Math 1201

7.4 Solving a System of Equations Using Substitution

To solve a system of equations algebraically means to solve without using a graph. There are two ways we will do this:

- Substitution
- Elimination

Solving by Substitution

Steps

- Label the equations as 1 and 2.
- Solve one of the equations for one of the variables.
- Take what you came up with and substitute it into the **other equation**.
- Solve for the single variable in the new equation.
- Substitute the value of the variable that you just came up with into the other equation and solve for the other variable.
- State your solution as an ordered pair.

Example 1:

Solve using substitution:

Sub ① into ②

① $y = 9 - x$

② $2x + 3y = 11$

$2x+3(9-x)=11$

$2x+27-3x=11$

$2x-3x=11-27$

$-x=-16$ or $16=x$

$\frac{-x}{-1} = \frac{-16}{-1}$

$x=16$

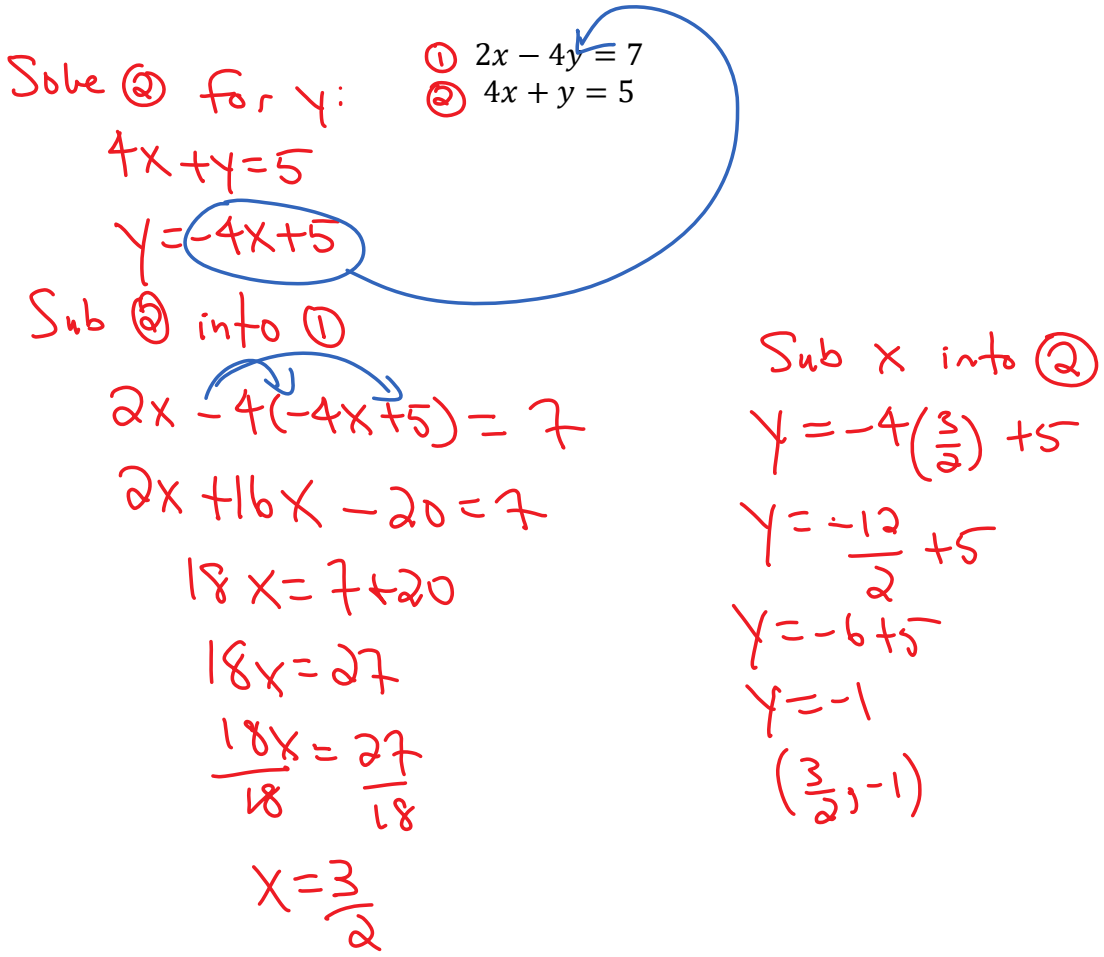
Sub x into ①

$y=9-16$

$y=-7$

$(16,-7)$

Example 2:
Solve using substitution:



Example 3:
Solve:

① $2x + 3y = 11$
② $4x - y = 13$

Solve ② for y:

$4x - y = 13$

$4x - 13 = y$

Sub ② into ①:

