Math 1201

6.5 Slope-Point Form of the Equation for a Linear Function

Slope-Point Form

This form of a linear function consists of the slope, $\boldsymbol{m}$, and one other point on the line, (x_1, y_1). This equation actually comes directly from the formula for finding the slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Instead of calling the second point (x_2, y_2), we simply call it (x, y):

$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = \frac{y - y_1}{\cancel{(x - x_1)}}\cancel{(x - x_1)}$$

$$m(x - x_1) = y - y_1 \rightarrow y - y_1 = m(x - x_1)$$

The equation of a line that passes through $\mathrm{P}(x_1, y_1)$ and has slope m is:

$$y - y_1 = m(x - x_1)$$

Where $\boldsymbol{m}$ is the slope of the line and and $(\boldsymbol{x_1}, \boldsymbol{y_1})$ is any point on that line.

Note: x_1 & y_1 will have the opposite sign in the equation.

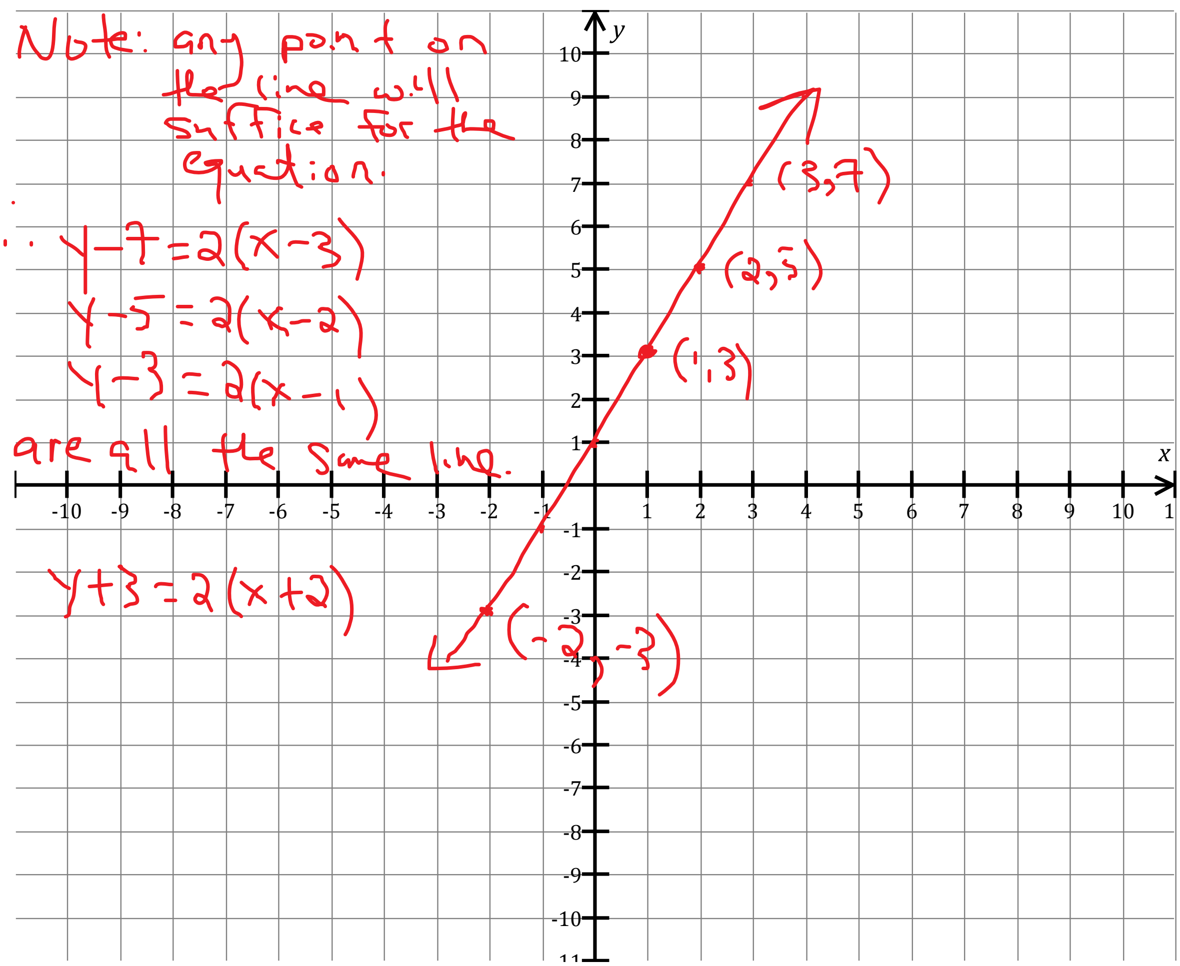
Example 1:
Identify the slope and a point on the line for each of the following equations and then graph the resulting line:

