Math 2200 **Date:**____________________

2.1A Angles in Standard Position

Trigonometry from Greek *trigōnon*, "triangle" and *metron*, "measure" is a branch of mathematics that studies relationships involving lengths and angles of triangles. The field emerged during the 3rd century BC from applications of geometry to astronomical studies.

Recall from Mathematics 1201:

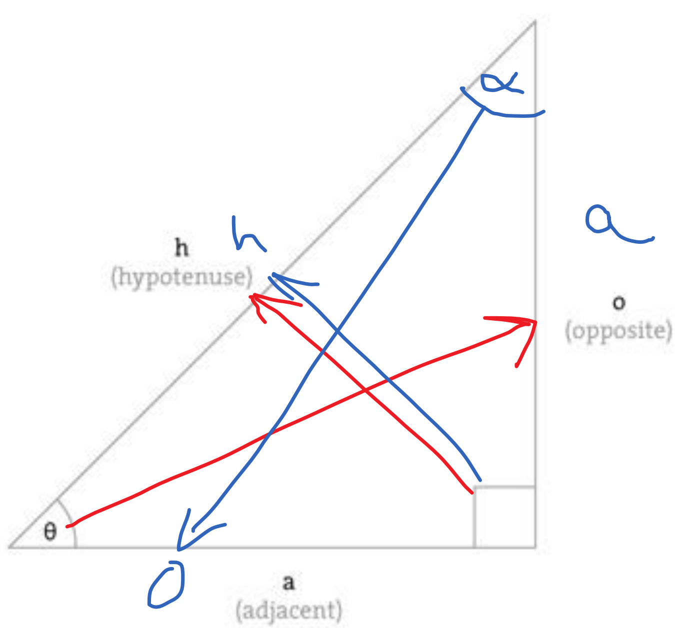
soh-cah-toa
sine cosine tangent

$$\text{sine}(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{tangent}(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cosine}(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent}(\theta) = \frac{\text{sine}(\theta)}{\text{cosine}(\theta)}$$

θ : theta

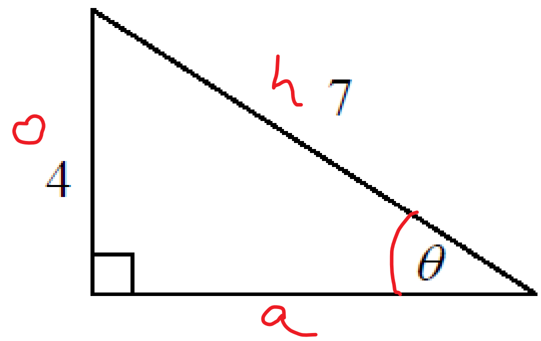
α : alpha



Remember that we use trigonometric functions to find the sides of right triangles and **inverse** trigonometric functions to find angles.

Example 1:
What is the value of θ in degrees?

$\boxed{\text{Soh}}$ ~~cah~~ ~~toa~~



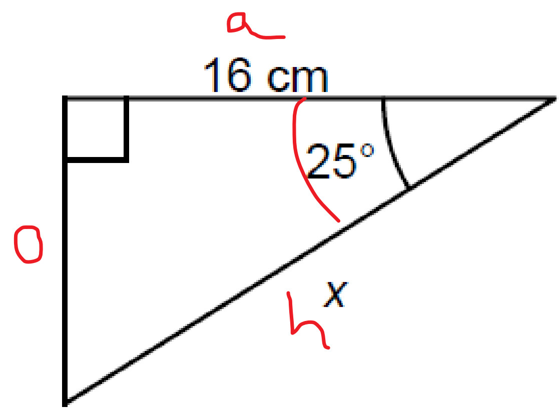
$\sin\theta = \frac{o}{h}$

$\sin\theta = \frac{4}{7}$

$\theta = \sin^{-1}\left(\frac{4}{7}\right) \rightarrow \theta = 35^\circ$

Example 2:
What is the value x?