$2x + 3(4x - 13) = 11$

$2x + 12x - 39 = 11$

$14x = 39 + 11$

$14x = 50$

$\frac{14x}{14} = \frac{50}{14}$

$x = \frac{25}{7}$

Sub x into ②

$y = 4\left(\frac{25}{7}\right) - 13$

$y = \frac{100}{7} - \frac{13}{1} \cdot \frac{7}{7}$

$y = \frac{100}{7} - \frac{91}{7}$

$y = \frac{9}{7}$

$\left(\frac{25}{7}, \frac{9}{7}\right)$

Example 4:
Solve:

① $5x - 3y = 18$
② $4x - 6y = 18$

Solve ① for x:

$5x - 3y = 18$

$5x = 3y + 18$

$\frac{5x}{5} = \frac{3y}{5} + \frac{18}{5}$

$x = \frac{3}{5}y + \frac{18}{5}$

Sub ① into ②

$4\left(\frac{3}{5}y + \frac{18}{5}\right) - 6y = 18$

$\frac{12}{5}y + \frac{72}{5} - 6y = 18$

$\frac{12}{5}y - 6y = 18 - \frac{72}{5}$

$\frac{12}{5}y - \frac{6 \cdot 5}{1 \cdot 5}y = \frac{18 \cdot 5}{1 \cdot 5} - \frac{72}{5}$

$\frac{12}{5}y - \frac{30}{5}y = \frac{90}{5} - \frac{72}{5}$

$-\frac{18}{5}y = \frac{18}{5}$

$\frac{-\frac{18}{5}y}{-\frac{18}{5}} = \frac{\frac{18}{5}}{-\frac{18}{5}}$

$y = -1$

Sub y into ①

$x = \frac{3}{5}(-1) + \frac{18}{5}$

$x = -\frac{3}{5} + \frac{18}{5}$

$x = \frac{15}{5}$

$x = 3$

$(3, -1)$

Solving Linear Systems with Fractional Coefficients
When a linear system has any fractional coefficients, it's best to get rid of the denominators first. This is done by finding the lowest common denominator, LCD, and multiplying each term by the LCD.

Example 5:
Solve:

① $\frac{1}{2}x + \frac{2}{3}y = -1$ LCD: $2 \cdot 3 = 6$

② $y = \frac{1}{4}x - \frac{5}{3}$ LCD: $4 \cdot 3 = 12$

$6①\ \ 6 \cdot \frac{1}{2}x + 6 \cdot \frac{2}{3}y = 6(-1)$

$3x + 4y = -6$

Sub ② into ①:

$3x + 4\left(\frac{1}{4}x - \frac{5}{3}\right) = -6$

$3x + x - \frac{20}{3} = -6$

$4x = \frac{20}{3} - \frac{6 \cdot 3}{1 \cdot 3}$

$4x = \frac{20}{3} - \frac{18}{3}$

$4x = \frac{2}{3}$

$\frac{4x}{4} = \frac{\frac{2}{3}}{\frac{4}{1}}$

$x = \frac{2}{3} \cdot \frac{1}{4}$

$x = \frac{2}{12}$

$x = \frac{1}{6}$

Sub x into ②:

$y = \frac{1}{4}\left(\frac{1}{6}\right) - \frac{5}{3} \cdot \frac{8}{8}$

$y = \frac{1}{24} - \frac{40}{24}$

$y = -\frac{39}{24}$

$y = -\frac{13}{8}$

$\left(\frac{1}{6}, -\frac{13}{8}\right)$

Example 6:
Solve:

① $\frac{1}{2}x - \frac{4}{5}y = -2$ LCD: 10

② $\frac{1}{4}x - y = \frac{3}{8}$

$10 \cdot ① \quad 10 \cdot \frac{1}{2}x - 10 \cdot \frac{4}{5}y = 10(-2)$

$5x - 8y = -20$

Solve ② for y:

$\frac{1}{4}x - \frac{3}{8} = y$

Sub ② into ①:

$5x - 8\left(\frac{1}{4}x - \frac{3}{8}\right) = -20$

$5x - 2x + 3 = -20$

$3x = -20 - 3$

$3x = -23$

$\frac{3x}{3} = \frac{-23}{3}$

$x = -23/3$

Sub x into ②:

$y = \frac{1}{4}\left(\frac{-23}{3}\right) - \frac{3}{8}$

$y = \frac{-23}{12} - \frac{3}{8}$

$y = \frac{-23 \cdot 2}{12 \cdot 2} - \frac{3 \cdot 3}{8 \cdot 3}$

$y = \frac{-46}{24} - \frac{9}{24}$

$y = \frac{-5}{24}$

$\left(\frac{-23}{3}, \frac{-5}{24}\right)$

Textbook Questions: page 425 #4, 5, 6, 8, 19