Math 2200 **Date:**____________________

3.1 Investigating Quadratic Functions in Vertex Form

Degree of a Function - refers to the highest exponent on the variable in an expression or equation.

In Math 1201, you learned about linear functions. These have a degree of 1. When the exponent on the variable is 1, we don't include it in the equation.

Examples of Linear functions:

$y = 2x^{1} + 5$ $\qquad$ $y = -7x + 2$ $\qquad$ $10x - 3y = 12$

Notice that in each of these equations, the exponent on the variable x is 1.

Quadratic Functions
These have a degree of 2. That is, highest power of any x is 2.

Examples of Quadratic functions:

$y = 2x^2 + 5x - 4$ $\qquad$ $y = -3x^2 - 7x + 1$ $\qquad$ $y = (x - 3)(x + 4)$

$y = x(x + 4)$ $\qquad$ $y = -3(x + 4)^2 + 2$ $\qquad$ $y = (3x - 2)^2$

Example 1:
Describe the reasoning used to decide whether each statement is true or false.

Polynomial Function	Classification	True or False	Explain/Justify
$y = 5(x + 3)$	Linear	T	highest power 1
$y = 5(x^2 + 3)$	Quadratic	T	" " 2
$y = 5^2(x + 3)$	Quadratic	F	" " 1
$y = 5x(x + 3)$	Linear	F	" " 2
$y = (5x + 1)(x + 3)$	Quadratic	T	" " 2
$y = 5(x + 3)^2 + 2$	Quadratic	T	" " 2

The Parabola

Parabola: the shape of the graph of any quadratic relation. It is a u-shaped graph, that can open upward or downward.

Watch “Projectile Motion – Science of NFL Football”

https://www.youtube.com/watch?v=HB4ws7RoA3M

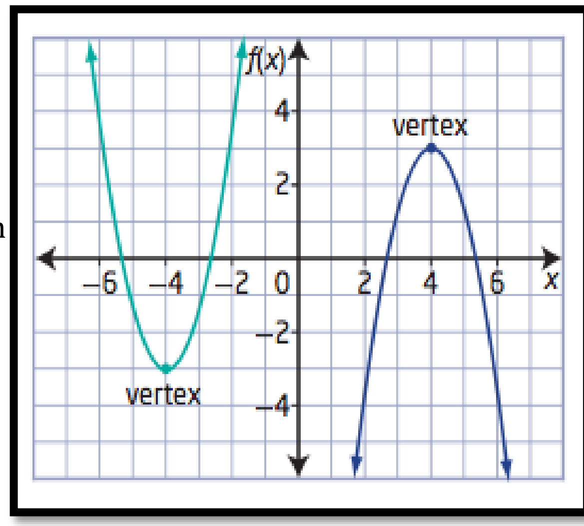
What are some other natural events that involve parabolas?

Bullet fired, archway, etc.

A **quadratic function** is a function, f, whose value $f(x)$at x is given by a polynomial of degree two. $f(x) = x^2$, the simplest form of a quadratic function

A **parabola** is the symmetrical curve of the graph of a quadratic function

The **vertex** of a parabola is the lowest point of the graph if the graph opens upward, or the highest point of the graph if the graph opens downward.

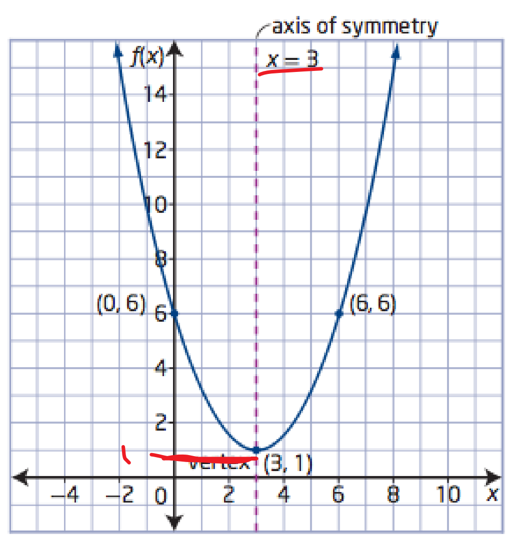


Y

The **minimum value** of a function is the least value in the range of a function.