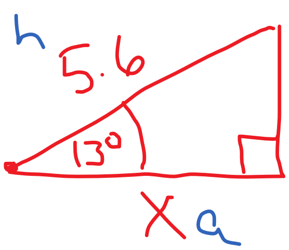
~~soh~~ $\boxed{\text{cah}}$ ~~toa~~



$\cos 25^\circ = \frac{16}{x} \rightarrow x = 18\text{cm}$

$x = \frac{16}{\cos 25^\circ}$

Example 3:
A pilot starts his takeoff and climbs steadily at an angle of 13˚. Determine the horizontal distance the plane has travelled when it has climbed 5.6 km along its flight path. Express your answer to the nearest tenth of a kilometre.



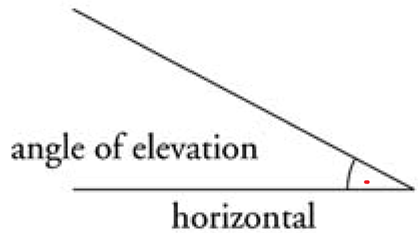
~~soh~~ $\boxed{\text{cah}}$ ~~toa~~

$5.6 \cdot \cos 13^\circ = \frac{x}{\cancel{5.6}} \cdot \cancel{5.6}$

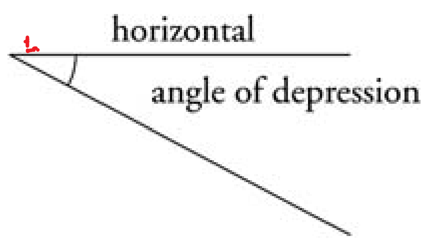
$x = 5.6 \cdot \cos 13^\circ$

$x = 5.5$ km

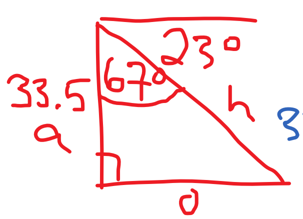
The Angle of Elevation
The angle formed by the horizontal and a line of sight above the horizontal is called the angle of elevation.



The Angle of Depression
The angle formed by the horizontal and a line of sight below the horizontal is called the angle of depression.



Example 4:
A tourist at Point Amour sees a fishing boat at an angle of depression of 23˚ and a sailboat at an angle of depression of 9˚. If the tourist is 33.5 m above the water, determine how far apart the two vessels are.

