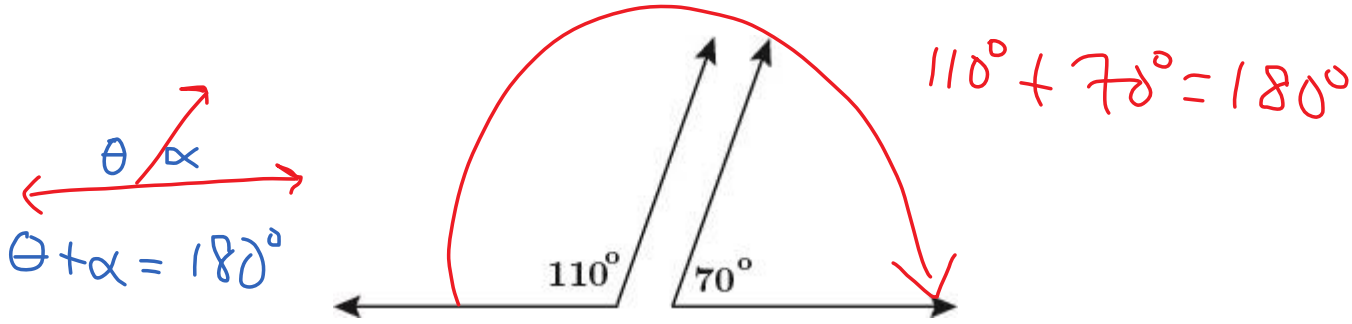
Math 2200

2.3B The Ambiguous Case

Supplementary angles

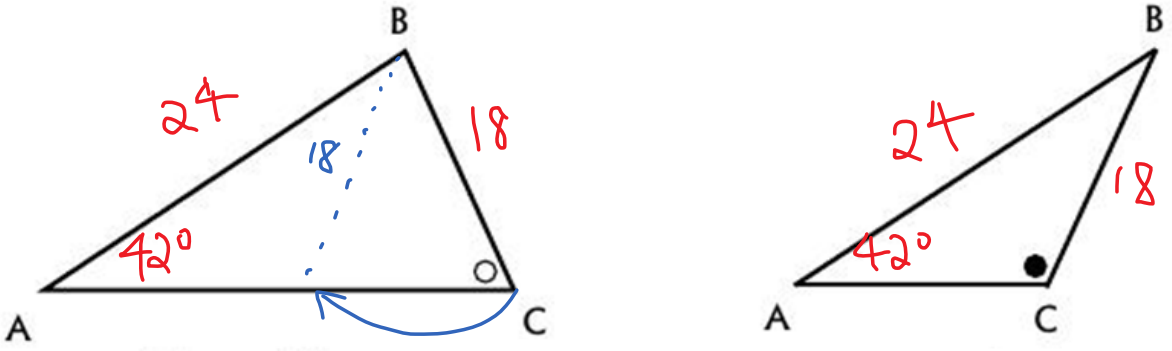
Two Angles are Supplementary if they add up to 180 degrees. For example:



The Ambiguous Case

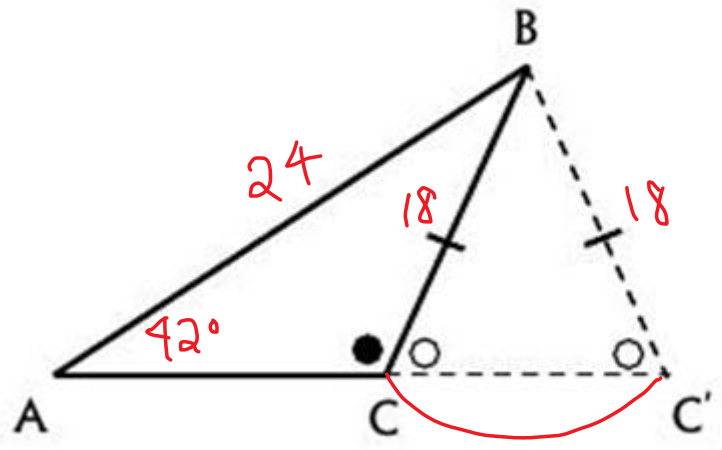
When solving a triangle, you must analyze the given information to determine how many solutions exist. If you are given two sides and an angle opposite one of those sides (SSA), the ambiguous case may occur. What this means is given two sides and an angle, I may be able to draw two distinct triangles with the same dimensions.

To demonstrate, use the following information to complete both diagrams: $\angle A = 42^{o}$, $c = 24$ and $a = 18$.



As you can see, it is possible to construct two different triangles from the given information and as a result, there are two possible values for $\angle C$ and b.

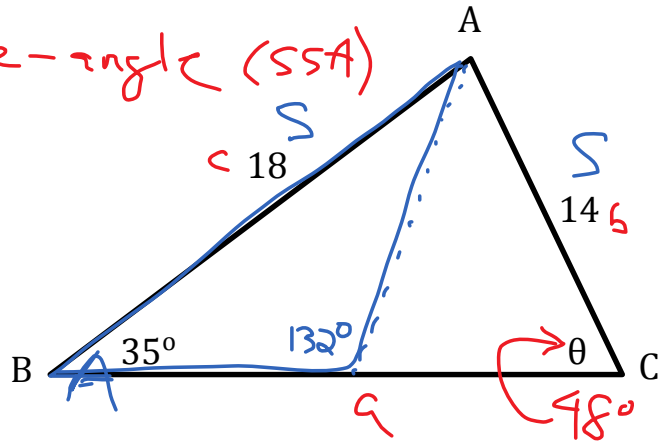
So the question is, are the two possible triangles somehow related? Can we use this relation to determine the missing angles and side?