The **maximum value** of a function is the greatest value in the range of a function.

The **axis of symmetry** is a line through the vertex that divides the graph of a quadratic function into two congruent halves. It is defined by the x-coordinate of the vertex.



Vertex Form

Quadratics written in vertex form:

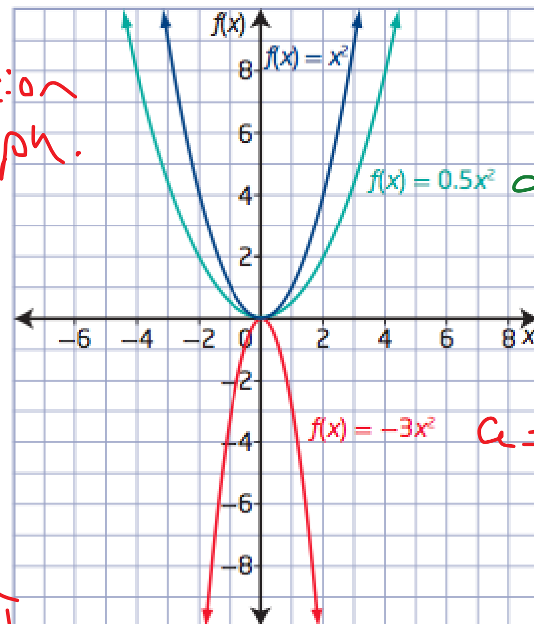
$$f(x) = a(x - p)^2 + q$$

are useful when graphing. The vertex form tells you the location of the vertex, (p, q), the shape of the parobala and the direction of opening.

$f(x) = a \cdot x^2$, $a = 1$

The Effect of Parameter *a*

- determines the orientation and shape of the graph.
- $a > 0$ opens upwards
- $a < 0$ opens downwards
- ignoring the negative
  - the larger a is, the more narrow the graph
  - the smaller a is, the more wide the graph



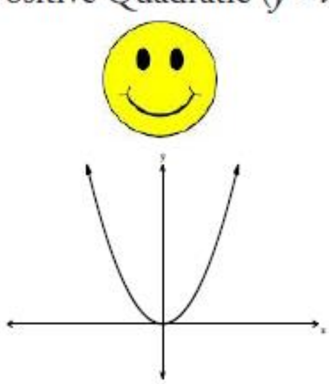
$a = 0.5$

$a = -3$

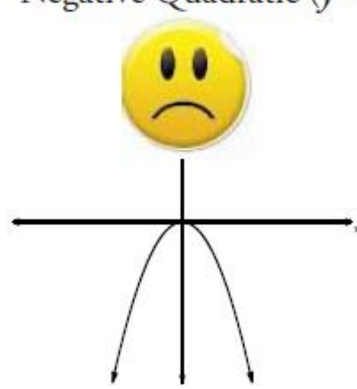
Summary:

One way to remember how to determine direction of opening is shown here.

Positive Quadratic ($y = x^2$)



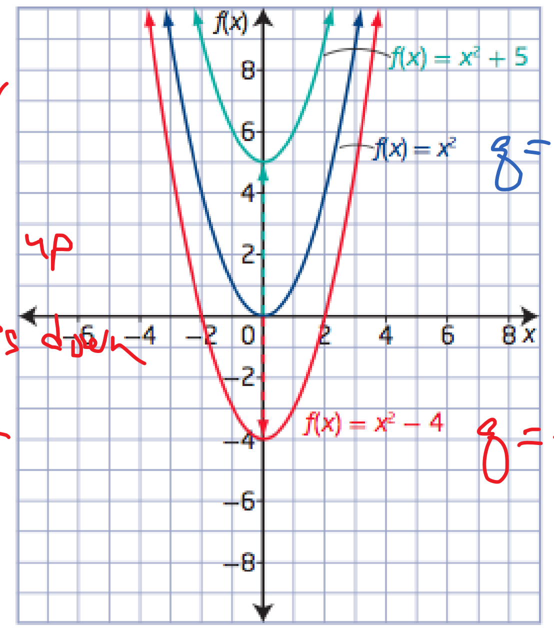
Negative Quadratic ($y = -x^2$)



