I

Geometric Sequence: a sequence in which the ratio of consecutive terms is constant.

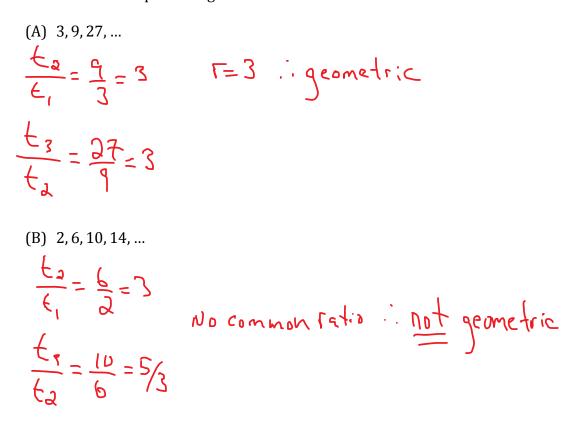
Common ratio: the ratio of successive terms in a geometric sequence. The ratio may be positive or negative. n = 4

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{t_4}{24} = \frac{t_4}{4}$$
2, 4, 8, 16, ..., the ratio is:

For example, for the following sequence, 2, 4, 8, 16, ..., the ratio is:

$$\frac{t_3}{t_2} = \frac{4}{2} = 2$$
, $\frac{t_4}{t_3} = \frac{16}{8} = 2$ $r = 2$ is geometric

Example 1: Determine if each sequence is geometric:



To develop the formula for a geometric sequence, consider a sequence such as 1, 3, 9, 27, 81,

i. What is the common ratio?

$$\frac{t_2}{t_1} = \frac{3}{1} = 3$$

$$\frac{t_2}{t_4} = \frac{3}{21} = 3$$

$$\frac{t_4}{t_4} = \frac{3}{21} = 3$$

ii. Can this sequence be rewritten to show the pattern of the first term and the common ratio

$$\frac{1}{1.5}, \frac{3}{1.5}, \frac{3}{1.5}, \frac{3}{1.5}, \frac{3}{1.5}, \frac{3}{1.5}, \frac{1}{3.5}, \frac{3}{5}, \frac{1}{5}, \frac{3}{5}, \frac{1}{5}, \frac{3}{5}, \frac{3}{5}, \frac{1}{5}, \frac{3}{5}, \frac{3}{$$

iii. Can you write the pattern in general terms for any first term and common ratio?

$$t_n = t_i \cdot r^{n-i}$$

The general term of a geometric sequence where *n* is a positive integer is

$$t_n = t_1 r^{n-1} \qquad \text{Provided}$$

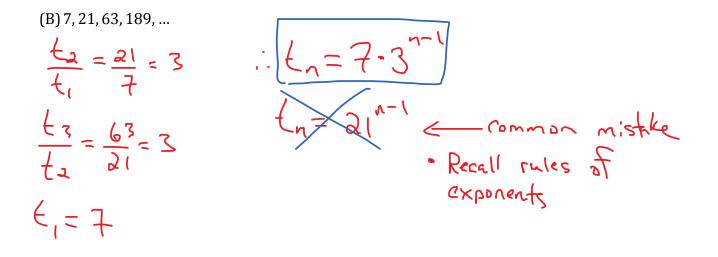
where t_1 is the first term of the sequence n is the number of terms *r* is the common ratio t_n is the general or n^{th} term

Example 3: -1 -1-{ Determine the formula for the n^{th} term:

$$L_{n} = L_{n} \cdot \Gamma$$

(A) 184, -92, 46, -23, ...

$$\begin{aligned}
\xi_{1} &= (84) \\
\Gamma &= ? \\
\frac{\xi_{2}}{\xi_{1}} &= -\frac{1}{2} \\
\frac{\xi_{3}}{\xi_{1}} &= -\frac{1}{2} \\
\frac{\xi_{3}}{\xi_{2}} &= -\frac{1}{2} \\
\frac{\xi_{3}}{\xi_{3}} &= -\frac{1}{2}
\end{aligned}$$



Example 4:

The first three terms of a geometric sequence are $\{x - 1, 2x, 3x + 9, ...\}$. Algebraically determine the value of x.

$$\frac{t_2}{t_1} = \frac{3x}{x-1} + \frac{t_3}{t_2} = \frac{3x+9}{3x}$$

$$\frac{2x}{x-1} = \frac{3x+9}{3x}$$

$$(3x)(3x) = (3x+9)(x-1)$$

$$4x^2 = 3x^2 + 6x - 9$$

$$4x^3 - 3x^3 - 6x + 9 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$x=3$$

Common Exponents

If an exponential equation has the same exponent, the bases must be equal as well. We can use this fact to help us solve equations involving geometric sequences. We can also use the ie $if n^4 = 3^4$, then n = 3properties of n^{th} roots.

Example 5:

Determine the formula for the n^{th} term: $t_3 = 5, t_6 = 135$

Algebraically:

$$t_n = t_1 \cdot r^{n-1}$$

 $5 = t_1 \cdot r^{n-1}$
 $0 = t_1 \cdot r^{n-1}$
 $1 = t_1 - t_1$

l

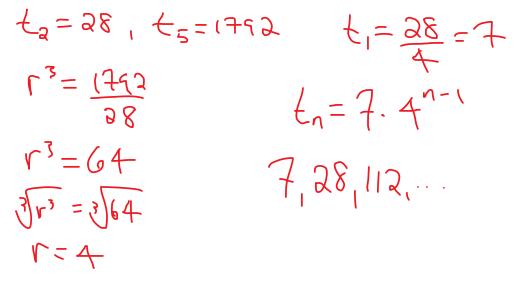
Example 6:

In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r, and list the first three terms of the sequence.

$$t_{s} = 54, t_{b} = -1458 \qquad t_{a} = 54 = -18 -3 -1458 = -27 \qquad t_{1} = -18 = 6 -3 r_{1} = -27 \qquad t_{1} = -18 = 6 -3 r_{2} = 3 -27 \qquad t_{n} = 6 (-3)^{n-1} 3 r_{1} = 3 -27 \qquad t_{n} = 6 (-18) -162 \qquad alternating r_{2} -3 \qquad f_{1} = -3 \qquad f_{2} = -18 -3 \qquad f_{1} = -18 = 6 -3 \qquad f_{2} = -18 = -18 -3 \qquad f_{1} = -18 = 6 -3 \qquad f_{2} = -18 = -18 -3 \qquad f_{1} = -18 = 6 -3 \qquad f_{2} = -18 = -18 -3 \qquad f_{2} = -18 = -18 = -18 -3 \qquad f_{1} = -18 = -18 = -18 = -18 -3 \qquad f_{2} = -18 =$$

Example 7:

In a geometric sequence, the second term is 28 and the fifth term is 1792. Determine the values of t_1 and r, and list the first three terms of the sequence.



Textbook Questions: page: 39 - 44; #1335 6,77, 10, 14, 16,23 25