Math 1201

7.6A Solve a System of Linear Equations Algebraically

Special Cases

Solving a System of Equations that Involves Fractions

Steps:
For the equation(s) that involves a fraction, we will rewrite the equation **without** the fraction. To do this:

- Multiply each term in the equation by the denominator of the fraction. This will give us an equivalent form of the equation. The new equation will represent the same line as the original equation that is written without fractions.
- Solve the system of equations using either substitution or elimination.

Example 1:
Solve using elimination or substitution:

$$3x + \frac{1}{2}y = 12$$
$$-2x + y = 8$$

$2 \cdot 3x + 2 \cdot \frac{1}{2}y = 2 \cdot 12$

① $6x + y = 24$

$-$ ② $-2x + y = 8$

$8x = 16$

$\frac{8x}{8} = \frac{16}{8}$

$x = 2$

Sub x into ②:

$-2(2) + y = 8$

$-4 + y = 8$

$y = 8 + 4$

$y = 12$

$(2, 12)$

Example 2:
Solve using elimination or substitution:

$$\frac{3}{4}x - y = -4$$
$$4x + 3y = -13$$

$4 \cdot \frac{3}{4}x - 4 \cdot y = 4(-4)$

① $3x - 4y = -16$

② $4x + 3y = -13$

$4$① $- 3$②

$12x - 16y = -64$

$-\ 12x + 9y = -39$

$-25y = -25$

$\frac{-25y}{-25} = \frac{-25}{-25}$

$y = 1$

Sub y into ②:

$4x + 3(1) = -13$

$4x + 3 = -13$

$4x = -13 - 3$

$4x = -16$

$\frac{4x}{4} = \frac{-16}{4}$

$x = -4$

$(-4, 1)$

Example 3:
Solve using elimination or substitution:

$$\frac{1}{2}x + \frac{2}{3}y = -1 \quad \text{LCM: } 6$$

$$y = \frac{1}{4}x - \frac{5}{3} \quad \text{LCM: } 12$$

$6 \cdot \frac{1}{2}x + 6 \cdot \frac{2}{3}y = 6(-1)$

① $3x + 4y = -6$

$12y = 12 \cdot \frac{1}{4}x - 12 \cdot \frac{5}{3}$

$12y = 3x - 20$

$20 = 3x - 12y$

② $3x - 12y = 20$

① ~~$3x$~~ $+4y = -6$

$-$ ② ~~$3x$~~ $-12y = 20$

$16y = -26$

$\frac{16y}{16} = \frac{-26}{16}$

$y = -\frac{13}{8}$

$\left(\frac{1}{6}, -\frac{13}{8}\right)$

Sub y into ②:

$3x - 12\left(\frac{-13}{8}\right) = 20$

$3x + \frac{39}{2} = \frac{20}{1} \cdot 2$

$3x = \frac{40}{2} - \frac{39}{2}$

$3x = \frac{1}{2}$

$\frac{1}{3} \cdot 3x = \frac{1}{2} \cdot \frac{1}{3}$

$x = \frac{1}{6}$

Word Problems

Example 4:
Jill earns $40 plus $10 per hour. Tony earns $50 plus $5 per hour. Create a system of linear equations that represents this situation and solve algebraically using either substitution or elimination.

Jill: $y = 10x + 40$

Tony: $y = 5x + 50$

"Comparison Method": a type of substitution

$y = y$

$10x + 40 = 5x + 50$

$10x - 5x = 50 - 40$

$5x = 10$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

$y = 5(2) + 50$

$y = 10 + 50$

$y = 60$

They both make $60 when they work 2 hours.

Example 5:
Mitchell solved the linear system $2x + 3y = 6$ and $x - 2y = -6$. His solution was $(2, 4)$. Verify whether Mitchell's solution is correct. Represent Mitchell's results on a graph.

① $2x + 3y = 6$
② $x - 2y = -6$

Solve ② for x:
$x = 2y - 6$

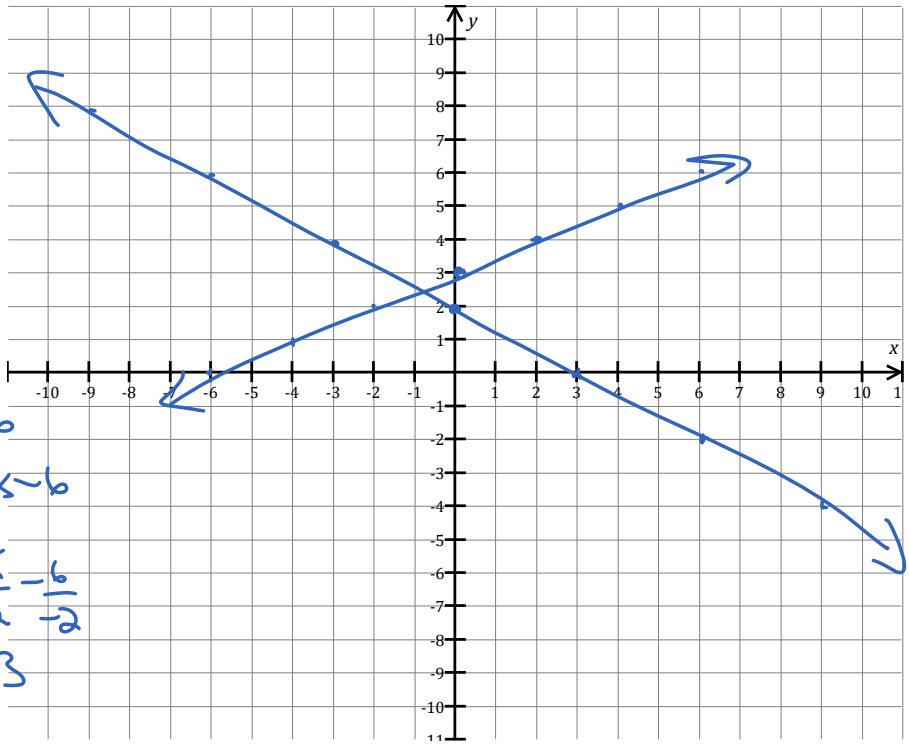
Sub ② into ①:
$2(2y - 6) + 3y = 6$
$4y - 12 + 3y = 6$
$7y = 12 + 6$
$7y = 18$
$\frac{7y}{7} = \frac{18}{7}$
$y = 18/7$

Sub y into ②:
$x = 2\left(\frac{18}{7}\right) - 6$
$x = \frac{36}{7} - \frac{6 \cdot 7}{1 \cdot 7}$
$x = \frac{36}{7} - \frac{42}{7}$
$\rightarrow x = \frac{-6}{7}$

$\left(-\frac{6}{7}, \frac{18}{7}\right)$

Graph:
$2x + 3y = 6$
$3y = -2x + 6$
$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$
$y = -\frac{2}{3}x + 2$

$x - 2y = -6$
$-2y = -x - 6$
$\frac{-2y}{-2} = \frac{-x}{-2} - \frac{6}{-2}$
$y = \frac{1}{2}x + 3$



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