$y = a(x-p)^2 + q$

The Effect of Parameter *q*

- translates (shifts) the parabola vertically q units.
- $q > 0$, parabola translates up
- $q < 0$, parabola translates down
- q is the y-coordinate of the vertex.



$q = 5$

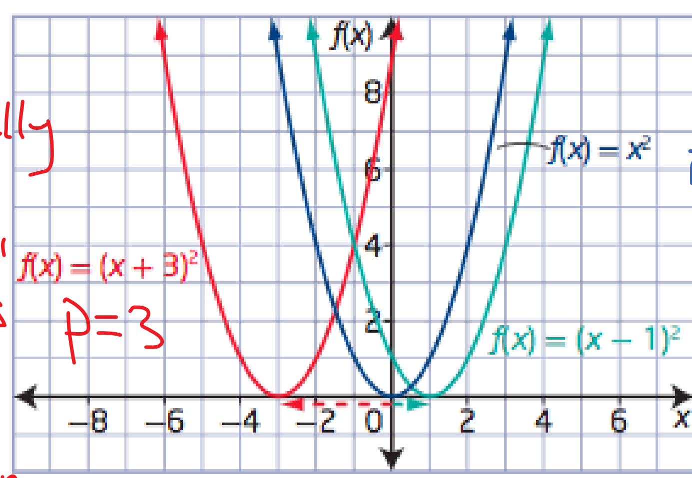
$q = 0$

$q = -4$

$y = a(x-\underline{p})^2 + q$

The Effect of Parameter *p*

- translates the parabola horizontally
- $p > 0$ in the equation, the parabola translates left or negative
- $p < 0$ in the equation, the parabola translates right or positive
- p is the x-coordinate of the vertex.



