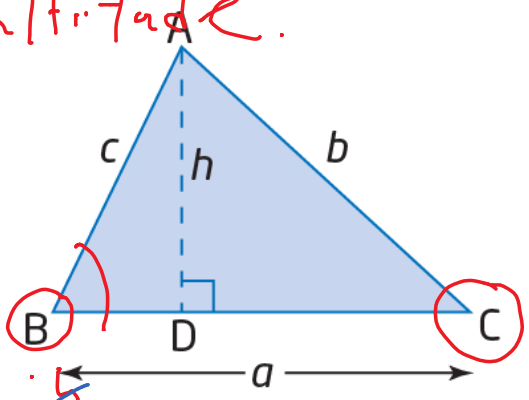
Math 2200

2.3A The Sine Law

The Sine Law is a relationship between the sides and angles in any triangle. Let's take a look how the formula is derived:

Let h be the altitude.

$\therefore AD \perp BC$



$\triangle ABD$ $\triangle ACD$

$c \cdot \sin B = h$ $b \sin C = h$

$h = c \cdot \sin B$ $h = b \sin C$

$h = h$

$\frac{1}{c} \cdot c \sin B = b \sin C \cdot \frac{1}{c}$

$\frac{1}{b} \sin B = \frac{b \sin C}{c} \cdot \frac{1}{b}$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

Same meth will show how a relates to SinA.

The Sine Law

Let ΔABC be any triangle, where a, b and c represent the measures of the sides opposite ∠A, ∠B and ∠C, respectively. Then:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$ ← finding sides.

or

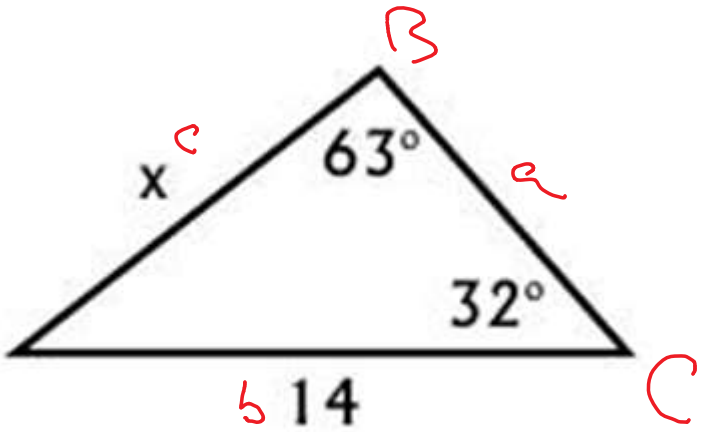
$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$ ← finding angles.

- you need a complete ratio for the sine law to work.
- if find a side, you need the opposite angle and vice versa.

Example 1

What is the measure of side x?

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$



$$\frac{x}{\sin 32^\circ} = \frac{14}{\sin 63^\circ} \cdot \sin 32^\circ$$

$$x = \frac{14 \sin 32^\circ}{\sin 63^\circ}$$

$$x = 8.3 \text{ units}$$

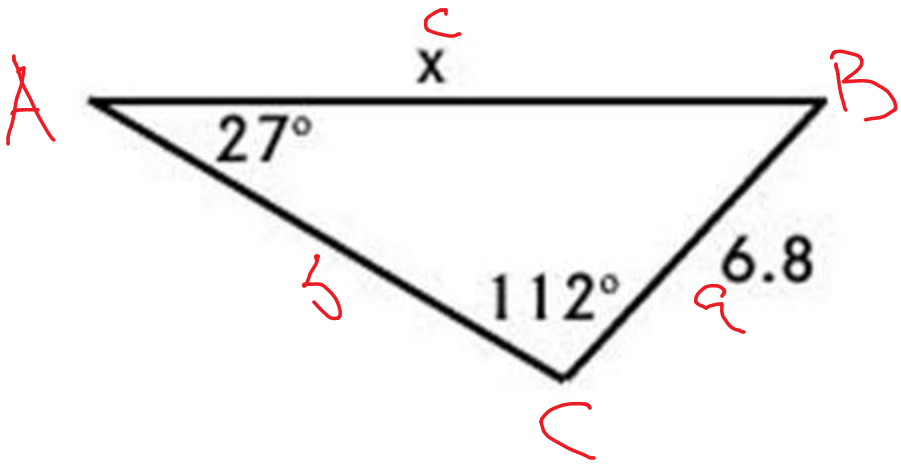
Example 2
What is the measure of side x?