$y - y_1 = m(x - x_1)$

(A) $y - 3 = 2(x - 1)$

$m = \frac{2}{1} \frac{\text{rise}}{\text{run}}$

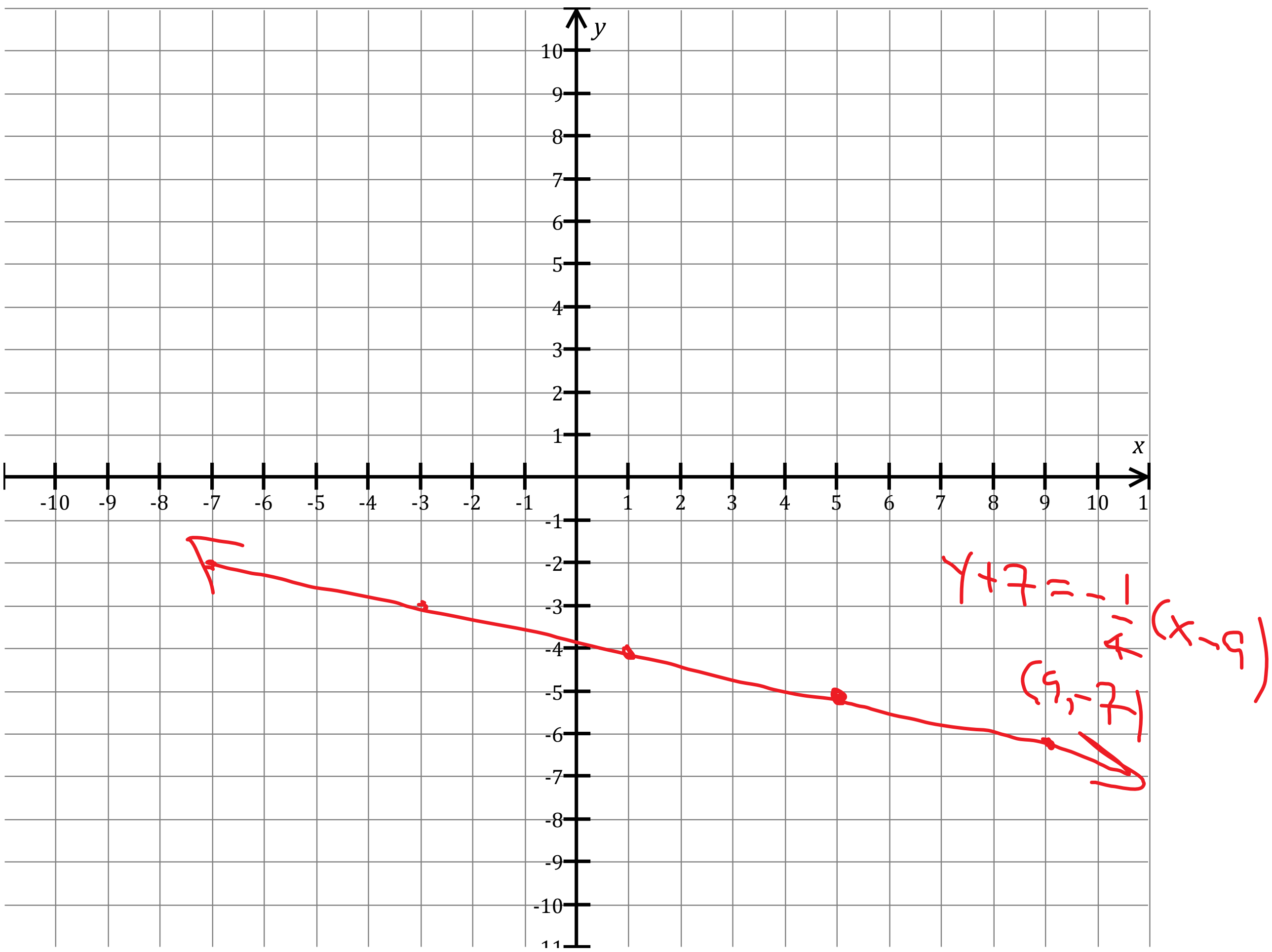
$(x_1, y_1) = (1, 3)$



(B) $y + 6 = -\frac{1}{4}(x - 5)$

$m = -\frac{1}{4} = \frac{-1}{4} \frac{\text{rise}}{\text{run}}$

$(x_1, y_1) = (5, -6)$



Example 2:
Write the equation of the line given the following information:

(A) The line has a slope of $\frac{1}{2}$ and passes through $(-3,\ 6)$.

m $\qquad$ x_1, y_1

$$y-6=\frac{1}{2}(x+3)$$

(B) The line passes through $(-2, 6)$ and $(4, -6)$.

$x_1\ y_1 \qquad x_2\ y_2$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{-6-6}{4-(-2)}$$

$$=\frac{-12}{6} = -2$$

$(-2,6)$: $y-6=-2(x+2)$

or

$(4,-6)$: $y+6=-2(x-4)$

(C) The line passes through $(4, 10)$ and is parallel to the line $y = 2x - 4$.

$x_1\ y_1 \qquad m$

Parallel lines have the same slope.

$$m=2$$

$$\therefore\ y-10=2(x-4)$$

