Math 2200 **Date:**____________________

3.2 Investigating Quadratic Functions in Standard Form

The **standard form** of a quadratic is:

$$f(x) = ax^2 + bx + c$$

$y = mx + b$

or

$$y = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

The Effect of Parameter *c*

$f(x) = -x^2 + 4x + 10$

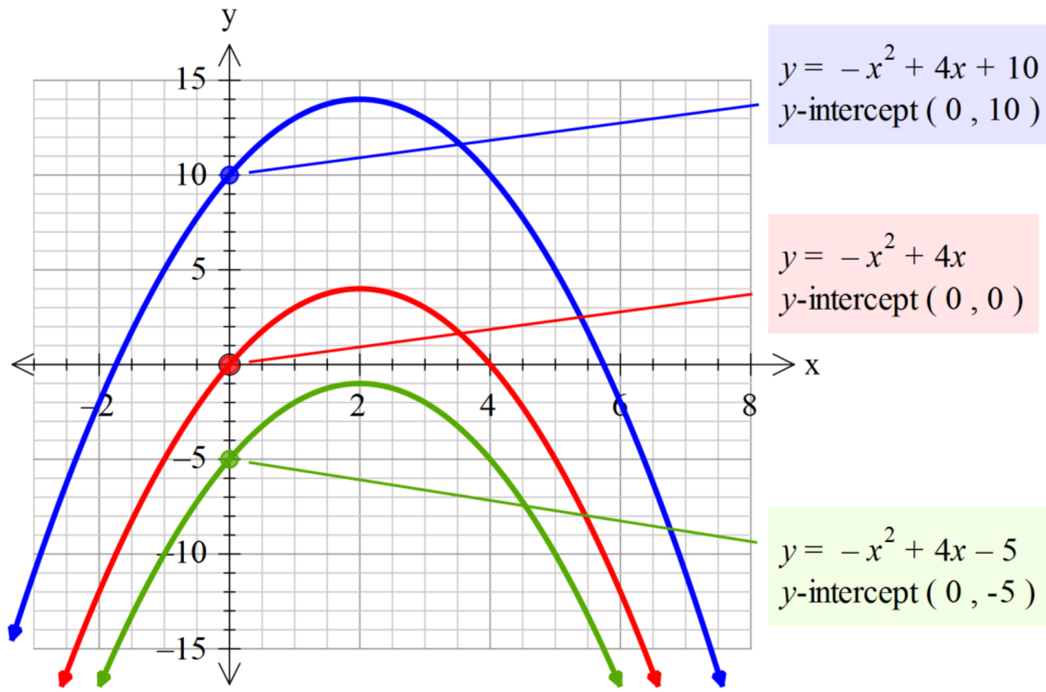
$c = 10$

$f(x) = -x^2 + 4x$

$c = 0$

$f(x) = -x^2 + 4x - 5$

$c = -5$



What do you think the parameter *c* affects?

c is the y-intercept.