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{x}{\sin 112^\circ} = \frac{6.8}{\sin 27^\circ}$$

$$x = \frac{6.8 \sin 112^\circ}{\sin 27^\circ}$$

$$x = 13.9 \text{ units}$$



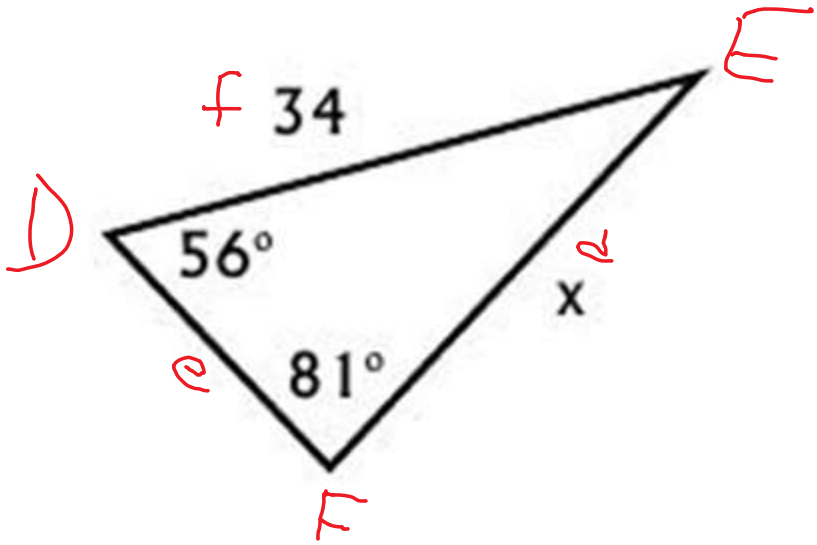
Example 3
What is the measure of side x?

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{x}{\sin 56^\circ} = \frac{34}{\sin 81^\circ}$$

$$x = \frac{34 \sin 56^\circ}{\sin 1^\circ}$$

$$x = 29 \text{ units}$$



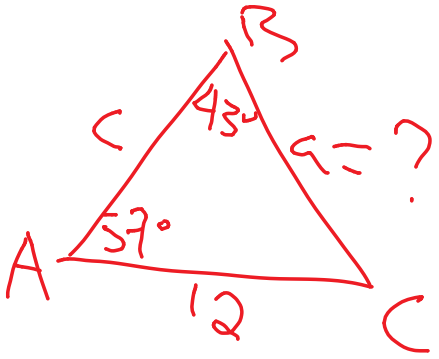
Example 4

Consider ΔABC. Determine the length of side a, given $b = 12$, $\angle A = 57^\circ$ and $\angle B = 43^\circ$.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 57} = \frac{12}{\sin 43^\circ}$$