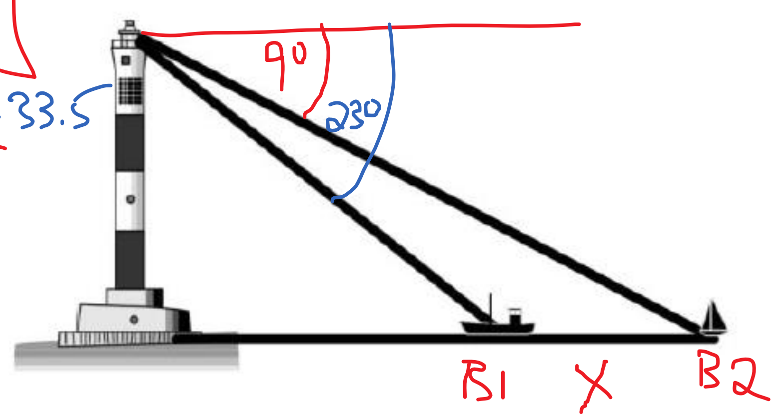


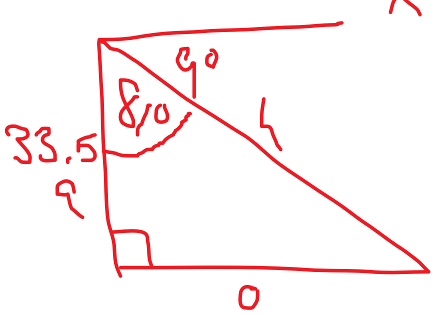
~~SOh~~ ~~CAh~~ $\boxed{\text{Toa}}$

$33.5 \cdot \tan 67° = \frac{x}{33.5} \cdot 33.5$

$x = 33.5 \tan 67°$

$x = 78.9\text{ m}$





$\tan 81° = \frac{x}{33.5}$

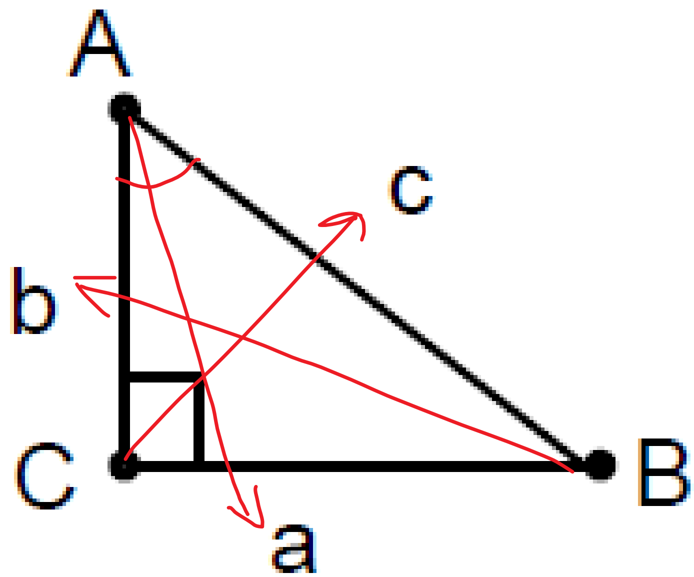
$x = 33.5 \tan 81°$

$x = 211.5\text{ m}$

$d = 211.5\text{ m} - 78.9\text{ m}$

$d = 132.6\text{ m}$

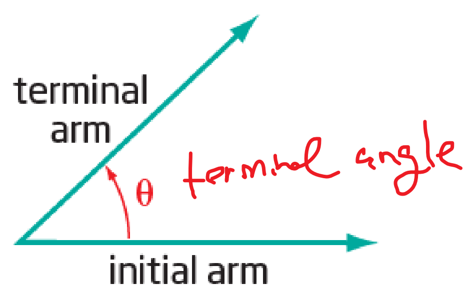
Labelling a Triangle
In a properly labelled triangle, the angles are named with capital letters and the sides opposite those angles are named with the corresponding lower case letter.



Angles in Standard Position
In geometry, and angle is formed by two rays with a common endpoint. In trigonometry, angles are often interpreted as rotations of a ray. The starting positions and the final position are called the initial arm and the terminal arm, respectively. Positive angles result in a counter-clockwise rotation. For this course, we will only deal with positive angles.

Initial arm: the arm of an angle in standard position that lies on the x-axis.

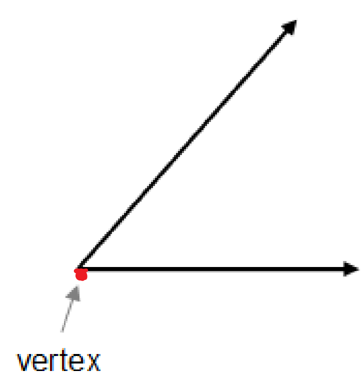
Terminal arm: the arm of an angle in standard position that meets the initial arm at the origin to form an angle. We will denote the terminal angle as θ.



