Math 1201

7.2/7.3 Solving Systems Linear Equations Graphically

System of Linear Equations: collection of linear equations involving the same set of variables.

Solving a system of equations involves finding the **intersection point** of multiple graphs. This is also called the **solution** of the system of linear equations

Solving a System of Equations by Graphing

One way to solve a system of equations is by graphing both lines and identifying the intersection point. This method also gives a visual representation of the situation.

Example 1:
Solve the linear system:

① $x + y = 8$
② $3x - 2y = 14$

$y = mx + b$

① $x + y = 8$

$y = -x + 8$

$m = \frac{rise}{run} = \frac{-1}{1}$

$b = 8$

② $3x - 2y = 14$

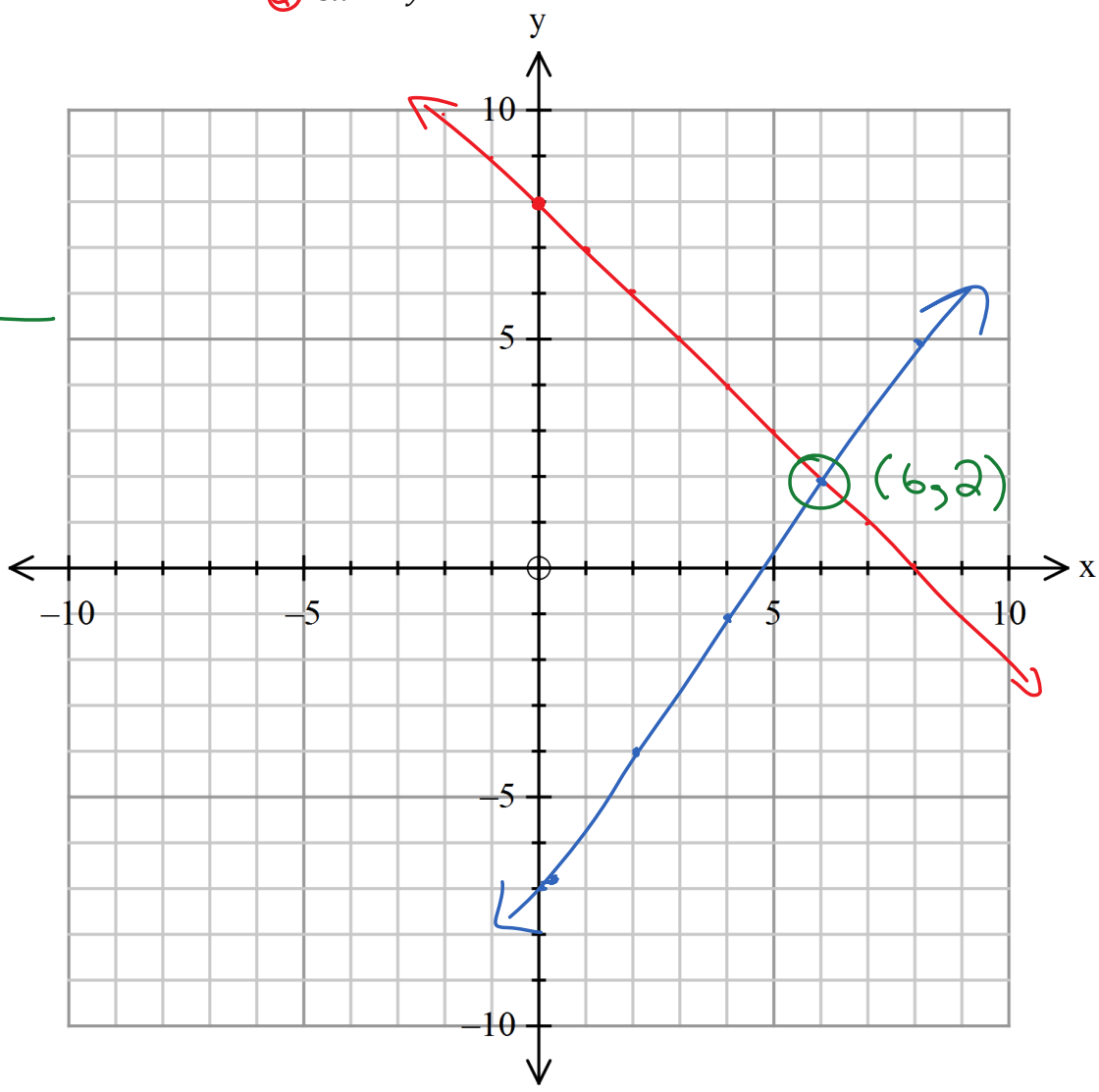
$-2y = -3x + 14$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{14}{-2}$