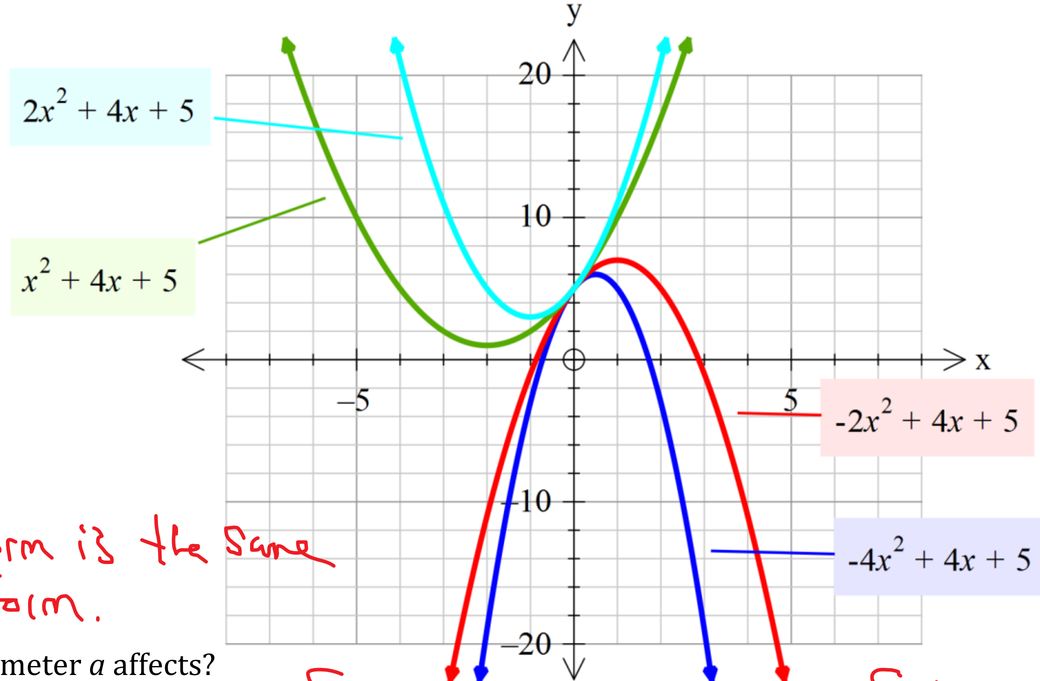
The Effect of Parameter *a*

$f(x) = -4x^2 + 4x + 5$

$f(x) = -2x^2 + 4x + 5$

$f(x) = x^2 + 4x + 5$

$f(x) = 2x^2 + 4x + 5$



The a in vertex form is the same a in standard form.

What do you think parameter *a* affects?

- a affects the direction of opening and the shape of the graph.
- a > 0, opens up
- a < 0, opens down
- ignoring the negative, the larger a is, the more narrow the graph.