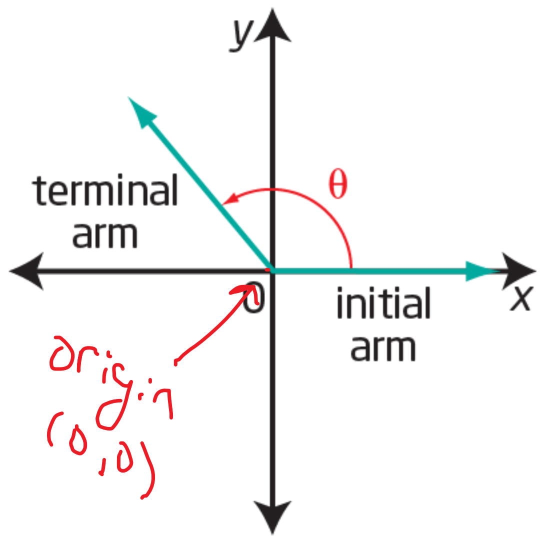
ray: $\longrightarrow$

line: $\longleftrightarrow$

Vertex: each angular point of a polygon, polyhedron, or other figure.

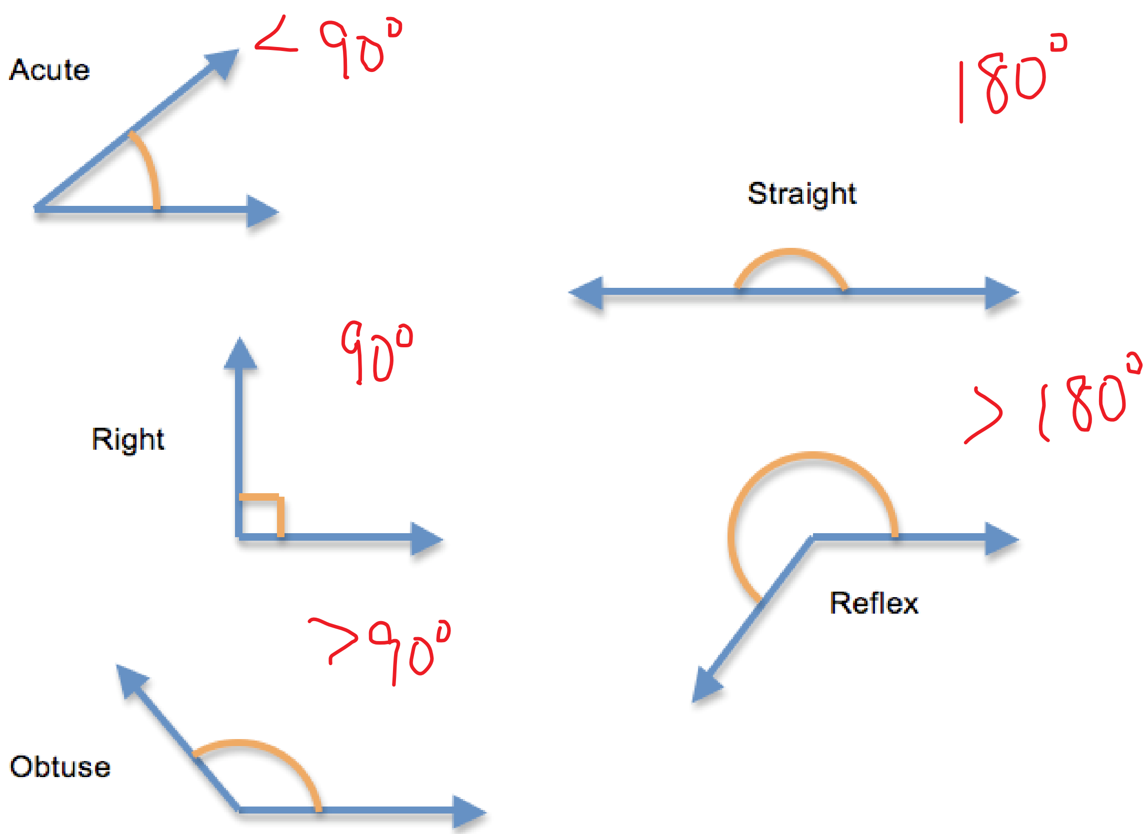


Angle in Standard Position: the position of an angle when its initial arm is on the positive x-axis and its vertex is at the origin.