$y = \frac{3}{2}x - 7$

$m = \frac{3}{2}$

$b = -7$



Example 2:
Solve the following system of equations graphically.

$y = mx + b$

$3x + 2y = 12$

$2y = -3x + 12$

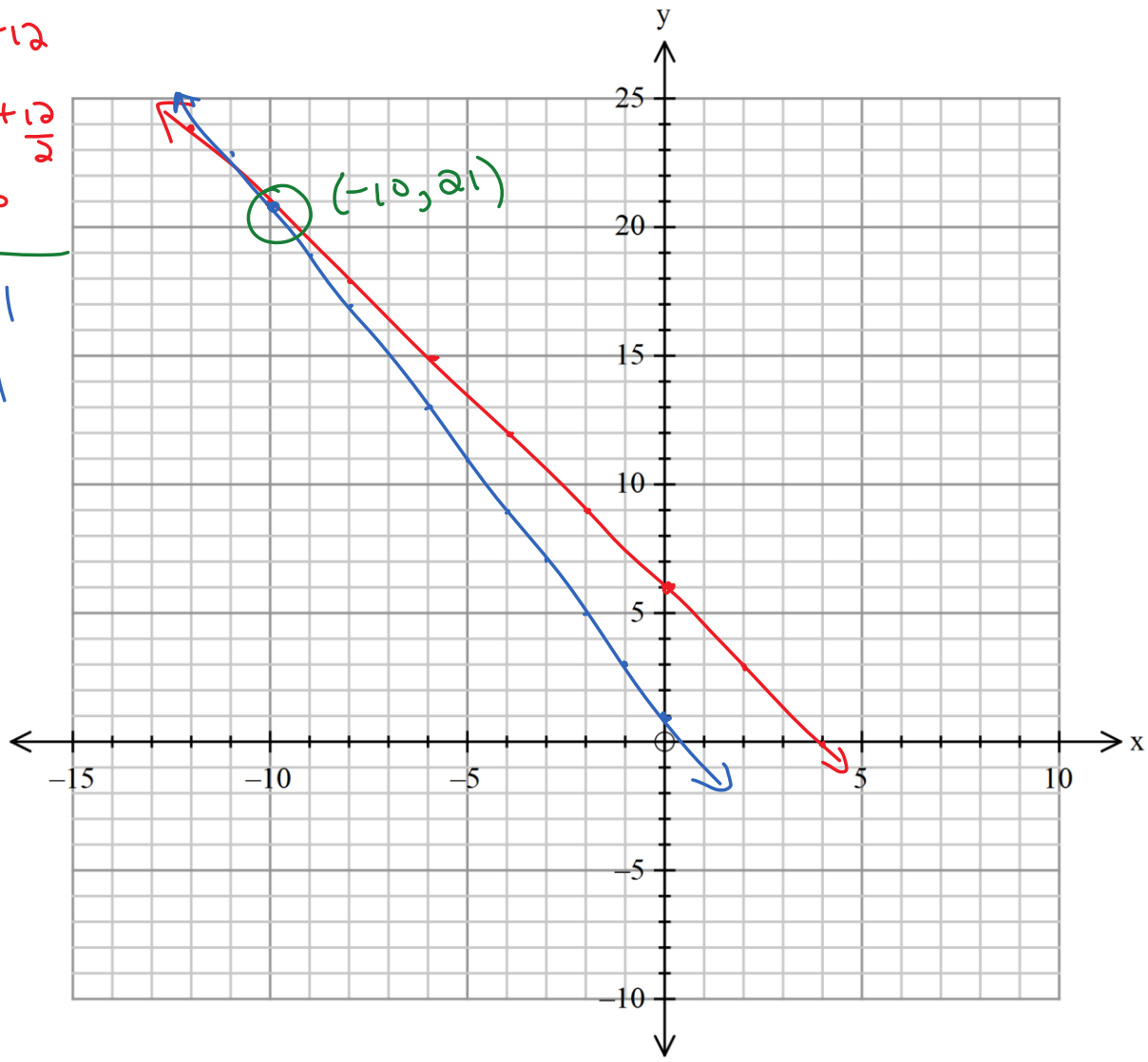
$\frac{2y}{2} = \frac{-3x}{2} + \frac{12}{2}$

