Math 2200 Date:______________________

2.1B Trigonometric Ratios for Special Angles

Special Right Triangles

For angles of 30°, 45° and 60°, you can determine the **exact values** of trigonometric ratios.

Exact Value

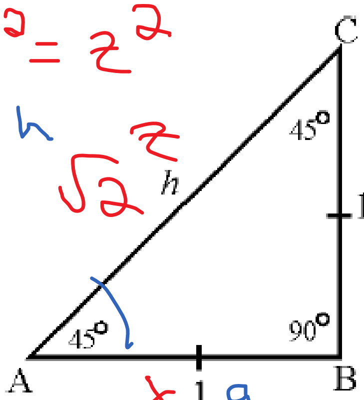
Answers involving fractions and/or radicals are exact as opposed to approximated decimal values. For example, $\frac{\sqrt{2}}{2}$ is an exact value and 0.7071067… is the approximation.

For simplicity, lets look at an right, isosceles triangle with side lengths of 1. Let's use the Pythagorean Theorem to find the hypotenuse, *c*.

$x^2 + y^2 = z^2$

h

$\sqrt{2}$ z



y o

x a

$(1)^2 + (1)^2 = h^2$

$1 + 1 = h^2$

$2 = h^2$

$\sqrt{h^2} = \sqrt{2}$

$h = \sqrt{2}$

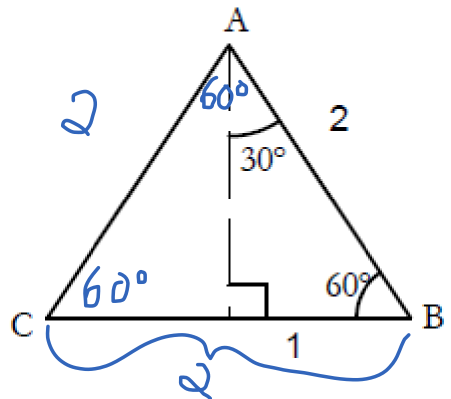
$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} =$ [illegible] $\quad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{r} \quad \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{y}{x}$

$\sin 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

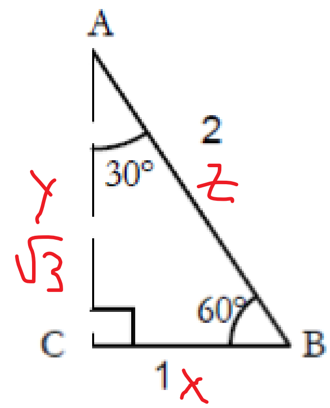
$\tan 45° = \frac{1}{1} = 1$

Now lets draw the altitude of an equilateral triangle with a side length of 2.



Let’s use the Pythagorean Theorem to find the altitude.

$$x^2 + y^2 = z^2$$
$$(1)^2 + y^2 = (2)^2$$
$$1 + y^2 = 4$$
$$y^2 = 4 - 1$$
$$y^2 = 3$$
$$\sqrt{y^2} = \sqrt{3}$$
$$y = \sqrt{3}$$

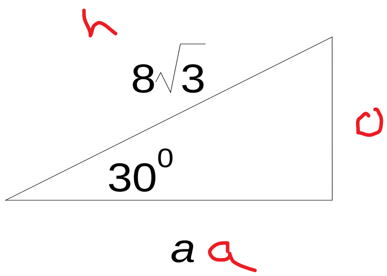


$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y}{r} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{r} \qquad \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{y}{x}$$

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Example 2:
What is the EXACT length of side a in this triangle?