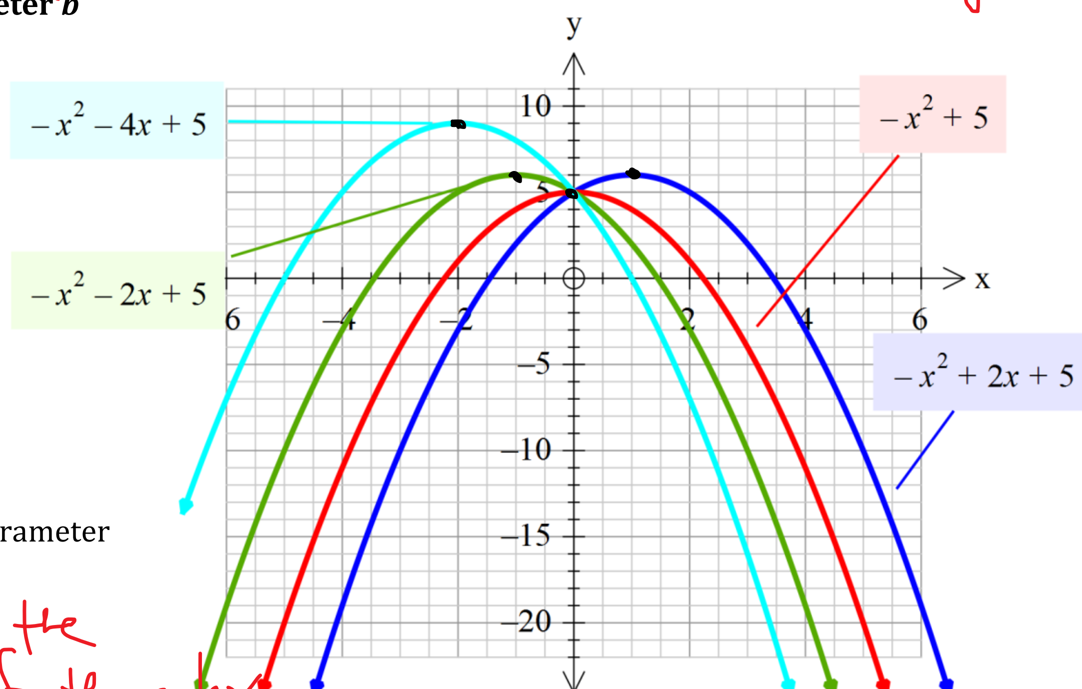
The Effect of Parameter *b*

$f(x) = -x^2 + 2x + 5$

$f(x) = -x^2 + 5$

$f(x) = -x^2 - 2x + 5$

$f(x) = -x^2 - 4x + 5$



What do you think parameter *b* affects?

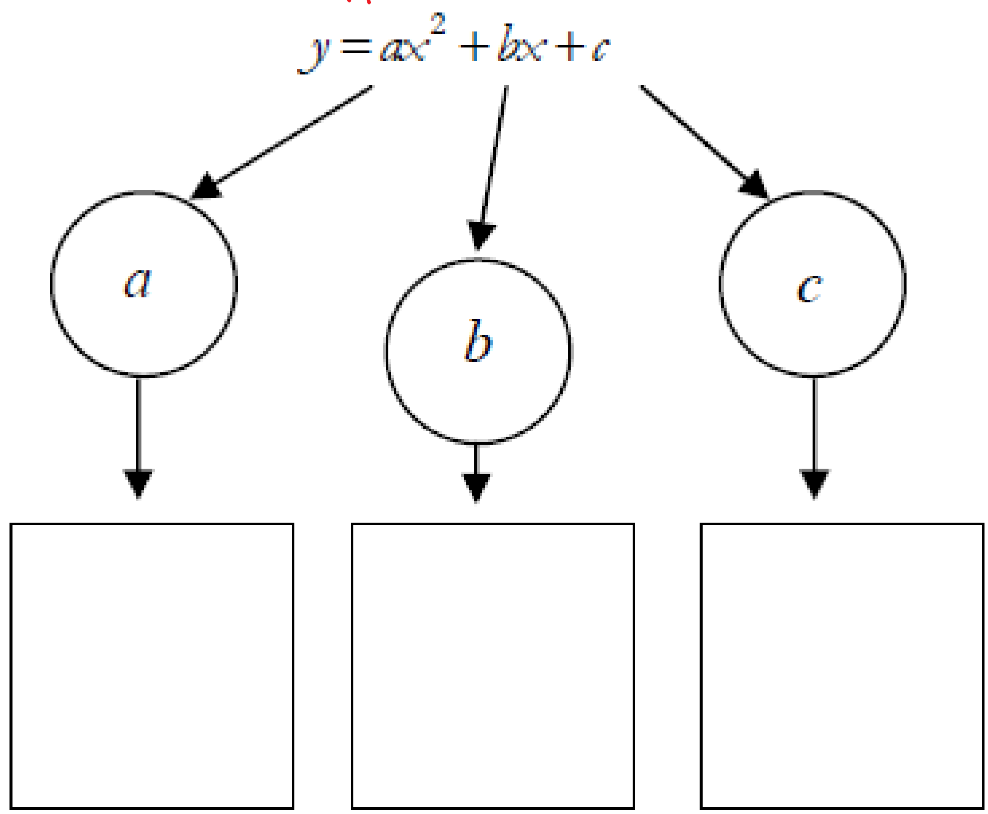
- b affects the position of the vertex.

For the standard form, $f(x) = ax^2 + bx + c$:

- a determines the shape and whether the graph opens upward (positive) or downward (negative)
- b influences the position of the graph (vertex)
- c determines the y-intercept of the graph

Example 1:
Complete the following webbing to describe the effects of the parameters a, b and c on the quadratic function $y = ax^2 + bx + c$.

Homework:



Example 2:
Explain how you could determine whether a quadratic function has either a maximum or minimum value without graphing.

