Math 2200

2.4 The Cosine Law

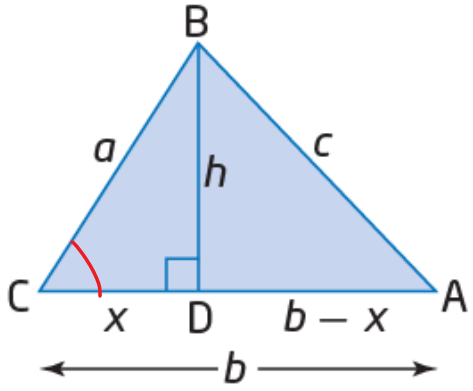
The cosine law describes the relationship between the cosine of an angle and the lengths of the three sides of any angle. Lets take a look at how this formula was developed:

ΔBCD

$a \cdot \cos C = \frac{x}{a}$

$x = a\cos C$

$a^2 = x^2 + h^2$



ΔABD

$c^2 = h^2 + (b-x)^2$

$c^2 = h^2 + b^2 - 2bx + x^2$

$c^2 = h^2 + x^2 + b^2 - 2bx$

$c^2 = a^2 + b^2 - 2ba\cos C$

$= (b-x)(b-x)$

$= b(b-x) - x(b-x)$

$= b^2 - bx - xb + x^2$

$= b^2 - bx - bx + x^2$

$= b^2 - 2bx + x^2$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

The Cosine Law

For any triangle, ΔABC, where a, b, and c are the lengths of the sides opposite ∠A, ∠B and ∠C, respectively. Then

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$ ← provided

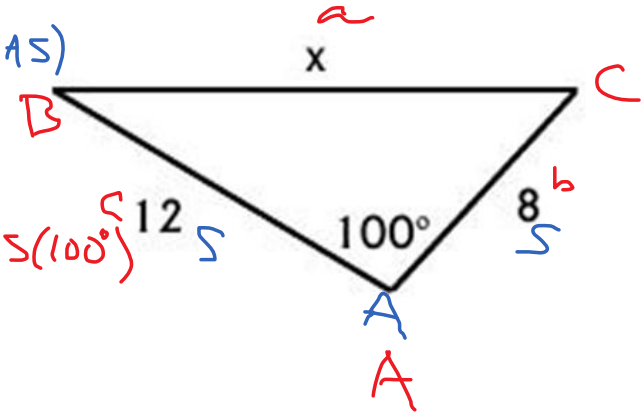
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Example 1

Determine the measure of x.

- Side-angle-side (SAS)



$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = (8)^2 + (12)^2 - 2(8)(12)\cos(100°)$

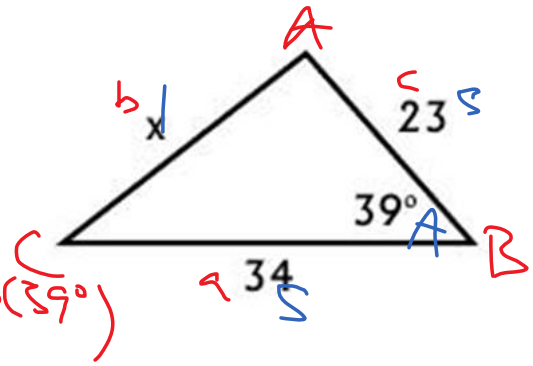
$x^2 = 241.3405$

$\sqrt{x^2} = \sqrt{241.3405}$

$x = 16$ units

Example 2

Determine the measure of x.