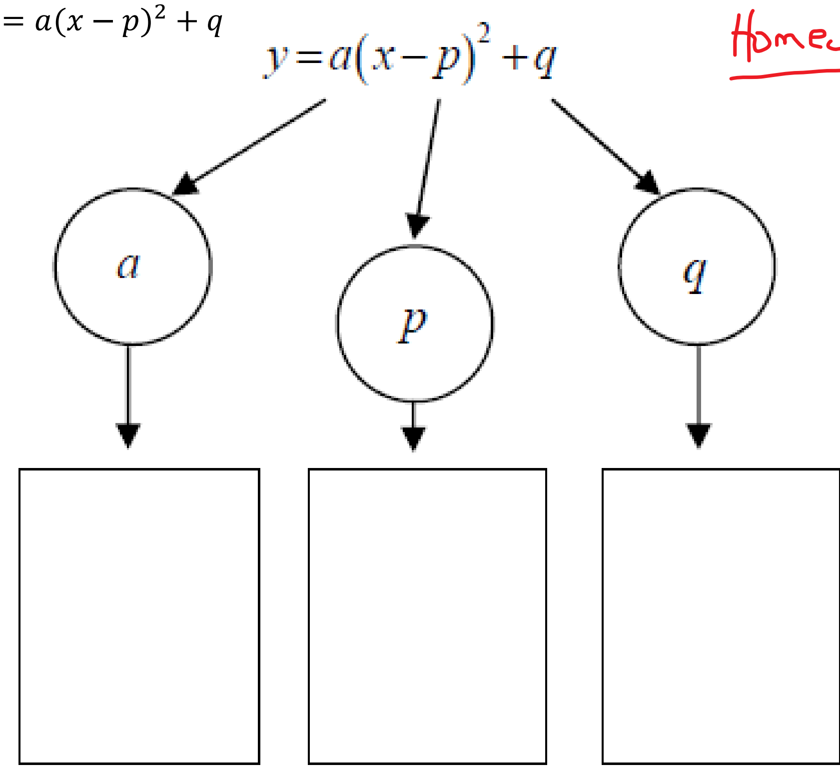
$p = 0$

$p = 3$

$p = -1$

Example 2:
Complete the webbing to describe the effects of the parameters a, p and q on the quadratic function $f(x) = a(x - p)^2 + q$

Homework



Example 3:
Determine the following characteristics for $y = \underset{a}{2}(x + \underset{p}{1})^2 - \underset{q}{3}$.

- the vertex
- the domain and range
- the direction of the opening
- the equation of the axis of symmetry

Then, sketch each graph.

vertex: $(-1, -3)$

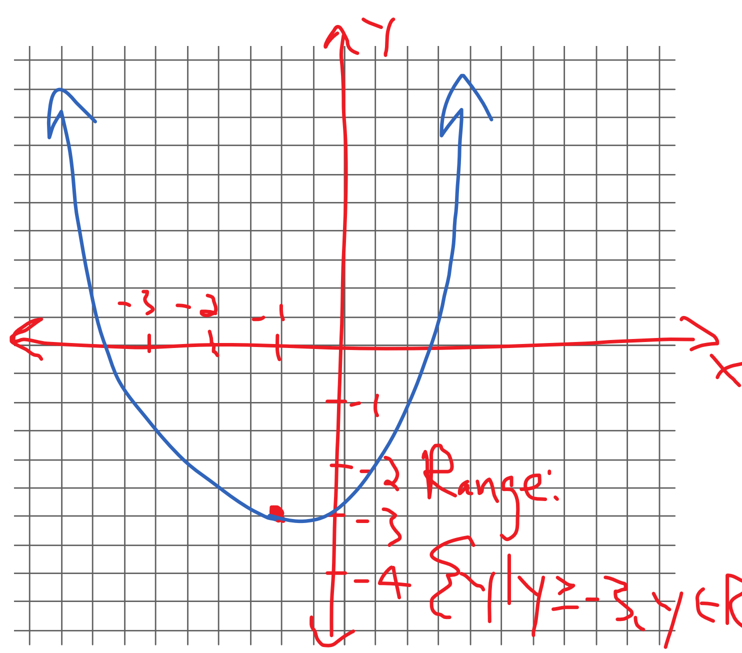
a: $2 > 0$ opens up

aos: $x = -1$

Domain: $\{x \mid x \in R\}$

$\mid$: such that

$\in$: belongs to



Range: $\{y \mid y \geq -3, y \in R\}$

Example 4: Very Important Question!!

Determine a quadratic function in vertex form for the graph.

$p \quad q$

$(0, 3)$

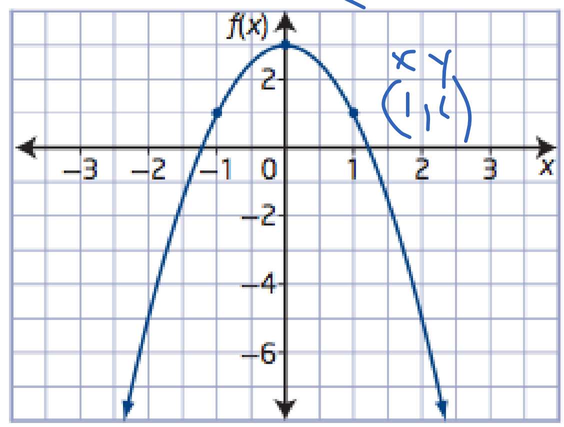
$y = \underline{a}(x - \underline{p})^2 + \underline{q}$

$1 = a(1 - 0)^2 + 3$

$1 = a + 3$

$1 - 3 = a$

$a = -2$



$y = -2(x - 0)^2 + 3 \longrightarrow y = -2x^2 + 3$

Example 5:

Try this on your own. Determine a quadratic function in vertex form for the graph.