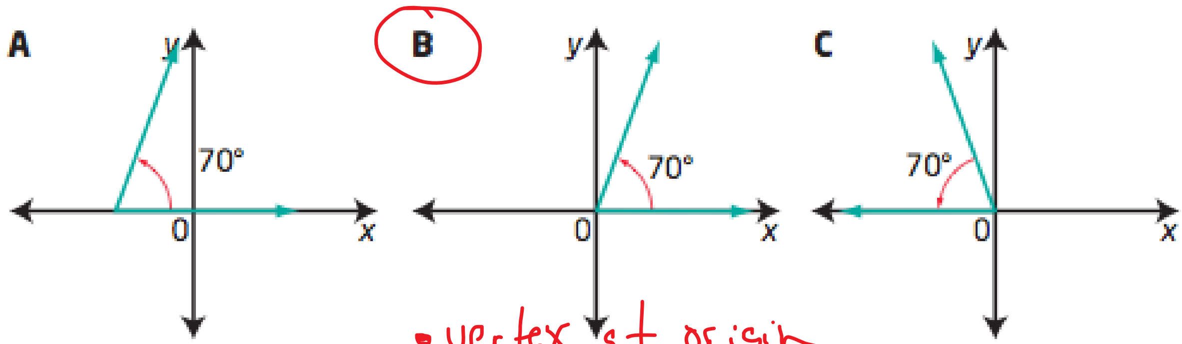
Types of Angles

By rotating the terminal arm we get the various types of angles:



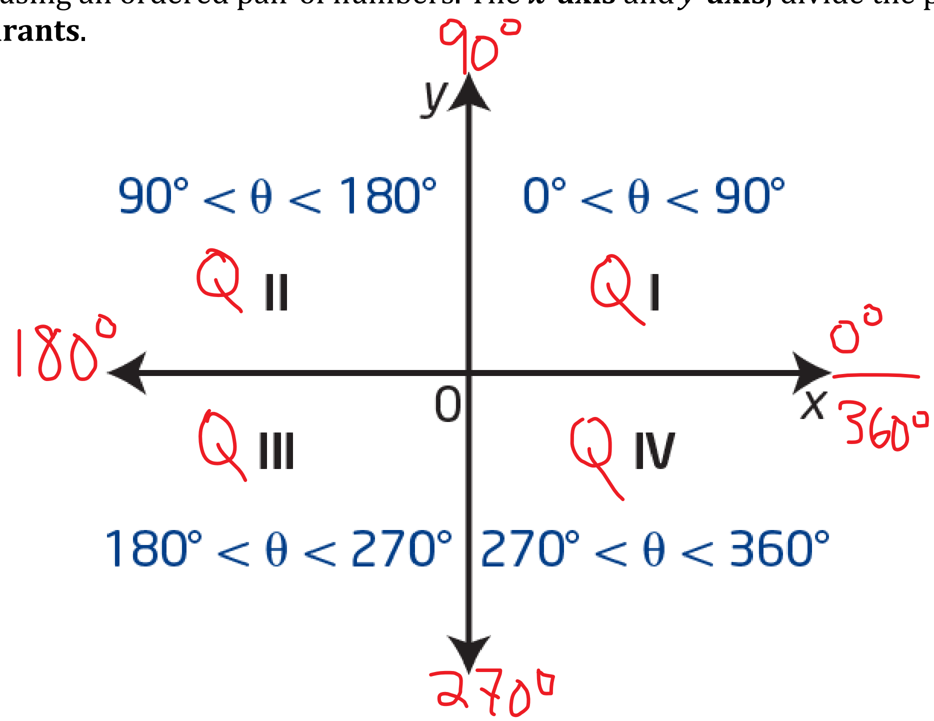
Example 5:

Which diagram shows an angle of 70° in standard position?



