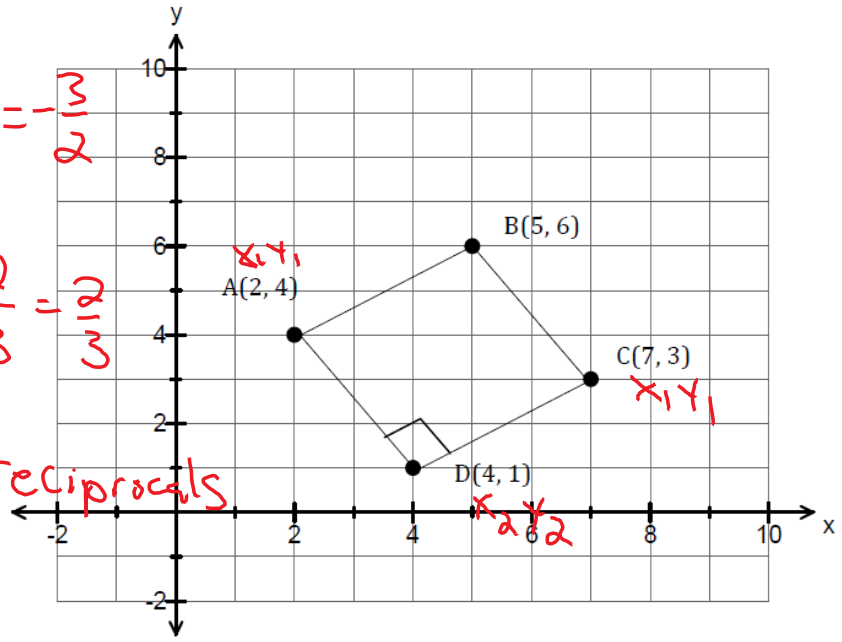
Same slope $\therefore \overline{MH} \parallel \overline{AT}$



Example 2:
Prove that $\overline{AD}$ and $\overline{DC}$ are perpendicular:

$m_{\overline{AD}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{4-2} = -\frac{3}{2}$

$m_{\overline{DC}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{4-7} = \frac{-2}{-3} = \frac{2}{3}$



Slopes are negative reciprocals

$\therefore \overline{AD} \perp \overline{DC}$

Example 3:
The slope of $\overline{AB}$ is $-\frac{4}{5}$. The slope of $\overline{CD}$ is $\frac{w}{35}$. Given $\overline{AB}$ is parallel to $\overline{CD}$, determine the value of w.

$m_{\overline{AB}} = m_{\overline{CD}}$

$-\frac{4}{5} = \frac{w}{35}$

$5w = -140$

$\frac{5w}{5} = \frac{-140}{5}$

$w = -28$

Example 4:
The slope of $\overline{AB}$ is 3. The slope of $\overline{CD}$ is $\frac{x}{12}$. Given $\overline{AB}$ is perpendicular to $\overline{CD}$, determine the value of x.

If $\overline{AB} \perp \overline{CD}$, then $m_{\overline{CD}} = -\frac{1}{3}$

$-\frac{1}{3} = \frac{x}{12}$

$3x = -12$

$\frac{3x}{3} = \frac{-12}{3}$

$x = -4$

Example 5:
Line $\overline{AB}$ has a slope of 2. Line $\overline{CD}$ is parallel to line $\overline{AB}$. The points $(1, k)$ and $(4, 12)$ lie on line $\overline{CD}$. Determine the value of k.

$x_1 y_1 \quad x_2 y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$2 = \frac{12 - k}{4 - 1}$

$3 \cdot 2 = \frac{12 - k}{3} \cdot 3$

$6 = 12 - k$

$k = 12 - 6$

$k = 6$

Example 6:
Is ΔABC a right triangle? Justify your answer.

Right Δ → 90°

$m_{\overline{AB}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-2}{4-(-4)}$

$= \frac{2}{8}$

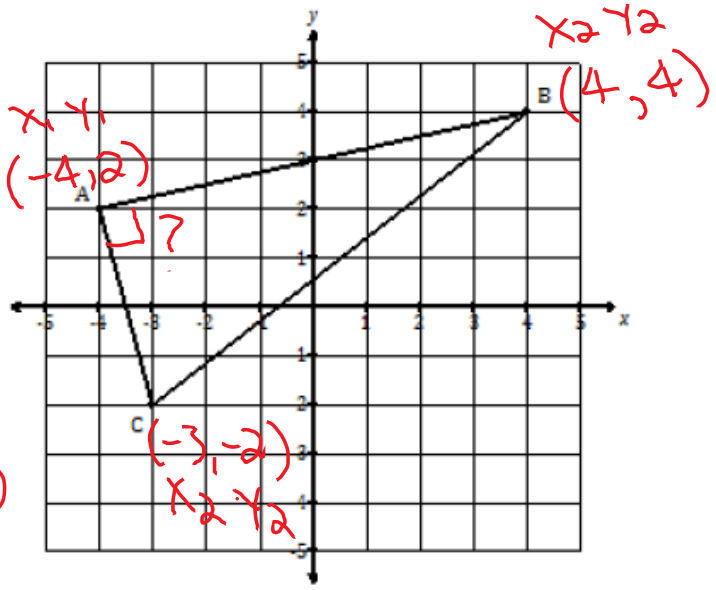
$= \frac{1}{4}$

$m_{\overline{AC}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-2}{-3-(-4)}$

$= \frac{-4}{1}$

$= -4$



Since $\overline{AB} \perp \overline{AC}$

ΔABC is right.

Textbook Questions: page 348 - 350 #3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 17