- $a > 0$, opens up and has a minimum value of q.
- $a < 0$, opens down and has a maximum value of q.

Relationship Between Vertex form and Standard Form

By expanding $f(x) = a(x-p)^2 + q$, and comparing the resulting coefficients with the standard for we can see the relationship between the two.

$$y = a(x-p)^2 + q$$
$$y = a(x-p)(x-p) + q$$
$$y = a(x^2 - px - px + p^2) + q$$
$$y = a(x^2 - 2px + p^2) + q$$
$$y = ax^2 - 2apx + ap^2 + q$$
$$y = ax^2 + bx + c$$

Some equations that relate a, b, and c to p and q:

$$b = -2ap \quad \text{or} \quad \boxed{p = -\frac{b}{2a}}$$ Most important (Not provided)

$$c = ap^2 + q \quad \text{or} \quad q = c - ap^2$$

The most useful of these is $p = -\frac{b}{2a}$, since p is the axis of symmetry or the x-coordinate of the vertex. We can then substitute this value for x in the equation in standard form to find the y-coordinate of the vertex which is q.

Example 2:
Complete the following table:

Function	Vertex	Equation of Axis of Symmetry	a	b	c	$-\frac{b}{2a}$
$y = x^2 - 4x + 7$	(2, 3)	$x=2$	1	-4	7	$\frac{-(-4)}{2(1)}=2$
$y = -2x^2 - 16x - 34$	(-4, -2)	$x=-4$	-2	-16	-34	$\frac{-(-16)}{2(-2)}=-4$
$y = 3x^2 - 6x + 10$	(1, 7)	$x=1$	3	-6	10	$\frac{-(-6)}{2(3)}=1$

Example 3:

(A) What is $y = -2(x-3)^2 + 5$ in standard form?

$y=-2(x-3)(x-3)+5$

$y=-2(x^2-3x-3x+9)+5$

$y=-2(x^2-6x+9)+5$

$y=-2x^2+12x-18+5$

$y=-2x^2+12x-13$

(B) What is $y = 3(x+2)^2 + 4$ in standard form?

$y=3(x+2)(x+2)+4$

$y=3(x^2+2x+2x+4)+4$

$y=3(x^2+4x+4)+4$

$y=3x^2+12x+12+4$

$y=3x^2+12x+16$

Example 4:
What is the vertex form of the quadratic function, $f(x) = -x^2 + 2x + 8$?

$y = a(x-p)^2 + q$

$a: -1$

$p = -\frac{b}{2a} = -\frac{2}{2(-1)} = -\frac{2}{-2} = 1$

$q = c - ap^2 = 8 - (-1)(1)^2 = 9$

or $q = -(1)^2 + 2(1) + 8 = -1 + 2 + 8 = 9$

$y = -(x-1)^2 + 9$

Homework: p175 #6

Example 5:
For each graph of a quadratic function, identify:

- the direction of the opening
- the coordinates of the vertex
- the maximum/minimum value
- the equation of the axis of symmetry
- the number of x-intercepts and the y-intercept
- the domain and range

(A) $f(x) = x^2 - 2x$

$a = 1 > 0$, opens up

$p = -\frac{b}{2a} = \frac{-(-2)}{2(1)} = 1$

$q = (1)^2 - 2(1) = 1 - 2 = -1$

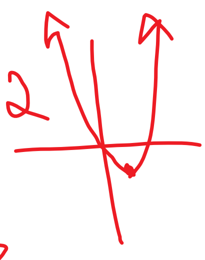
Vertex: $(1, -1)$

minimum: -1

aos: $x = 1$

x-intercepts: 2

y-intercept: 0



D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq -1, y \in \mathbb{R}\}$

(B) $f(x) = -2x^2 - 12x + 25$

$a = -2 < 0$ ∴ opens down

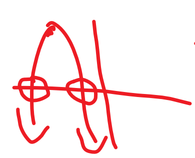
$p = -\frac{b}{2a} = \frac{-(-12)}{2(-2)} = \frac{12}{-4} = -3$