(D) The line passes through the point $(2, 5)$ and is perpendicular to $y = -\frac{1}{4}x + 9$.

$x_1\ y_1$

Perpendicular lines have slopes that are negative reciprocals.

$$\therefore\ m=4$$

$$y-5=4(x-2)$$

Changing from Slope-Point to Slope-Intercept Form

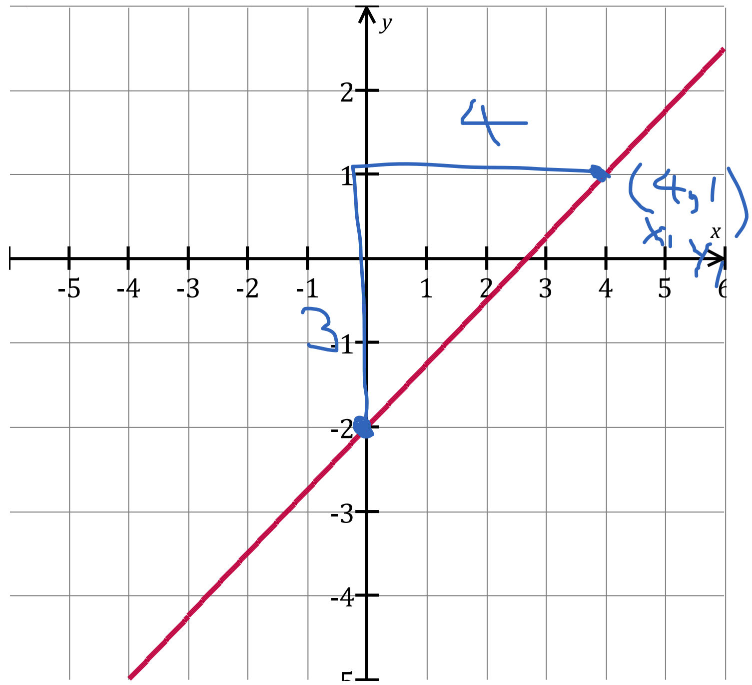
Example 3:

(A) Write an equation in slope-point form:

$m = \frac{3}{4}$

$(x_1, y_1) = (4, 1)$

$y - 1 = \frac{3}{4}(x - 4)$



(B) Write the eqeuation from part (A) in slope-intercept form: $y = mx + b$

$y - 1 = \frac{3}{4}(x - 4)$

$y = \frac{3}{4}(x - 4) + 1$

$y = \frac{3}{4}x - \frac{3}{\cancel{4}} \cdot \cancel{4} + 1$

$y = \frac{3}{4}x - 3 + 1$

$y = \frac{3}{4}x - 2$

Example 4: (just touches)

The line $\overleftrightarrow{BD}$ is tangent to the circle at point $A(2, 4)$. If the centre of the circle is $C(-1, 1)$, write the equation of the tangent line $\overleftrightarrow{BD}$.