- vertex at origin
- initial arm extends on positive x-axis

Cartesian Plane
A Cartesian plane is defined by two perpendicular number lines: the ***x*-axis**, which is horizontal, and the ***y*-axis**, which is vertical. Using these axes, we can describe any point in the plane using an ordered pair of numbers. The ***x*-axis** and *y*-**axis**, divide the plane into four **quadrants.**



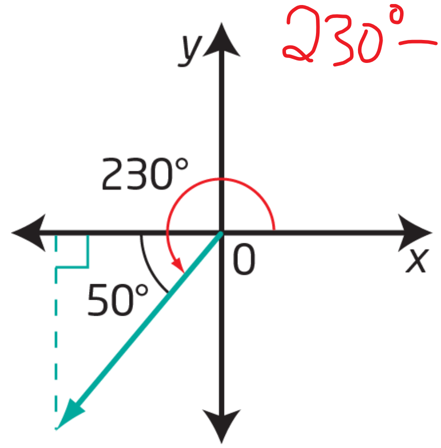
Reference Angles
The acute angle whose vertex is the origin whose arms are the terminal arm of the angle and the x-axis. The **reference angle** is denoted by $\boldsymbol{\theta_R}$.

$230^{\circ} - 180^{\circ} = 50^{\circ}$

For example, the example to the right has a terminal angle, $\theta = 230^{\mathrm{o}}$, and a reference angle of $\theta_R = 50^{\mathrm{o}}$.

So we say the reference angle for 230^{o} is 50^{o}.

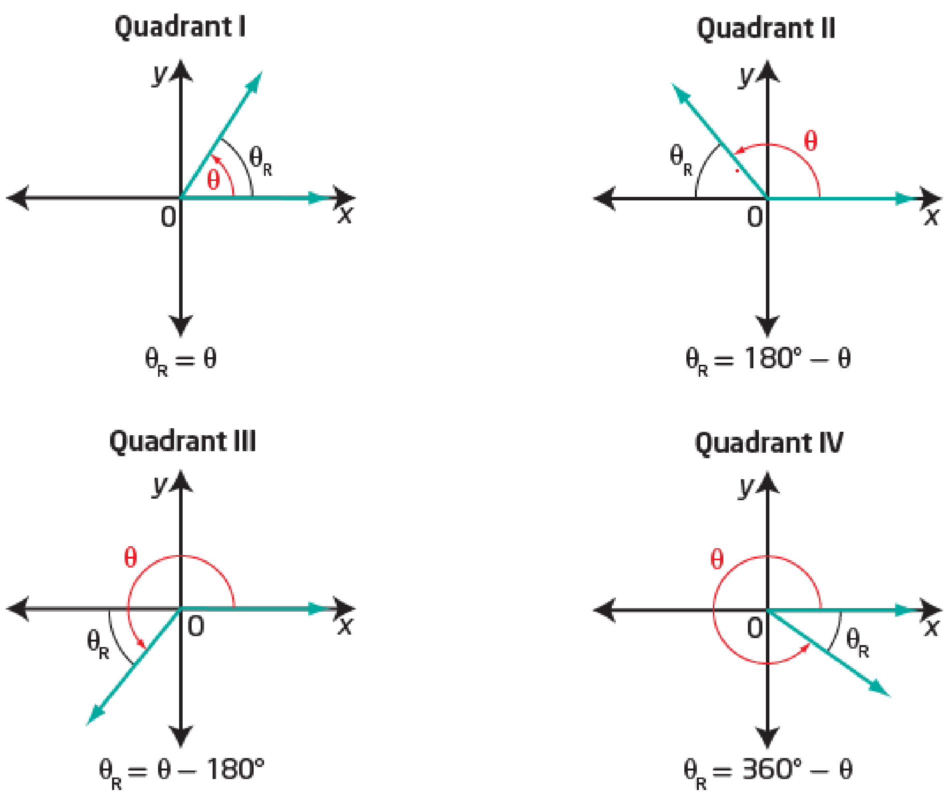
$\theta_R = 50^{\circ}$



Which quadrant is θ_R in?