$q = -2(-3)^2 - 12(-3) + 25$
$= -18 + 36 + 25$
$= 18 + 25$
$= 43$

Vertex: $(-3, 43)$

maximum: 43

aos: $x = -3$



x-ints: 2

y-int: 25

D: $\{x \mid x \in \mathbb{R}\}$, R: $\{y \mid y \leq 43, y \in \mathbb{R}\}$

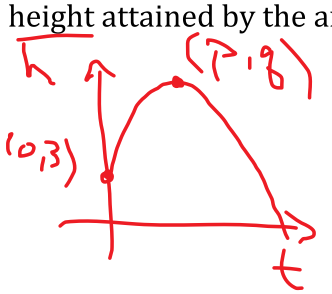
Maximization/Minimization (Optimization) Problems

There are two main types of "Max/Min" problems.

1. The equation is given. You have to find the maximum or minimum value of the function, depending on if the parabola opens upwards or downwards. This of course is the vertex, (p, q).

2. You have to make the equation yourself using information given and then use that equation to find the maximum or minimum value. Typically you are asked to find the maximum area of some square or rectangle such as a garden or floor space of a house.

Example 6:
An arrow is fired from a bow and its height, h, in metres above the ground, t seconds after being fired, is given by $h(t) = -5t^2 + 40t + 3$. Algebraically determine the maximum height attained by the arrow and the time taken to reach this height.



$$p = \frac{-b}{2a} = -\frac{40}{2(-5)} = -\frac{40}{-10} = 4\ s$$

$$q = -5(4)^2 + 40(4) + 3$$
$$= -80 + 160 + 3$$
$$= 83\ m$$

or $q = c - ap^2$

$q = 3 - (-5)^2(4)$

$q = 83$

Max height of 83m @ 4s.

Example 7:

A rancher has 100 m of fencing available to build a rectangular corral. (pen)

(A) Write a quadratic function in standard form to represent the area of the corral.

Step 1: Draw diagram

Step 2: Perimeter equation

$2l + 2w = 100$

Step 3: Area Equation

$A = l \cdot w$

Step 4: Solve perimeter equation for one variable.

$2w = -2l + 100$

$\frac{2w}{2} = \frac{-2l + 100}{2} \rightarrow w = -l + 50$

Step 5: Sub perimeter equation into area equation.

$A = l \cdot w$

$A = l(-l + 50)$

$A = -l^2 + 50l$

Think: $y = -x^2 + 50x$

(B) What are the coordinates of the vertex? What does the vertex represent in this situation?

$p = \frac{-b}{2a} = \frac{-50}{2(-1)} = \frac{-50}{-2} = 25\text{m}$ $\quad q = -(25)^2 + 50(25) = 625\text{m}^2$

