Math 2200

2.2B Trigonometric Algebraic Expressions and Quadrantal Angles

Trigonometric Algebraic Expressions

There are cases where a trigonometric ration may be in the form of an algebraic expression. Some techniques from Level 1 will help in solving this type of problem.

Recall from Math 1201, when solving an algebraic expression, the idea is to isolate the variable. For example:

$$-2x - 1 = 0$$

$$\frac{-2x}{-2} = \frac{1}{-2}$$

$$x = -\frac{1}{2}$$

If we treat trigonometric functions like variables, we can solve for the function and then apply the inverse function to the trigonometric ratio to find the associated angle(s).

Example 1

Solve for θ, where $0^o \leq \theta \leq 360^o$.

(A) $2\cos\theta - 1 = 0$

$$\frac{2\cos\theta}{2} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta_R = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta_R = 60^\circ$$

$$\theta = \theta_R = \underline{60^\circ}$$

$$\theta = 360^\circ - 60^\circ = \underline{300^\circ}$$

(B) $2\sin\theta + \sqrt{3} = 0$

$$\frac{2\sin\theta}{2} = \frac{-\sqrt{3}}{2}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta_R = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\theta = 180^\circ + 60^\circ = 240^\circ$$

$$\theta = 360^\circ - 60^\circ = 300^\circ$$

(C) $-2\cos\theta - 1 = 0$

$\frac{-2\cos\theta}{-2} = \frac{1}{-2}$

$\cos\theta = -\frac{1}{2}$

$\theta_R = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

$\theta = 180^\circ - 60^\circ = 120^\circ$

$\theta = 180^\circ + 60^\circ = 240^\circ$

(D) $\sqrt{2}\cos\theta + 2 = 1$

$\sqrt{2}\cos\theta = 1 - 2$

$\frac{\cancel{\sqrt{2}}\cos\theta}{\cancel{\sqrt{2}}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

$\theta_R = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

$\theta = 180^\circ - 45^\circ = 135^\circ$

$\theta = 180^\circ + 45^\circ = 225^\circ$

* (E) $3\cos\theta - 2\sqrt{3} = 4\sqrt{3}$

$3\cos\theta = 4\sqrt{3} + 2\sqrt{3}$

$\frac{3\cos\theta}{3} = \frac{6\sqrt{3}}{3}$

$\cos\theta = 2\sqrt{3}$

$\cancel{\theta_R = \cos^{-1}(2\sqrt{3})}$

θ = undefined.

Example 2

Solve for θ, where $0^o \leq \theta \leq 360^o$.

(A) $-5\cos\theta - 3 = 0$

$-5\cos\theta = 3$

$\frac{-5\cos\theta}{-5} = \frac{3}{-5}$

$\cos\theta = -\frac{3}{5}$

$\theta_R = \cos^{-1}\left(\frac{3}{5}\right)$

$\theta_R = 53^o$

$-+ \mid ++$

$-- \mid +-$

$\theta = 180^o - 53^o = 127^o$

$\theta = 180^o + 53^o = 233^o$

(B) $11\sin\theta + 12 = 0$

$11\sin\theta = -12$

$\frac{11\sin\theta}{11} = \frac{-12}{11}$

$\sin\theta = -\frac{12}{11}$

$\theta = \sin^{-1}\left(-\frac{12}{11}\right)$

undefined

(C) $-3\cos\theta - 1 = 0$

$-3\cos\theta = 1$

$\frac{-3\cos\theta}{-3} = \frac{1}{-3}$

$\cos\theta = -\frac{1}{3}$

$\theta_R = \cos^{-1}\left(\frac{1}{3}\right) = 71^o$

$-+ \mid ++$

$-- \mid +-$

$\theta = 180^o - 71^o = 109^o$

$\theta = 180^o + 71^o = 251^o$

(D) $\quad 7\cos\theta + 2 = -1$

$7\cos\theta = -1 - 2$

$\frac{7\cos\theta}{7} = \frac{-3}{7}$

$\cos\theta = -\frac{3}{7}$

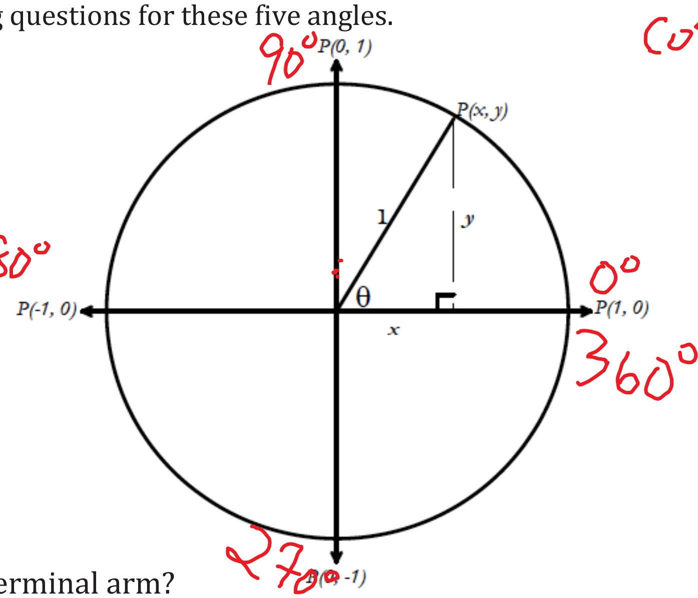
$\theta_R = \cos^{-1}\left(\frac{3}{7}\right) = 65^\circ$

$\theta = 180^\circ - 65^\circ = 115^\circ$

$\theta = 180^\circ + 65^\circ = 245^\circ$

Quadrantal Angles

Trigonometric ratios for angles whose measurements are 0°, 90°, 180°, 270° or 360° are also special cases and will now be explored. These five angles are called the **Quadrantal Angles**. Answer the following questions for these five angles.



$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\cos\theta = x$

$\sin\theta = \frac{y}{r} = \frac{y}{1}$

$\sin\theta = y$

Where is the terminal arm?

Either on the x-axis or y-axis.

Does this angle have a corresponding reference angle?

No. Same distance to both axis.

Can a reference triangle be drawn? Explain.

No. Can't have a triangle with 1 arm.

Use a calculator to verify the values of sine, cosine and tangent at all five quadrantal angles.

Angle	cosθ	sinθ	tanθ
0°	1	0	0
90°	0	1	undefined
180°	−1	0	0
270°	0	−1	undefined
360°	1	0	0