$y = a(x - p)^2 + q$

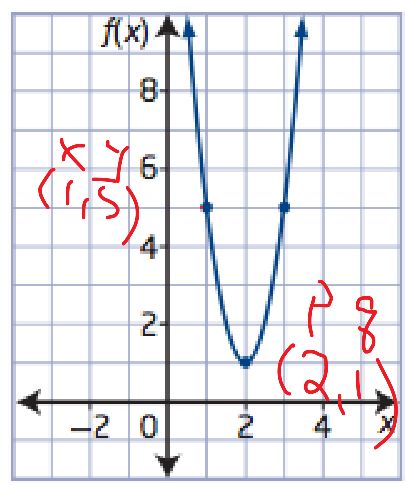
$5 = a(1 - 2)^2 + 1$

$5 = a(1) + 1$

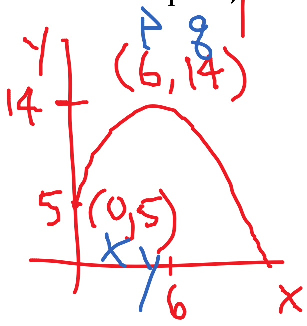
$5 - 1 = a$

$a = 4$

$y = 4(x - 2)^2 + 1$



Example 6:
A stream of water from a fountain forms a parabolic shape. Given the spout on the fountain is 5 cm high and the maximum height reached by the water is 14 cm at a distance of 6 cm from the spout, what is the height of the water when it is 8 cm from the spout?



BEDMAS

$y = a(x-p)^2 + q$

$5 = a(0-6)^2 + 14$

$5 - 14 = 36a$

$-9 = 36a$

$\frac{-9}{36} = \frac{36a}{36}$

$a = -\frac{1}{4}$

$y = -\frac{1}{4}(x-6)^2 + 14$

$x = 8$

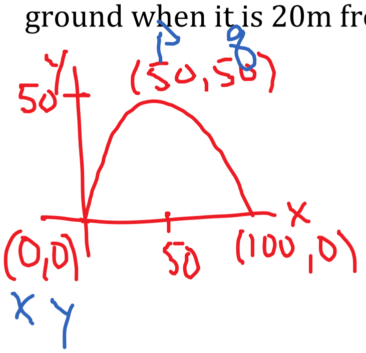
$y = -\frac{1}{4}(8-6)^2 + 14$

$y = -\frac{1}{4}(4) + 14$

$y = -1 + 14$

$y = 13$ cm high

Example 7:
A cannonball, fired from ground level, reaches a maximum height of 50m when it is fired at a ground-level target which it hits 100m away. How high is the cannonball above the ground when it is 20m from the target?