Vertex $(25, 625)$

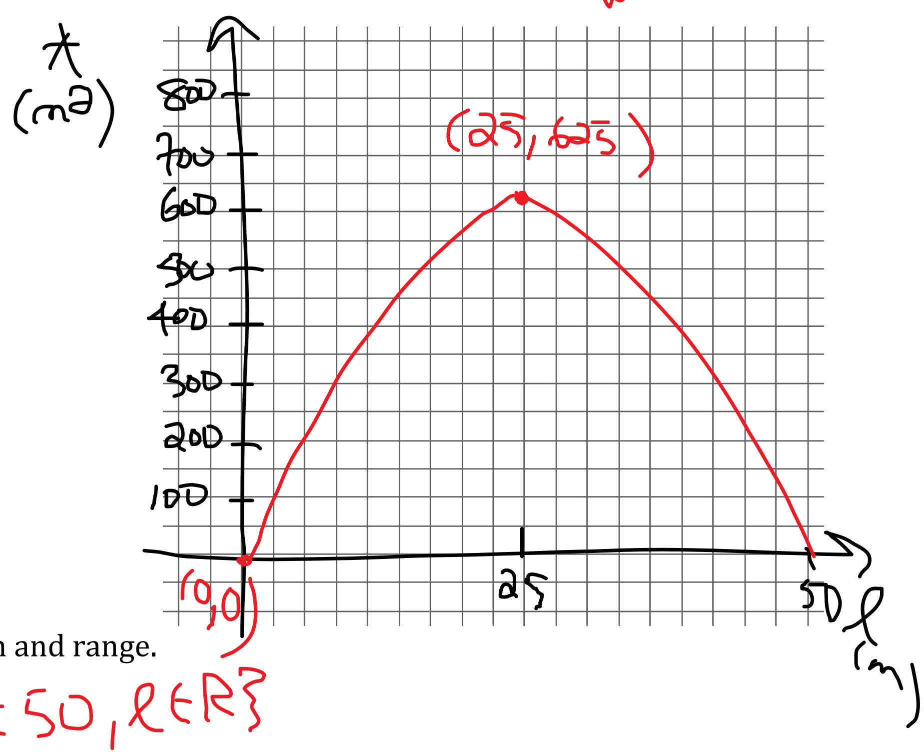
$l = 25\text{m}$

$w = -(25) + 50$

$w = 25\text{m}$

(C) Sketch the graph for the function you determined in part (A).

$l = 0, w = 50$

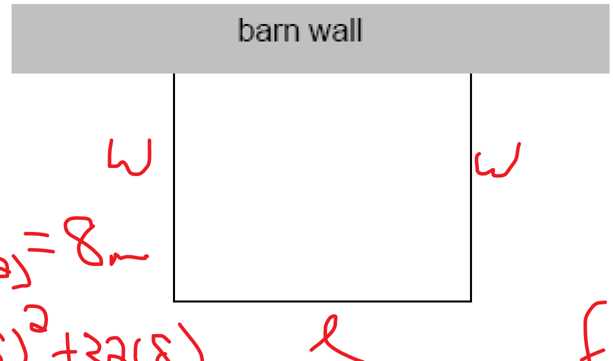


(D) Determine the domain and range.

D: $\{l \mid 0 \le l \le 50, l \in R\}$

R: $\{A \mid 0 \le A \le 625, A \in R\}$

Example 8:
A farmer is constructing a pig pen and is using his barn wall as one side of the pen. If he has 32 m of fencing and wants to use it all, write the quadratic function that models the area of the pig pen, and use it to determine the maximum area of the pen.



① $l + 2w = 32$

② $A = l \cdot w$

Solve ① for l:

$l = -2w + 32$

Sub ① into ②

$A = (-2w + 32) \cdot w$

$A = -2w^2 + 32w$

$p = \frac{-b}{2a}$

$p = \frac{-32}{2(-2)} = 8\text{m}$

$q = -2(8)^2 + 32(8)$

$= 128\ \text{m}^2$

$w = 8\text{m}$

$l = -2(8) + 32 = 16\text{m}$

The max area of the pen is 128m^2.

Example 9:
A student makes and sells necklaces at the beach during the summer months. The material for each necklace costs her $6.00 and she has been selling about 20 per day at $10.00 each. She has been wondering whether or not to raise the price, so she takes a survey and finds that for every dollar increase she would lose two sales a day. What price should she set for the necklaces to maximize profit?

Step 1: Current Equation.

R = (Price)(# sold)

$R = (4)(20)$

Step 2: Set variable for price increments

Let n be number of $1 increments.

Step 3: Adjust the equation.

$R = (4 + n)(20 - 2n)$

$R = 80 - 8n + 20n - 2n^2$

$R = -2n^2 + 12n + 80$

$p = \frac{-b}{2a} = \frac{-12}{2(-2)} = 3$

$q = -2(3)^2 + 12(3) + 80$

$= 98$

Max profit of $98 when necklaces are $13.

Textbook Questions: Page 174 - 179, # 6, 8, 9, 11, 13, 15, 18, 21-24