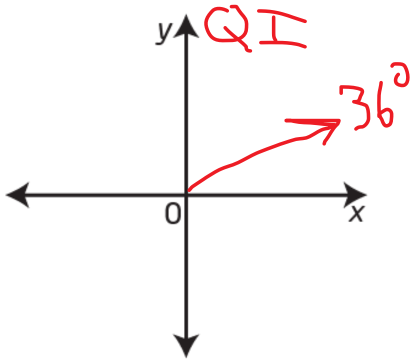
Q III

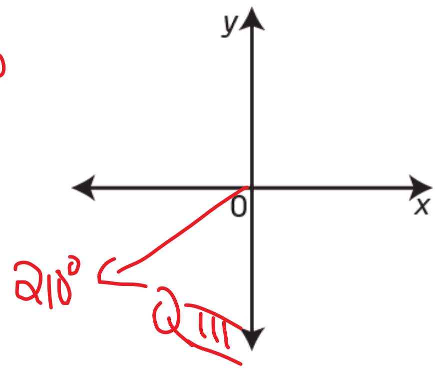
To find θ_R in any quadrant, you need to measure **how many degrees the terminal arm has rotated from the closest horizontal or x-axis line.** In the above example, the terminal arm is
$230^{\mathrm{o}} - 180^{\mathrm{o}} = 50^{\mathrm{o}}$.

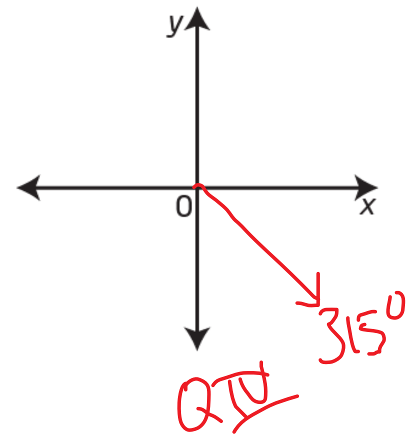


Example 6:
Sketch each angle in standard position and state the quadrant in which the terminal arm lies.

(A) 36^{o} (B) 210^{o} (C) 315^{o}

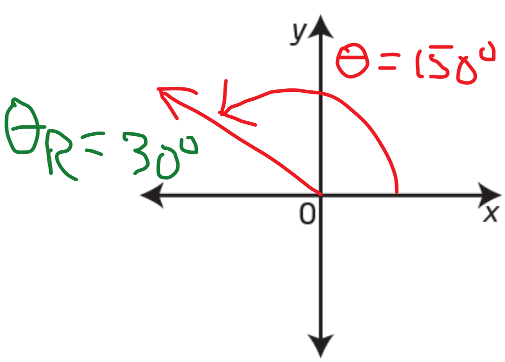


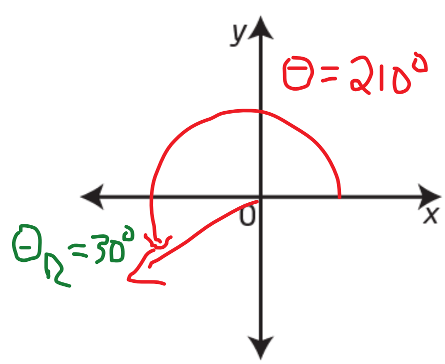


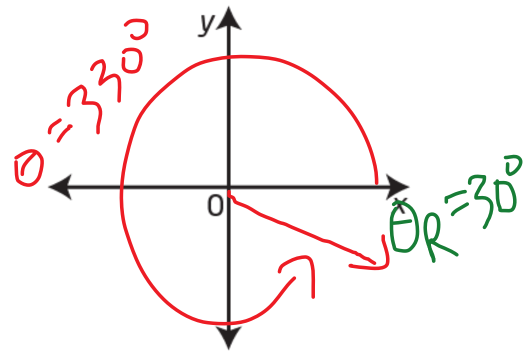


Example 7
Determine the reference angle θ_R for each angle θ. Sketch θ in standard position and label the θ_R.