$$a = \frac{12 \sin 57^\circ}{\sin 43^\circ} = 15 \text{ units}$$



Example 5

Determine the measure of θ.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

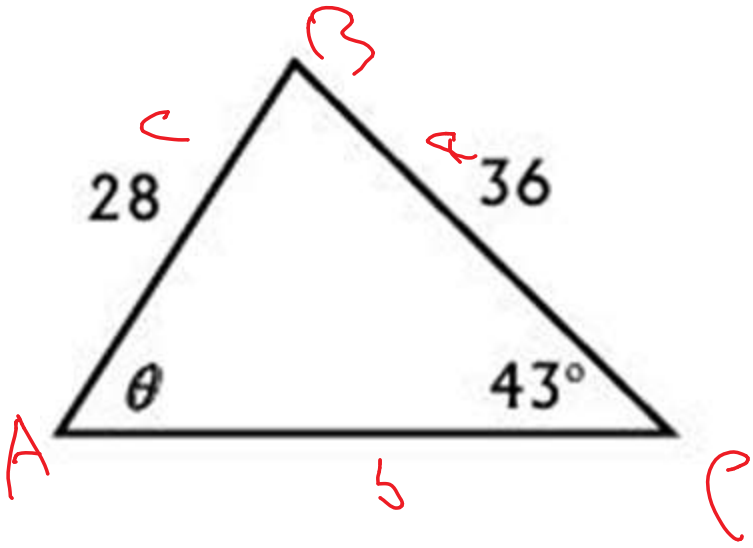
$$\frac{36 \sin\theta}{36} = \frac{\sin 43^\circ \cdot 36}{28}$$

$$\sin\theta = \frac{\sin(43^\circ) \cdot 36}{28}$$

$$\sin\theta = 0.8769$$

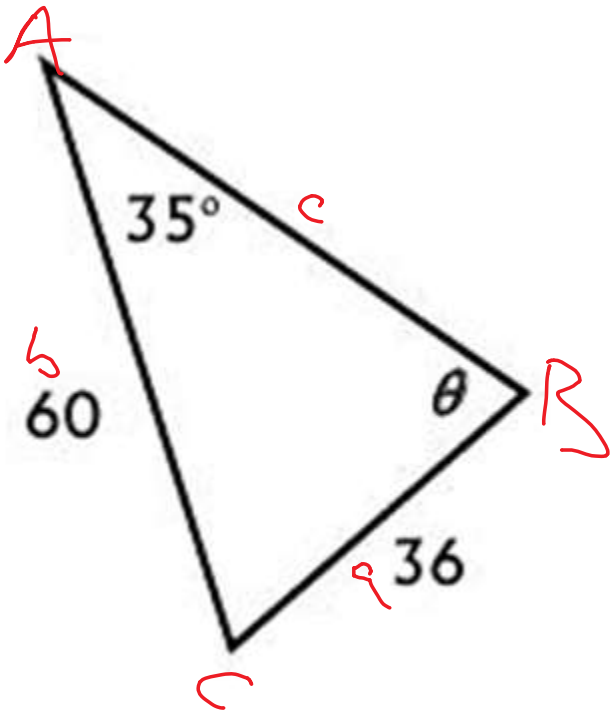
$$\theta = \sin^{-1}(0.8769)$$

$$= 61^\circ$$



Example 6

Determine the measure of θ.



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

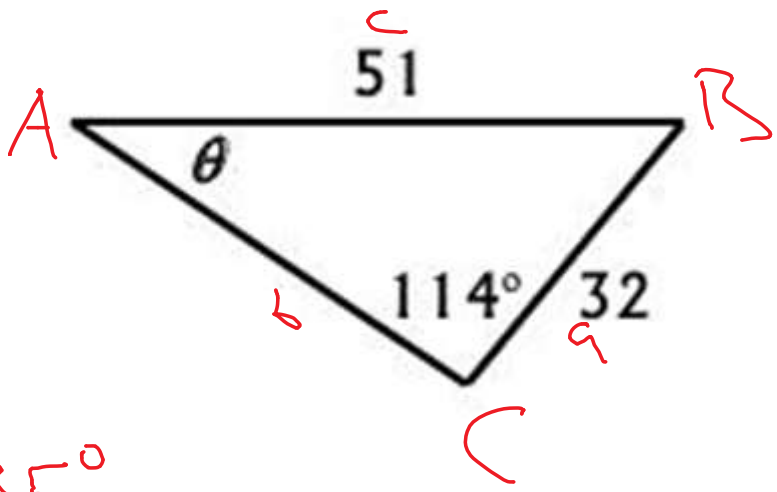
$$\frac{\sin\theta}{60} = \frac{\sin 35^\circ}{36}$$

$$\sin\theta = \frac{60\sin 35^\circ}{36}$$

$$\theta = \sin^{-1}(0.9560) = 73^\circ$$

Example 7

Determine the measure of θ.