If we combine the two possibilities, we see we get isosceles triangle $\Delta CBC'$. This shows that the two possible angles for each possible triangle are supplementary. Therefore, once we find one angle, we can find the other by subtracting it from 180^{o}.

Example 1

Find the possible values of θ.



Look for side-side-angle (SSA)

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin\theta}{18} = \frac{\sin 35^\circ}{14}$

$\sin\theta = \frac{18\sin 35}{14}$

$\sin\theta = 0.7375$

$\theta = \sin^{-1}(0.7375)$

$\theta = 48^\circ$

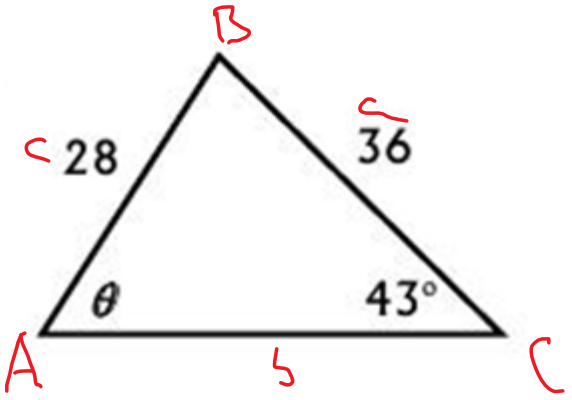
or $\theta = 180^\circ - 48^\circ = 132^\circ$

Check:

$132^\circ + 35^\circ = 167^\circ < 180^\circ$

Example 2

Find the possible values of θ.



$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin\theta}{36} = \frac{\sin 43^\circ}{28}$

$\sin\theta = \frac{36\sin 43^\circ}{28}$

$\sin\theta = 0.8769$

$\theta = \sin^{-1}(0.8769)$

$\theta = 61^\circ$

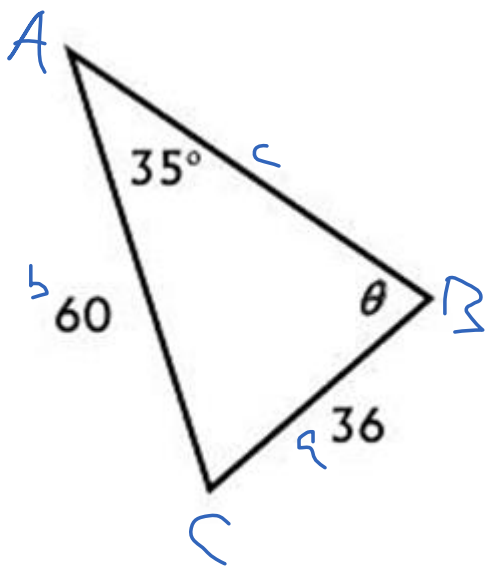
or $\theta = 180^\circ - 61^\circ = 119^\circ$ ✓

Check:

$119^\circ + 43^\circ = 162^\circ < 180^\circ$

Example 3

Find the possible values of θ.



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin\theta}{60} = \frac{\sin 35^\circ}{36}$$

$$\sin\theta = \frac{60\sin 35^\circ}{36}$$

$$\sin\theta = 0.9560$$

$$\theta = \sin^{-1}(0.9560)$$

$$\theta = 73^\circ$$

or $\theta = 180^\circ - 73^\circ = 107^\circ$ ✓

Check:

$107^\circ + 35^\circ = 142^\circ < 180^\circ$

Example 4

Find the possible values of θ.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

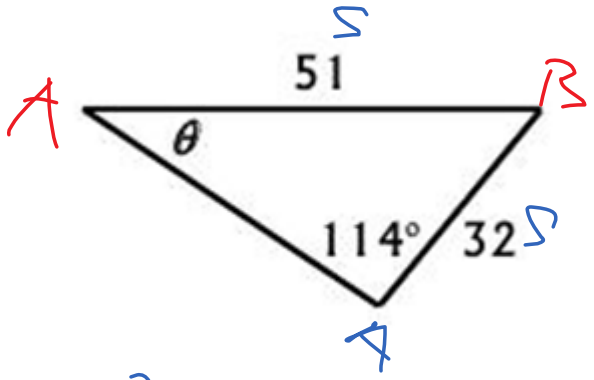
$\frac{\sin\theta}{32} = \frac{\sin 114^\circ}{51}$

$\sin\theta = \frac{32\sin 114^\circ}{51}$

$\sin\theta = 0.5732$

$\theta = \sin^{-1}(0.5732)$

$\theta = 35^\circ$



or $\theta = 180^\circ - 35^\circ$

$= 145^\circ$

Check:

$145^\circ + 114^\circ = 259^\circ > 180^\circ$

∴ no ambiguous case

Textbook Questions: page: 109 - 110; # 8, 11, 17