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$x^2 = (34)^2 + (23)^2 - 2(34)(23)\cos(39°)$

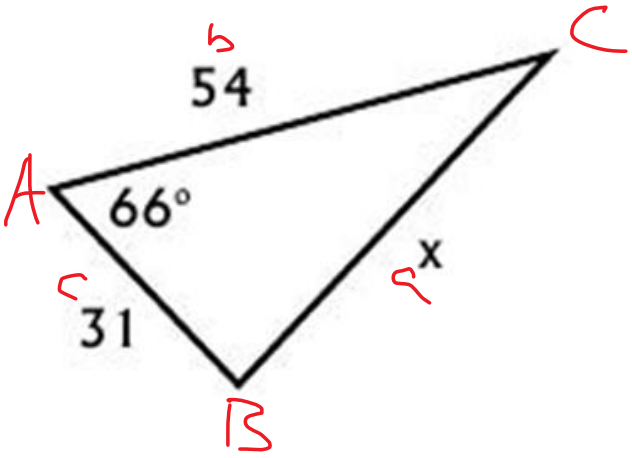
$x^2 = 469.5437$

$\sqrt{x^2} = \sqrt{469.5437}$

$x = 22 \text{ units}$

Example 3

Determine the measure of x.



$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = (54)^2 + (31)^2 - 2(54)(31)\cos 66°$

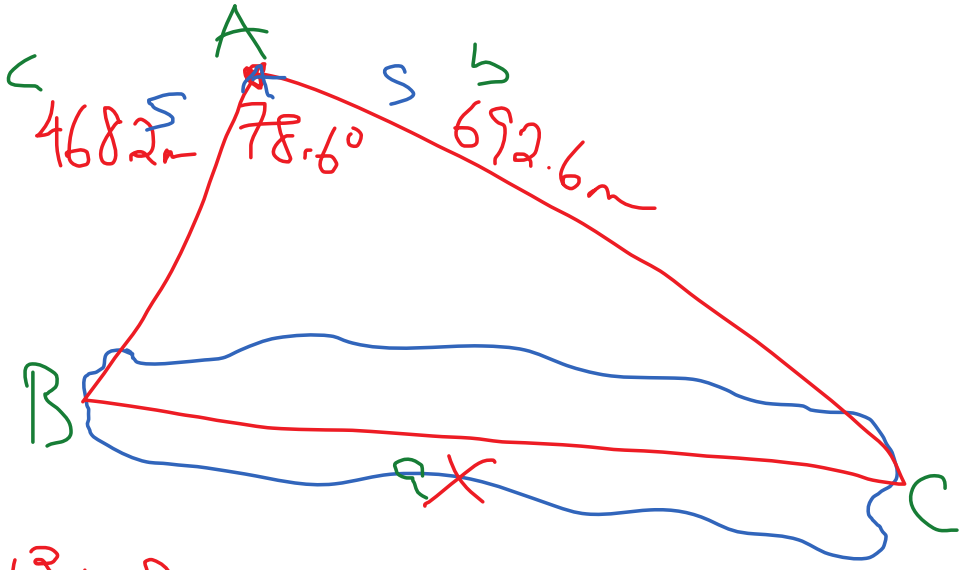
$x^2 = 2515.2457$

$\sqrt{x^2} = \sqrt{2515.2457}$

$x = 50 \text{ units}$

Example 4

A surveyor needs to find the length of a marshy area in Gros Morne National Park. The surveyor sets up her transit at a point A. She measures the distance to one end of the the marsh as 468.2 m, the distance to the opposite end of the swamp as 692.6 m, and the angle of sight between the two as 78.6°. Determine the length of the marshy area, to the nearest tenth of a metre.



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (692.6)^2 + (468.2)^2 - 2(692.6)(468.2)\cos(78.6^\circ)$

$a^2 = 570715.2054$

$\sqrt{a^2} = \sqrt{570715.2054}$

$a = 755.5\text{ m}$

The marsh is 755.5 m long.

We can also use the Cosine Law to solve for missing angles. We can use the formula as is, but it's better to rearrange the formula and isolate the angle.