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin\theta}{32} = \frac{\sin 114^\circ}{51}$$

$$\sin\theta = \frac{32\sin 114^\circ}{51}$$

$$\theta = \sin^{-1}(0.5732) = 35^\circ$$

Example 8

Consider ΔDEF. Determine the measure of ∠D given ∠E = 43°, d = 65 cm and e = 52 cm.

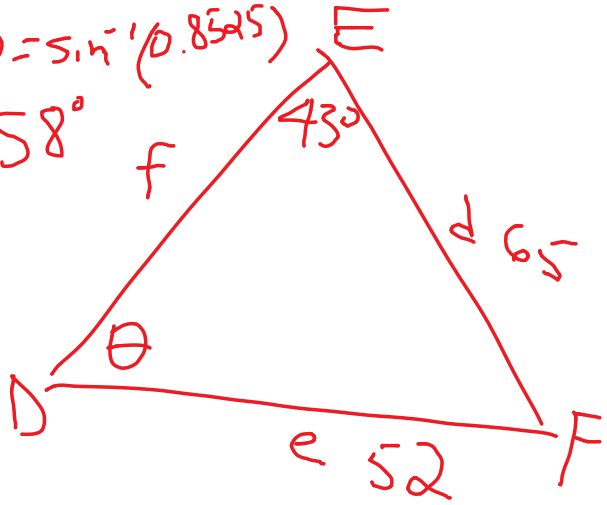
$\frac{\sin D}{d} = \frac{\sin E}{e}$

$\frac{\sin\theta}{65} = \frac{\sin 43°}{52}$

$\sin\theta = \frac{65\sin 43°}{52}$

$\theta = \sin^{-1}(0.8525)$

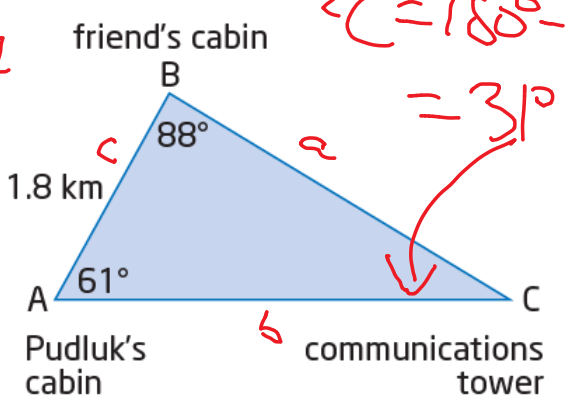
$\theta = 58°$



Example 9

Pudluk and his family own a cabin up at Bonne Bay pond. He and his friend wish to determine the distance from Pudluk's cabin to a communications tower they regularly ride to on snowmobile. Pudluk and his friend know the distance between their cabins is 1.8 km. Using a transit, they estimate the measure of the angles between their cabins and the communication tower as shown in the diagram below. Determine the distance from Pudluk's cabin to the tower to the nearest tenth of a kilometer.

* Sometimes you may have to find the angle that completes the ratio.



$\angle C = 180° - (61° + 88°)$

$= 31°$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 88°} = \frac{1.8}{\sin 31°}$

$b = \frac{1.8\sin 88°}{\sin 31°}$

$b = 3.5$ km

Pudluk's cabin is 3.5 km from the tower.

Textbook Questions: page: 108 - 112; # 1, 2, 3, 4, 5, 10, 12, 13, 24