(A) $\theta = 150^{o}$ (B) $\theta = 210^{o}$ (C) $\theta = 330^{o}$







What do you notice about the θ_R for each angle in standard position?

$\theta_R = 30°$ for all terminal angles.

Terminal Arms and Graphing Points

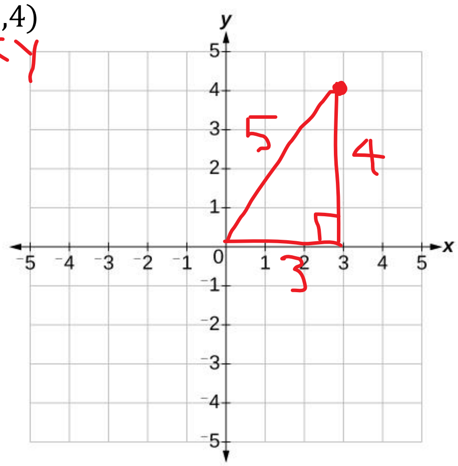
Now that you have sketched an angle in standard position and determined the quadrant in which it terminates, it is a natural extension to draw an angle given a point on its terminal arm. You will notice as the sign changes for each x and y coordinate the point will **reflect** in the x or y axis.

Example 8:

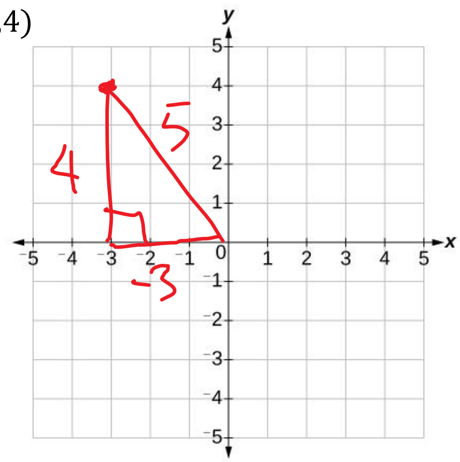
Determine which quadrant the terminal arm of the angle is located based on the point $P(x, y)$ given.

(A) $(3,4)$

xy

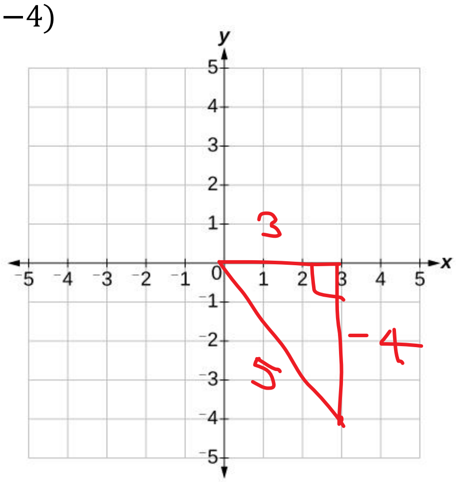


(B) $(-3,4)$



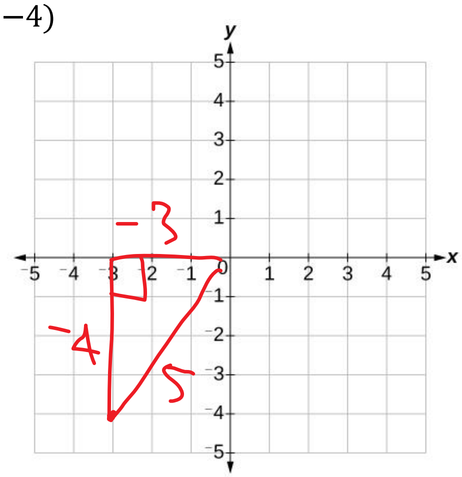
Reflection: y-axis

(C) $(3,-4)$



Reflection: x-axis

(D) $(-3,-4)$



Reflection: x & y-axis

Textbook Questions: page: 82 - 87; # 1, 2, 3, 4, 5, 6, 7, 9, 14