$y = -\frac{3}{2}x + 6$

① $3x + 2y = 12$
② $2x + y = 1$

$2x + y = 1$

$y = -2x + 1$



Example 3:
Mitchell solved the linear system $2x + 3y = 6$ and $x - 2y = -6$. His solution was $(2, 4)$. (x, y)
Verify whether Mitchell's solution is correct. Illustrate Mitchell's results on a graph.

① $2x+3y=6$

$2(2)+3(4)=6$

$4+12=6$

$16 \neq 6$

② $x-2y=-6$

$2-2(4)=-6$

$2-8=-6$

$-6=-6$

LHS = RHS

Doesn't satisfy both equations. ∴ Not a solution.

~~Intercept~~ method:

① $2x+3y=6$

x-int: $y=0$

$2x+3(0)=6$

$2x=6$

$\frac{2x}{2}=\frac{6}{2}$

$x=3$

$(3,0)$

y-int: $x=0$

$2(0)+3y=6$

$3y=6$

$\frac{3y}{3}=\frac{6}{3}$

$y=2$

$(0,2)$

② $x-2y=6$

x-int: $y=0$

$x-2(0)=6$

$x=6$

$(6,0)$

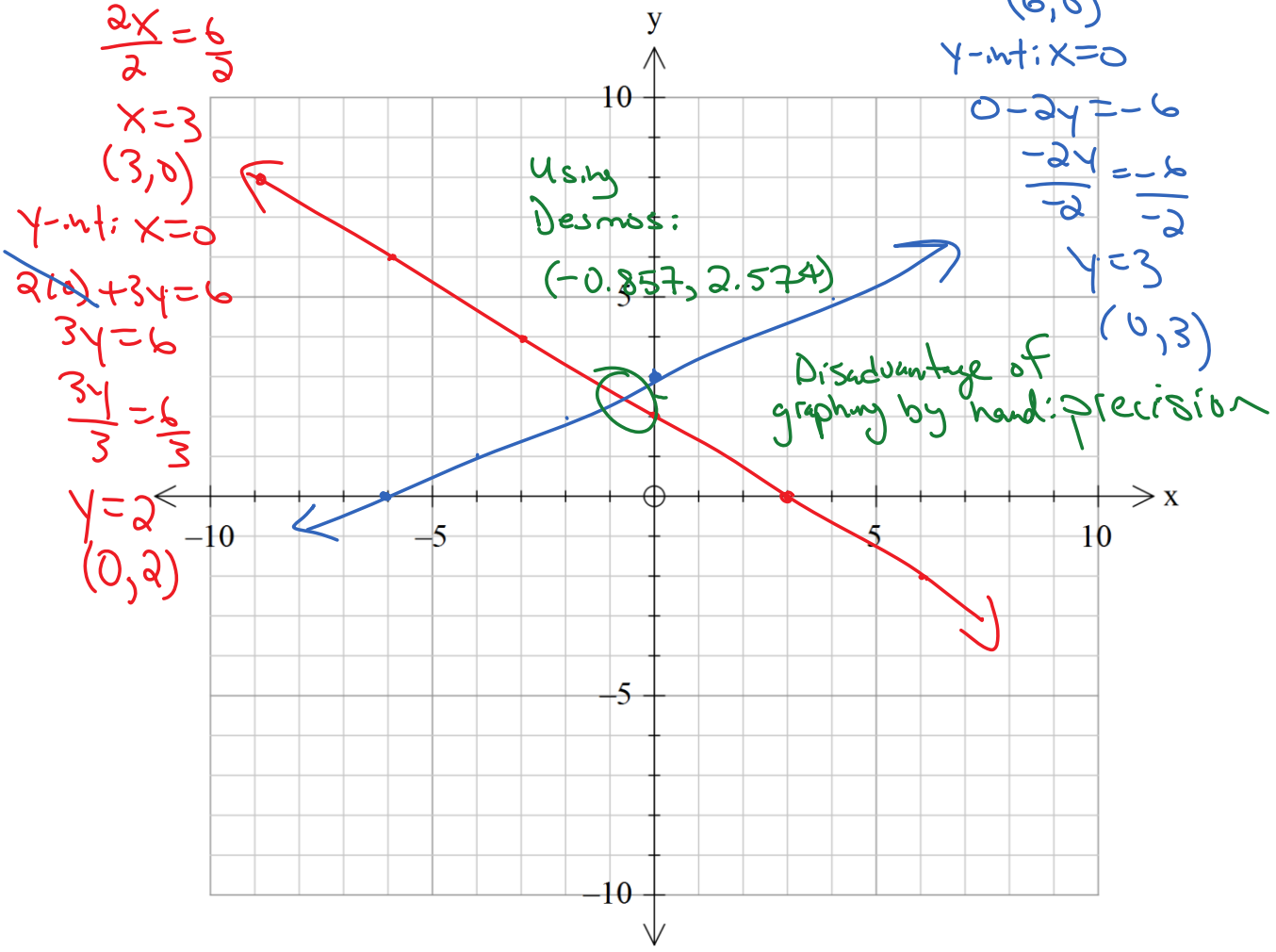
y-int: $x=0$

$0-2y=-6$

$\frac{-2y}{-2}=\frac{-6}{-2}$

$y=3$

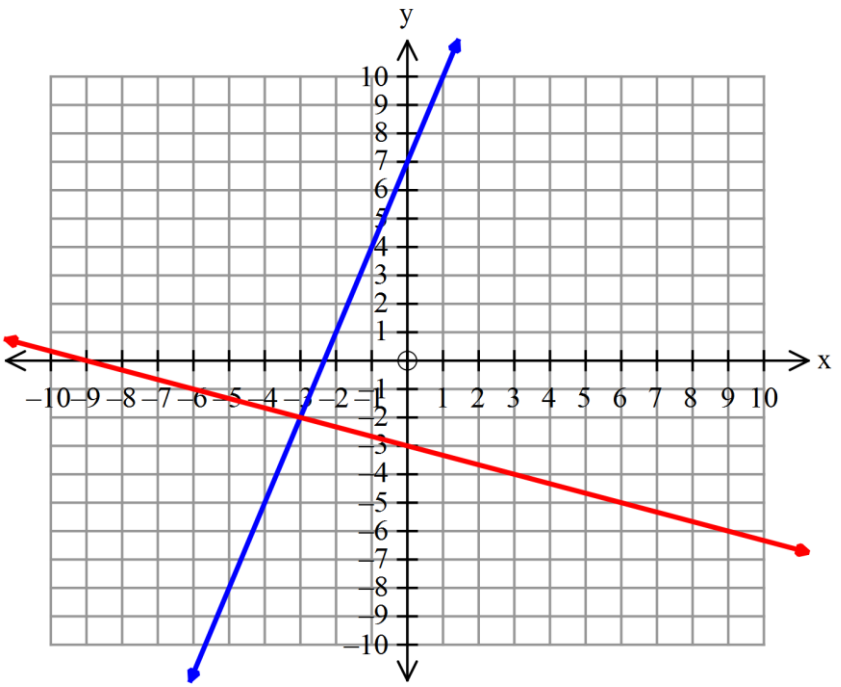
$(0,3)$



Determining the Number of Solutions to a System of Equations (without graphing)