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$$\cos 30^\circ = \frac{a}{8\sqrt{3}}$$

$$8\sqrt{3}\cdot\frac{\sqrt{3}}{2} = \frac{a}{\cancel{8\sqrt{3}}}\cdot\cancel{8\sqrt{3}}$$

$a = 4\cdot 3$

$a = 12$

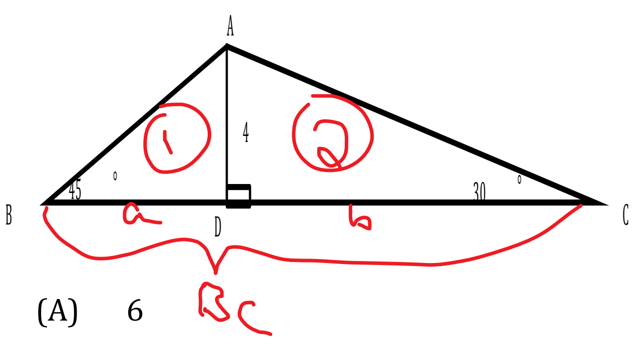
(A) $16\sqrt{3}$

(B) 12

(C) $4\sqrt{3}$

(D) $\frac{16}{\sqrt{3}}$

Example 3:
What is the exact length of BC?



① $\tan 45^\circ = \frac{4}{a}$

$a \cdot 1 = \frac{4}{\cancel{a}} \cdot \cancel{a}$

$a = 4$

② $\tan 30^\circ = \frac{4}{b}$

$\frac{1}{\sqrt{3}} = \frac{4}{b}$

$b = 4\sqrt{3}$

$BC = 4 + 4\sqrt{3}$

(A) 6

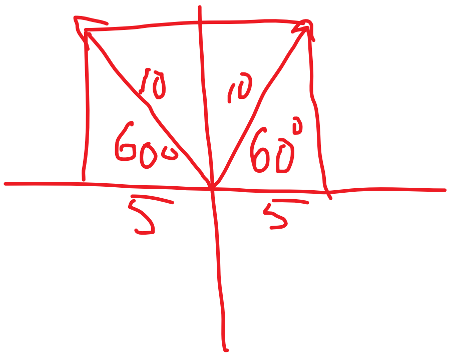
(B) 12

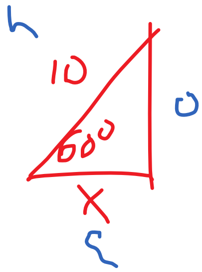
(C) $4 + 4\sqrt{3}$

(D) $4\sqrt{2} + 4\sqrt{3}$

Example 4:
A metronome is a device that helps music students keep time. Jimmy's metronome has a pendulum arm of 10 cm long. For one particular tempo, the settings result in the arm moving back and forth from a start position of 60° to 120°. What horizontal distance does the tip of the arm move in one beat? Give your answer in exact value.

$180° - 120° = 60°$





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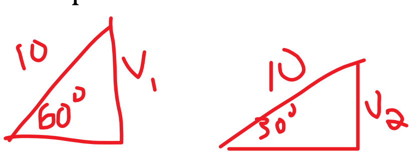
$$\cos 60° = \frac{x}{10}$$

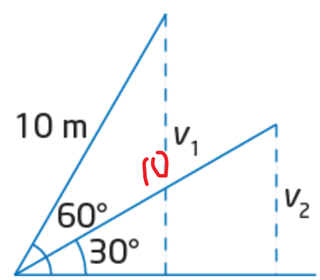
$$10 \cdot \frac{1}{2} = \frac{x}{\cancel{10}} \cdot \cancel{10}$$

$$x = 5$$

$$2x = 2(5) = 10\text{ cm}$$

Example 5:
A 10m boom lifts material onto a roof in need of repair. Determine the exact vertical displacement of the end of the boom when the operator lowers it from 60° to 30°.





$$\sin 60° = \frac{V_1}{10} \qquad \sin 30° = \frac{V_2}{10}$$

$$10 \cdot \frac{\sqrt{3}}{2} = \frac{V_1}{\cancel{10}} \cdot \cancel{10} \qquad 10 \cdot \frac{1}{2} = \frac{V_2}{\cancel{10}} \cdot \cancel{10}$$

$$V_1 = 5\sqrt{3} \qquad V_2 = 5$$

$$d = 5\sqrt{3} - 5\text{ m}$$

Textbook Questions: page: 82 - 87; #8, 11, 13, 22