$y = a(x-p)^2 + q$

$0 = a(0-50)^2 + 50$

$-50 = 2500a$

$\frac{-50}{2500} = a$

$a = -\frac{1}{50}$

$y = -\frac{1}{50}(x-50)^2 + 50$

$100 - 20 = 80$

$x = 80$

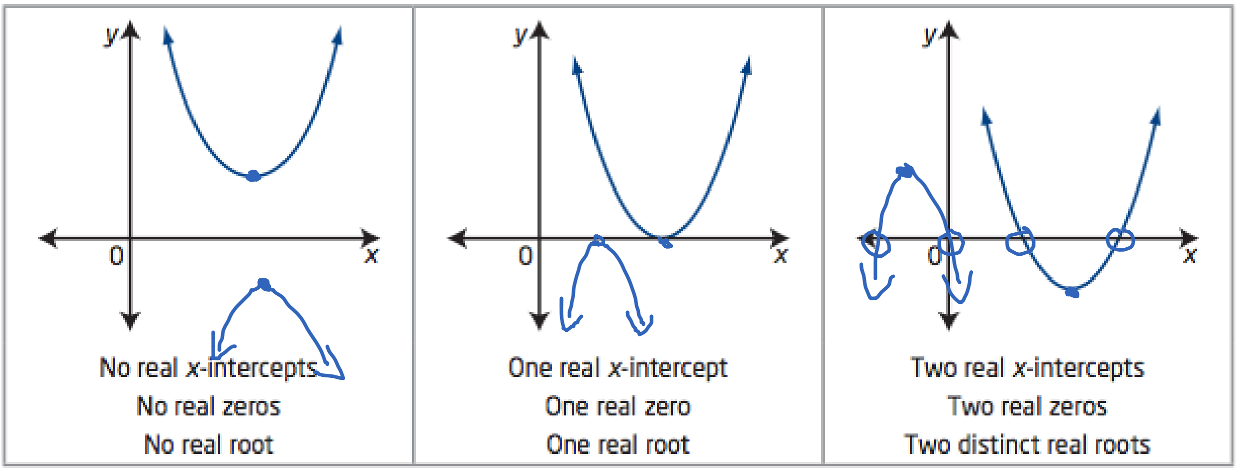
$y = -\frac{1}{50}(80-50)^2 + 50$

$y = 32$ m high

The Number of Zeros

What will affect the number of x-intercepts?

- The direction of the opening of the parabola
- The location of the vertex



To determine how many zeros a quadratic function has, make a rough sketch of the graph using the vertex and direction of opening.

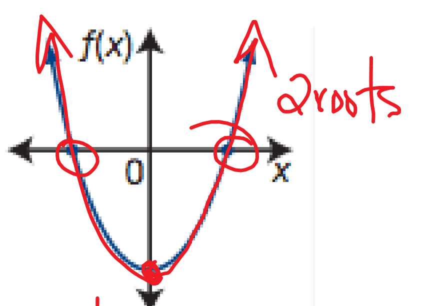
Example 8:

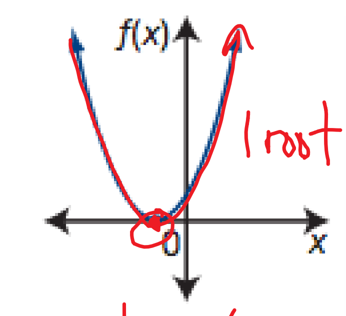
Determine the number of x-intercepts for each quadratic function.

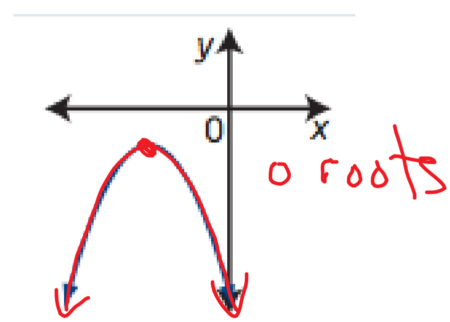
What will affect the number of x-intercepts?