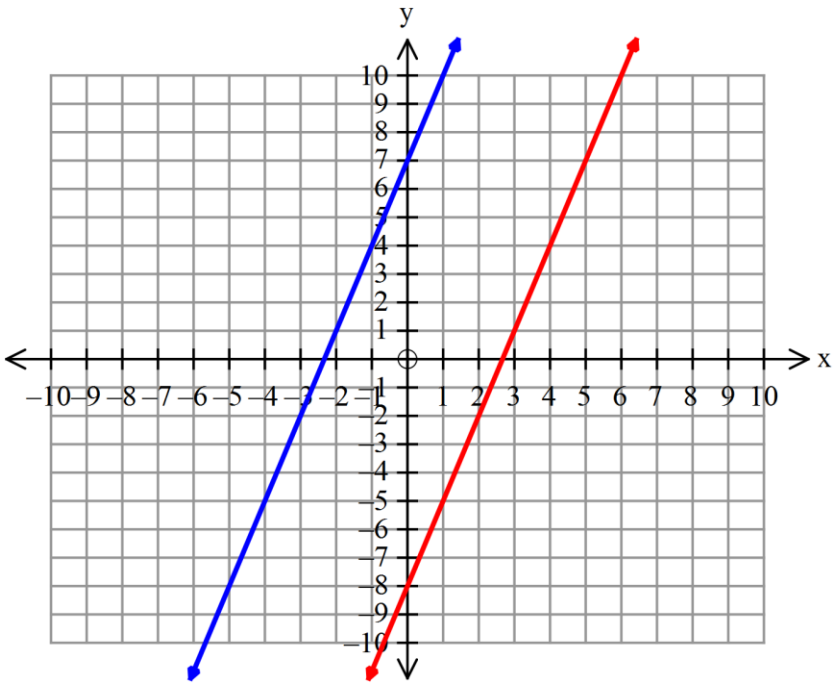
When finding solutions to a system of linear equation using a graph, there are three possible cases we could have to consider:

Case 1:
One intersection point – one solution.



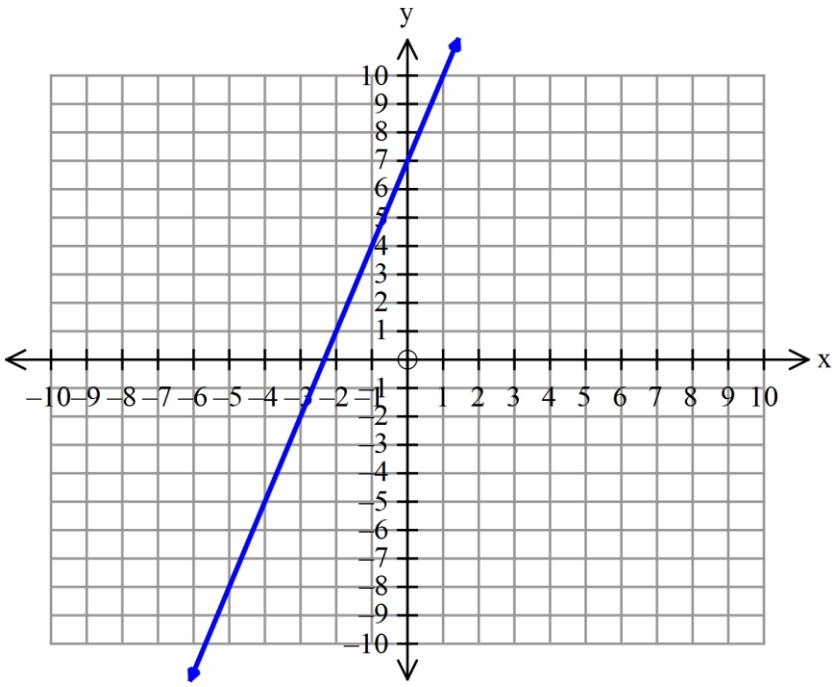
One Solution: This happens when the equations have different slopes and y-intercepts.

Case 2:
0 intersection points – no solution.