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

Isolate $\cos A$

$$b^2 + c^2 - 2bc \cdot \cos A = a^2$$

$$+2bc \cdot \cos A = a^2 - b^2 - c^2$$

$$-2bc$$

$$\cos A = \frac{(a^2 - b^2 - c^2)}{-2bc}(-1)(-1)$$

$$\cos A = \frac{-1 \cdot (a^2 - b^2 - c^2)}{2bc}$$

$$\cos A = \frac{-a^2 + b^2 + c^2}{2bc}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or

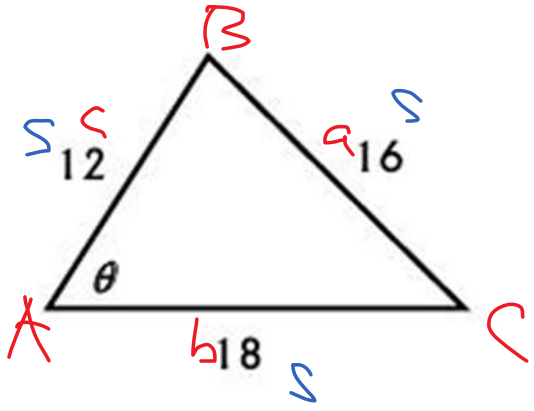
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 5

Determine the value of θ to the nearest degree.



Side - side - side

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{(18)^2 + (12)^2 - (16)^2}{2(18)(12)}$$

$$\cos\theta = \frac{212}{432}$$

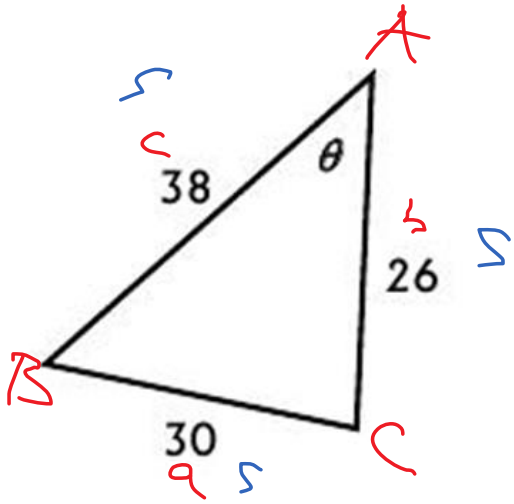
$$\cos\theta = 0.4907$$

$$\theta = \cos^{-1}(0.4907)$$

$$\theta = 61^\circ$$

Example 6

Determine the value of θ to the nearest degree.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{(26)^2 + (38)^2 - (30)^2}{2(26)(38)}$$

$$\cos\theta = \frac{1220}{1976}$$

$$\cos\theta = 0.6174$$

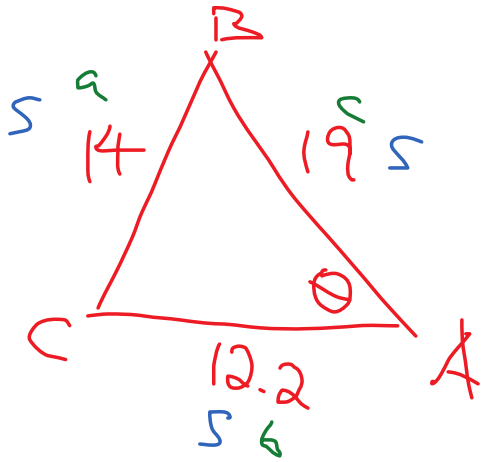
$$= \cos^{-1}(0.6174)$$

$$\theta = \cos^{-1}(0.6174)$$

$$\theta = 52^\circ$$

Example 7

The Lions' Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has lengths 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite the 14 m side, to the nearest degree.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{(12.2)^2 + (19)^2 - (14)^2}{2(12.2)(19)}$$

$$\cos\theta = 0.6770$$

$$\theta = \cos^{-1}(0.6770)$$

$$\theta = 47^\circ$$

The angle opposite the 14m side is 47°.

Textbook Questions: page: 119 - 124; # 1, 2, 4, 6, 8, 10, 12, 19, 23, 30