- The direction of the opening of the parabola
- The location of the vertex

(A) $f(x) = 0.8x^2 - 3$ (B) $f(x) = 2(x + 1)^2$ (C) $f(x) = -3(x + 2)^2 - 1$







Vertex: $(0, -3)$ Vertex: $(-1, 0)$ Vertex: $(-2, -1)$

$a > 0$ opens up $a > 0$ opens up $a < 0$ opens down

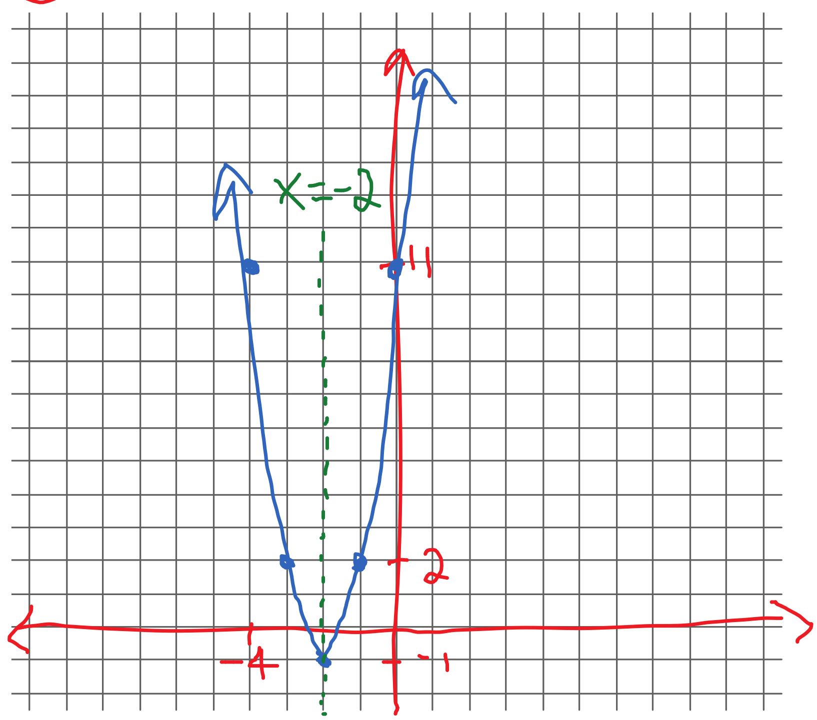
Table of Values and Vertex Form

If given a parabola in the form of a table of values, there are observations that can be made that can help determine the equation of the parabola in vertex form.

Example 9:

Graph the table of values below and answer the following questions:

x	-4	-3	-2	-1	0
y	11	2	-1	2	11



(A) What is the connection between the axis of symmetry of the graph and the vertex?

the p coordinate is the aos.

(B) What is the equation of the axis of symmetry?

$x=-2$

(C) What is the connection between the maximum or minimum point on the graph and the vertex?

The y coordinate is the minimum.

(D) How can the vertex be obtained directly from the vertex form of the quadratic function?

It's the maximum or minimum point.

Example 10:
Determine the vertex form of the quadratic function from the given table of values.

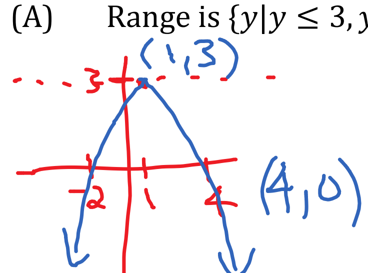
x	-6	-5	-4	-3	-2	-1	0
y	1	3	1	-5	-15	-29	-47

Vertex: $(-5,3)$
(x,y): $(-4,1)$
$y=a(x-p)^2+q$

$1=a[-4-(-5)]^2+3$
$1=a(1)^2+3$
$1-3=a$
$a=-2$
$y=-2(x+5)^2+3$

Example 11:
Determine the vertex form of the quadratic function given the following information.

(A) Range is $\{y|y \leq 3, y \in R\}$ and the x-intercepts are -2 and 4.



$\frac{-2+4}{2}=\frac{2}{2}=1$

$y=a(x-p)^2+q$
$0=a(4-1)^2+3$
$-3=9a$
$\frac{-3}{9}=\frac{9a}{9}$
$a=-\frac{1}{3}$
$y=-\frac{1}{3}(x-1)^2+3$

(B) Equation of the axis of symmetry is $x = 2$, the minimum value of y is -5, and the y-intercept is 3.

$(2,-5)$ p q
$(0,3)$ x y
$y=a(x-p)^2+q$
$3=a(0-2)^2-5$

$3+5=4a$
$8=4a$
$\frac{8}{4}=\frac{4a}{4}$
$a=2$
$y=2(x-2)^2-5$

Textbook Questions: page: 157-162; # 3, 7, 8, 10, 12, 15, 17, 20, 21, 22