No Solution: This happens for parallel lines. Equations have the same slope but different y-intercepts.

Case 3:
Infinite intersection points – infinite solutions.



Infinite Solutions: This happens when the equations have the same slopes and intercepts.

Example 4:
Determine the number of solutions for each system of equations.

(A) $y = 2x + 3$ and $y = 2x - 4$

Same slope, different y-int ∴ no solution

(B) $y = 5x - 1$ and $y = 3x + 4$

different slope, different y-int ∴ 1 solution

(C) ① $2x + 3y - 1 = 0$ and ② $4x + 6y - 2 = 0$

Since equation ② is a multiple of equation ①, infinite solutions

Graphing Systems of Equations Using Desmos Graphing Calculator

We can also use technology to graph system of equations and find the solutions to these systems. Desmos is a free graphing calculator created by Google and can be run from any tablet or smart phone. There is an app you can download for Apple or Android or you can simply use the web browser.

One of the benefits of graphing with technology is the level of precision the software can display. When constructing graphs by hand, you will often have to estimate what the solutions are if the coordinates are not integers. Desmos will give solutions to the nearest hundredth.

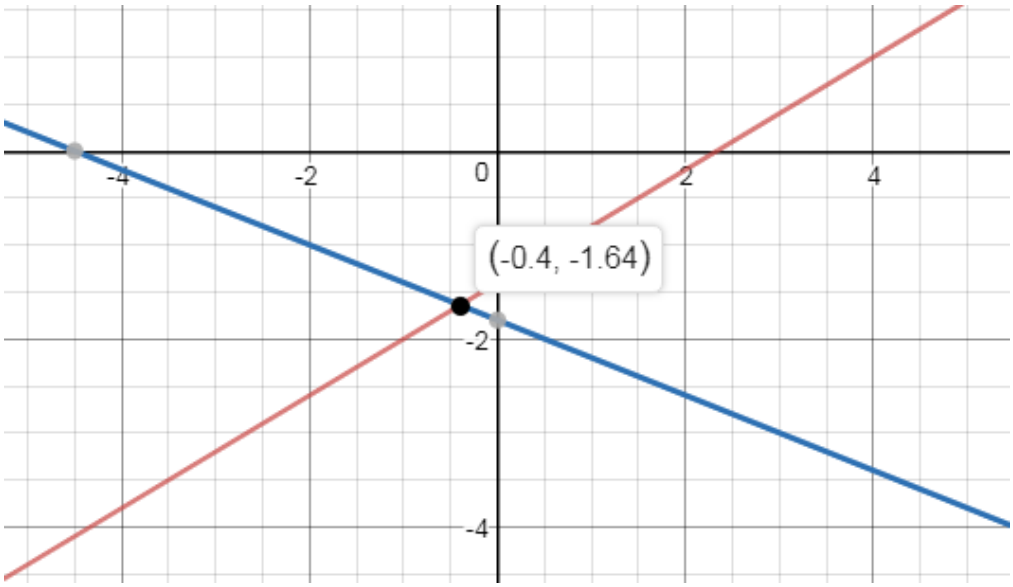
Example 5:
Graph the following system using technology:

$$3x - 5y = 7$$
$$-2x - 5y = 9$$

Once in Desmos, enter each linear equation into the equation field.

Desmos will graph the linear system. Simply click on the intersection point and the software will display the solution. You can also click on the x and y – intercepts.

The solution to the system of equations is:



Example 6:
Graph the following system using technology:

$$5x - 6y = -11$$
$$-3x - 7y = 3$$

$(-1.792, 0.34)$

Example 7:
Jill earns $40 plus $10 per hour. Tony earns $50 plus $5 per hour. Graphically represent the linear system relating Jill and Tony's earnings. Using technology, identify the solution to the linear system and explain what it represents.

Jill: $y = 10x + 40$

Tony: $y = 5x + 50$

$(2, 60)$

They both make $60 when they work 2 hours.

Graph by hand

Use Desmos

Textbook Questions: page 409 - 410 #3, 4, 5, 6, 7, 8, 9