$$m_{\overline{AC}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\overline{AC} \perp \overline{BD}$$

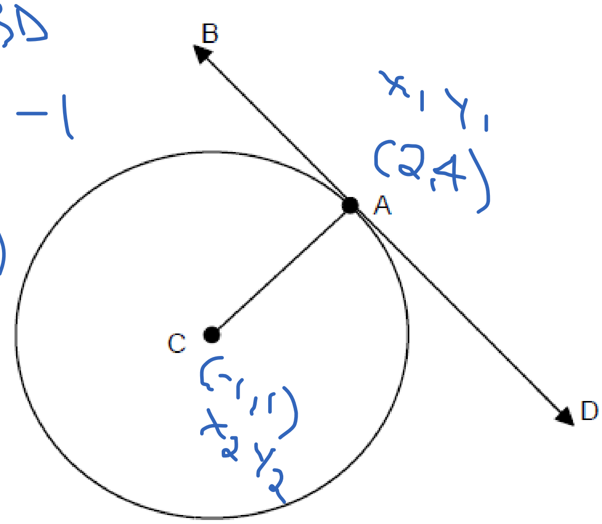
$$\therefore m_{\overline{BD}} = -1$$

$$m_{\overline{AC}} = \frac{1 - 4}{-1 - 2}$$

$$y - 4 = -(x - 2)$$

$$m_{\overline{AC}} = \frac{-3}{-3}$$

$$m_{\overline{AC}} = 1$$



Example 5:
The graph shown below is made up of linear segments A, B and C. Write an equation in slope-point form for the line that contains each segment.

A

$m = \frac{1}{1} = 1$

$y - 2 = (x - 1)$

B

$m = \frac{-3.5}{1} = -3.5$

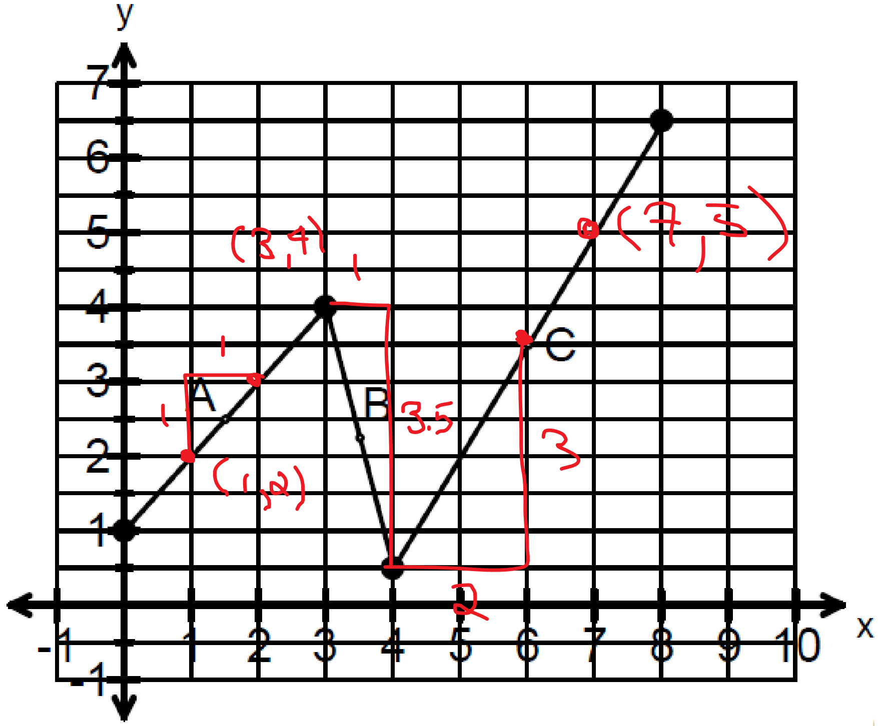
$\frac{-3.5}{1} = \frac{-35}{10} = -\frac{7}{2}$

$y - 4 = -\frac{7}{2}(x - 3)$

C

$m = \frac{3}{2}$

$y - 5 = \frac{3}{2}(x - 7)$



Textbook Questions: page 372 -374 #4, 5, 6(a, b), 7, 8, 9, 11, 12, 20, 22, 19, 20